



A variant of united multi-operator evolutionary algorithms with application to livestock feed ration optimization

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ABSTRACT

The United Multi-Operator Evolutionary Algorithms (UMOEAs) demonstrate high performance in evolutionary computation due to their combination of various operators for Differential Evolution (DE) variants and local search mechanisms, and have evolved into three versions over the last decade. In this work, a novel variant of the UMOEAs is proposed, named UMOEAs-IV. UMOEAs-IV employs a novel calculation method for the scaling factor F , which generates dynamic values to adaptively control the step size of mutation operators, a mutation strategy with complementary operators that better balance exploration and exploitation, an Estimation-of-Distribution Algorithm (EDA) to learn the probabilistic distribution of promising individuals, and a stagnation strategy to help individuals escape local optima. UMOEAs-IV is compared with 15 recently proposed DE-based algorithms and tested on the IEEE Congress on Evolutionary Computation 2017 (CEC2017) benchmark functions, showing superior performance over all of them. It is also applied to livestock feed ration optimization for beef and dairy cattle, where it shows top performance in the experimental results. The source code of UMOEAs-IV is provided at <https://github.com/microhard1999/CODES>.

1. Introduction

Since the 1990s, various metaheuristic algorithms have been proposed, with Differential Evolution (DE) [1] and Particle Swarm Optimization (PSO) [2] being two of the most representative. In the metaheuristics community, both single-objective [3–5] and multi-objective problems [6,7] are active topics of research. Each year, many DE- and PSO-based variants are introduced, contributing significant innovations to the metaheuristic community. In recent IEEE Congress on Evolutionary Computation (CEC) competitions, DE-based variants have consistently ranked among the top metaheuristics [8–11]. These DE-based variants demonstrate outstanding performance on both low- and high-dimensional single-objective problems compared to other metaheuristic algorithms. Recent surveys highlight the continued proliferation of DE variants that address parameter control, operator ensembles, scalability to high dimensions, multimodality, and automated algorithm design, all aimed at enhancing robustness across diverse benchmark suites and real-world problems [12]. One major research strand focuses on parameter adaptation and control, as fixed parameter settings (e.g., mutation factor, crossover rate, and population size) remain a significant limitation, particularly in dynamic or complex landscapes. A taxonomy of adaptation mechanisms, covering control-parameter adaptation, mutation-strategy adaptation, dynamic population sizing,

search-space adaptation, and composite adaptation frameworks, was proposed by [13]. A second major trend is the rise of ensemble and multi-operator frameworks. Rather than relying on a single mutation or crossover scheme, modern DE variants combine multiple candidate operators, often with adaptive selection or weighting, and sometimes integrate restart mechanisms or local search hybrids [14–17]. A third key direction concerns scalability and multimodality. As problem dimensionality and landscape complexity increase, DE must address issues of diversity loss and premature stagnation. Recent comparative studies examine how modern DE variants perform across varying dimensions and landscape types, noting that although progress has been made, challenges persist, especially in high-dimensional problems [18,19]. Finally, the field is witnessing growing interest in learning-based selection and automation of DE strategies. For instance, dynamic algorithm selection using deep reinforcement learning has been applied to DE, enabling an agent to observe landscape or algorithmic features and switch among candidate operators during runtime. Such approaches point to future DE frameworks that are less manually tuned and more responsive to problem feedback [20–22].

The United Multi-Operator Evolutionary Algorithms (UMOEAs) framework is DE-based and incorporates local search mechanisms in

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the later stages of evolution. The first and second generations of the UMOEAs were proposed by Elsayed et al. [14,15], and were later upgraded to a third generation by Hong et al. [16]. The UMOEAs framework integrates various evolutionary operators, such as DE variants, genetic algorithm operators, and local search mechanisms, into a single framework. This allows the algorithm to leverage the strengths of each operator, improving both exploration (to search new regions) and exploitation (to refine solutions in promising regions). Through self-adaptive multi-operator strategies, UMOEAs can adjust their search behavior dynamically when facing a wide range of problem landscapes without requiring extensive manual tuning. By combining global search with local search mechanisms, UMOEAs helps prevent premature convergence to local optima while ensuring that promising regions are thoroughly explored. The modular multi-operator framework also enables UMOEAs to adapt to various types of problems, making them versatile and effective in solving complex or high-dimensional optimization problems. By employing multiple operators and hybrid strategies, the algorithm maintains a diverse population while converging efficiently toward global optima, thereby achieving high-quality, well-distributed solutions for single-objective optimization [14–16]. Since UMOEAs are designed in a modular way, new operators or strategies can be integrated easily, allowing researchers to extend the framework to create more powerful variants without redesigning the entire system.

In UMOEAs-III [16], two mutation operators are employed, demonstrating complementary characteristics and an effective balance between exploration and exploitation. However, this strategy can still be improved to enable DE to adjust its search patterns more accurately. Moreover, in UMOEAs-III, the scaling factor F shows fluctuating amplitude and may even increase in the later stages of the evolutionary process for certain problems, which causes unnecessary exploration to some extent, since individuals are assumed to be close to the global optimum; hence, a smaller mutation step size is expected. According to our observation, Estimation of Distribution Algorithm (EDA) [23–25] generally performs well in low-dimensional problems because the probabilistic models can accurately capture dependencies between variables. Consequently, convergence is often faster, as the number of parameters in the model is manageable. In high-dimensional problems, however, EDA can struggle because the complexity of the probabilistic model grows rapidly. Sampling quality may degrade, and more samples may be required to reliably estimate distributions. To address this, simplified models can be used in high-dimensional EDA to reduce complexity, although this may result in the loss of some dependency information. In the current UMOEAs framework, the stagnation strategy is missing; thus, when individuals are trapped in local optima, there is a lack of effective means to escape. Based on the above observations and insights, we were motivated to develop a new generation of UMOEAs that builds on the strengths of existing mechanisms across various problems while overcoming their drawbacks. The proposed UMOEAs-IV has the following highlights:

- A novel calculation of the scaling factor F , which considers the Failure Rate (FR), is proposed; this F can better adapt to the landscape of the evolution.
- A mutation strategy that considers the historical promotion rate of each mutation operator and combines two mutation operators is proposed. Learning from elite individuals is also incorporated.
- A probability calculation for EDA is proposed, which takes the problem scale into account. When solving lower-dimensional problems, EDA has a high probability of being applied, and vice versa.
- A stagnation strategy that employs Gaussian learning and individual learning is integrated into the UMOEAs framework to help individuals escape local optima when they become trapped.
- The proposed UMOEAs-IV is compared with 15 recently proposed DE-based variants and is also applied to livestock feed ration optimization to demonstrate its advantages.

The structure of the remaining sections of this paper is as follows. Section 2 introduces related work, including UMOEAs-III, local search mechanisms, EDA, and the stagnation strategy. Section 3 presents the details of UMOEAs-IV, including the novel calculation method of the scaling factor F , the proposed mutation strategy, the probability calculation for the application of EDA, and the stagnation strategy. Section 4 outlines the parameter settings used for all DE-based variants examined in this study, along with the corresponding experimental results. This section also includes comparisons, analyses, ablation tests, search pattern analyses, and discussions. Section 5 describes the real-world application of the proposed UMOEAs-IV: livestock feed ration optimization. Section 6 analyzes the computational complexity of all compared algorithms. Finally, Section 7 summarizes the study and discusses directions for future research.

2. Related work

2.1. LSHADE-cnEpSin and UMOEAs-III

In the UMOEAs-III framework, an improved LSHADE-cnEpSin [26] is employed, where the original scaling factor $F_{i,g}$ is calculated using Eqs. (1) and (2). Since the Current Function Evaluation (cfe) serves as the stopping criterion and Nonlinear Population Size Reduction (NPSR) [27] is applied in UMOEAs-III, the population size becomes dynamic at each generation, which can cause the current generation number g to exceed the maximum generation number G_{max} in later stages of the evolutionary process. Consequently, both equations are replaced by Eqs. (3) and (4). These proposed equations ensure that $\frac{cfe}{FES_{max}}$ remains within the $[0, 1]$ range, where FES_{max} denotes the maximum number of function evaluations.

$$F_{i,g} = \frac{1}{2} \cdot (\sin(2 \cdot \pi \cdot freq_i \cdot g + \pi) \cdot (1 - \frac{g}{G_{max}}) + 1) \quad (1)$$

$$F_{i,g} = \frac{1}{2} \cdot (\sin(\pi \cdot g) \cdot \frac{g}{G_{max}} + 1) \quad (2)$$

where $freq_i$ represents the frequency of the sinusoidal function and is randomly selected from the memory history M_{freq} , which stores the successful mean frequencies from previous generations.

$$F_{i,g} = \frac{1}{2} \cdot (\sin(2 \cdot \pi \cdot freq_i \cdot g + \pi) \cdot (1 - \frac{cfe}{FES_{max}}) + 1) \quad (3)$$

$$F_{i,g} = \frac{1}{2} \cdot (\sin(\pi \cdot g) \cdot \frac{cfe}{FES_{max}} + 1) \quad (4)$$

UMOEAs-III employs a mutation strategy that combines two complementary operators, defined in Eqs. (5) and (6), allowing the algorithm to switch between them in different situations [16]. Eqs. (5) and (6) are referred to as ‘current-to-pbest with archive’ and ‘current-to-pbest without archive’, respectively.

$$v_{i,g}^d = x_{i,g}^d + F_{i,g} \cdot (x_{pbest,g}^d - x_{i,g}^d) + F_{i,g} \cdot (x_{r1,g}^d - \tilde{x}_{r2,g}^d) \quad (5)$$

$$v_{i,g}^d = x_{i,g}^d + F_{i,g} \cdot (x_{pbest,g}^d - x_{i,g}^d) + F_{i,g} \cdot (x_{r1,g}^d - x_{r3,g}^d) \quad (6)$$

where $\tilde{x}_{r2,g}$ is chosen from the union of the entire PS_{DE} and the archive, $x_{r1,g}^d$ and $x_{r3,g}^d$ are randomly selected individuals from the population at generation g , with $r1 \neq r2 \neq r3$, and $x_{pbest,g}^d$ is the p -best individual at generation g with $p = 10\%$, where $d = 1, \dots, D$ and D denotes the dimensionality.

2.2. Local search mechanisms

The Covariance Matrix Adaptation Evolution Strategy (CMA-ES) [28] and the Sequential Quadratic Programming (SQP) method [29,30] have been employed in various metaheuristics and have demonstrated excellent exploitation capabilities [4,5,31,32]. To reinforce exploitation capability, UMOEAs-III employs CMA-ES and the SQP method: CMA-ES is applied throughout the evolution process, while the SQP method

is specifically used when the evolution reaches $0.75 \times FES_{max}$, where FES_{max} denotes the maximum number of function evaluations. CMA-ES models the distribution of successful candidate solutions using a covariance matrix, which captures correlations between variables. This enables the algorithm to search along promising directions and adapt to the local landscape. CMA-ES also automatically adjusts the step size based on the success of previous steps, naturally reducing it to fine-tune solutions around local optima. This self-adaptive mechanism makes the algorithm highly efficient for local exploitation. The core principle of the SQP method lies in its ability to transform a nonlinear problem into a quadratic approximation [33,34]. Essentially, the SQP method aims to determine a suitable search direction and formulate a quadratic optimization problem. The integration of CMA-ES and the SQP method greatly enhances the exploitation capability of UMOEAs-III.

2.3. Estimation of distribution algorithm

The EDA is a special class of Evolutionary Algorithms (EAs) that does not rely on traditional crossover or mutation operators. Instead, EDA generates new solutions by sampling from a probability distribution constructed using the dominant individuals from previous generations. This model-based approach has attracted considerable attention, and EDA and its variants have achieved significant success in both combinatorial and continuous optimization problems [35–38].

In [25], an Improved DE hybridized with EDA (IDE-EDA) was proposed. The proposed IDE-EDA employs EDA as a model-based exploitation engine to complement DE's exploration. EDA builds a probabilistic model from selected high-quality solutions, samples offspring from this model to intensify the search in promising regions, and, together with a control parameter, enables the hybrid to balance exploitation and exploration. The EDA can be represented by Eqs. (7) to (11).

$$PS_e = \begin{cases} 0.5 \cdot PS_{DE}, & PS_{DE} \geq 2 \cdot D \\ PS_{DE}, & PS_{DE} < 2 \cdot D \end{cases} \quad (7)$$

$$\mu = \frac{1}{PS_e} \sum_{i=1}^{PS_e} x_i, \quad x_i \in PS_e \quad (8)$$

$$C = \frac{1}{PS_e} \sum_{i=1}^{PS_e} (x_i - \mu)(x_i - \mu)^T, \quad x_i \in PS_e \quad (9)$$

$$x_i = N(\mu, C) \quad (10)$$

Eq. (11) ensures that the newly generated values remain within bounds.

$$x_i^d = x_{min}^d + rand \cdot (x_{max}^d - x_{min}^d), \quad \text{if } x_i^d < x_{min}^d \text{ or } x_i^d > x_{max}^d \quad (11)$$

here, $rand$ follows a uniform distribution in $[0,1]$.

In IDE-EDA, the EDA builds a probabilistic model of the most promising individuals in the current population. By capturing the statistical distribution of high-quality solutions, EDA focuses the search on regions likely to contain better solutions. Once the model is constructed, EDA samples new individuals from the learned distribution, enabling IDE-EDA to generate offspring guided by learned patterns rather than relying solely on random mutations or crossover. While DE is effective at exploring the search space through mutation and crossover, it may converge slowly near local optima. EDA complements DE by providing exploitation, intensifying the search in promising regions and accelerating convergence. In this framework, DE handles diversification (exploration), whereas EDA ensures intensification (exploitation). This synergy allows IDE-EDA to avoid premature convergence while efficiently refining solutions.

2.4. Stagnation strategy

The stagnation strategy is employed by many metaheuristics to avoid being trapped in local optima, and its implementation varies [3, 39–41]. In [17], a stagnation strategy is employed to help individuals escape from local optima. It uses a counter and diversity metrics to record stagnancy. Once stagnation occurs, horizontal and vertical crossovers are applied using Eqs. (12) and (13), respectively.

$$x_{r1,g+1}^d = rand1 \cdot x_{r1,g}^d + rand2 \cdot x_{r2,g}^d + rnds \cdot (x_{r1,g}^d - x_{r2,g}^d) \quad (12)$$

here, $x_{r1,g}^d$ and $x_{r2,g}^d$ are two different individuals randomly selected from the population. $rand1$ ($rand2 = 1 - rand1$) and $rnds$ are random numbers in $[0,1]$ and $[-1,1]$, respectively.

$$x_{ss,g+1}^d = rand \cdot x_{ss,g}^{d1} + rand \cdot x_{ss,g}^{d2} \quad (13)$$

where $x_{ss,g}^{d1}$ and $x_{ss,g}^{d2}$ represent different dimensions of the stagnated individual, with $d1$ and $d2 \in [1, D]$, and $rand$ follows a uniform distribution over $[0, 1]$.

Horizontal and vertical crossovers, inspired by genetic algorithms, implement a straightforward competition mechanism to enhance global search and reduce blind spots [42–44]. Horizontal crossover partitions the solution space into semi-group hypercubes and performs edge searches to strengthen global exploration. Individuals of the same dimension are randomly pre-screened and paired for iterative updates. Vertical crossover, on the other hand, enables crossovers of varying sizes, allowing specific dimensions to escape local optima without impacting others.

3. UMOEAs-IV

In UMOEAs-IV, a novel method for calculating the scaling factor F is proposed, which takes the FR into account. A new mutation strategy is employed, combining two complementary mutation operators. A probability formula for applying EDA is also proposed; this formula considers the problem scale and integrates the dimensionality of the problem. Additionally, a novel stagnation strategy is introduced to prevent being trapped in local optima when stagnation occurs. The pseudocode of UMOEAs-IV is presented in Algorithm 1.

3.1. Scaling factor F

In UMOEAs-IV, a novel method for calculating scaling factor $F_{i,g}$ is proposed, as defined by Eqs. (14) and (15).

$$\mu = 0.35 + 0.3 \left(\frac{2}{1 + e^{-10 \cdot (1-FR)}} - 1 \right) \quad (14)$$

where the FR is calculated as $FR = \frac{NF_{offspring}}{PS_{DE}}$, where $NF_{offspring}$ is the number of offspring with fitness values worse than their parents, and PS_{DE} is the population size of DE.

$$F_{i,g} = N(\mu, 0.02) \quad (15)$$

where $i = 1, \dots, PS_{DE}$, PS_{DE} is the population size of DE, and N is the Gaussian distribution. The design of Eq. (14) takes the following factors into account:

- The generated scaling factor should shrink as the evolution proceeds, thereby promoting strong exploration in the early stages and exploitation in the later stages.
- Empirical value (e.g., 0.35) can be applied, determined from extensive experimental results.
- Compared with the scaling factor calculation method used by UMOEAs-III, the proposed method exhibits a narrower fluctuation range and further enhances the exploitation capability in the later stages of evolution.

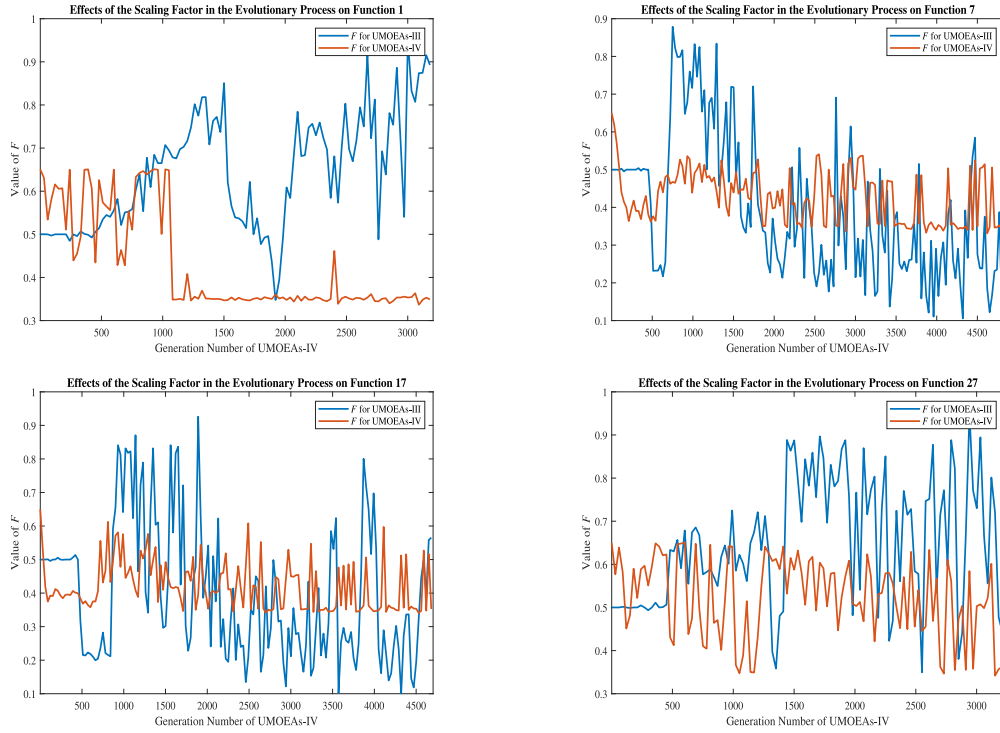


Fig. 1. The distribution of the scaling factor F in UMOEAs-III and UMOEAs-IV.

Fig. 1 illustrates the distribution of the scaling factor F used in UMOEAs-III and UMOEAs-IV. As shown, F , calculated by Eqs. (14) and (15), takes larger values in the early stage of evolution, resulting in larger mutation step sizes, and smaller values in the later stage, leading to reduced mutation step sizes. This design aims to allow the mutation operators to emphasize exploration in the early stage and exploitation in the later stage of evolution. Compared with the original calculation method of F in UMOEAs-III, the smaller fluctuation amplitude of F in UMOEAs-IV ensures a more stable balance between exploration and exploitation.

3.2. Mutation strategy

In UMOEAs-IV, the mutation strategy combines two mutation operators, Eqs. (5) and (16), and switches between them according to $Prob_{DE-MUT_w}$.

$$v_{i,g}^d = x_{i,g}^d + F_{i,g} \cdot (x_{elite,g}^d - x_{i,g}^d) + F_{i,g} \cdot (x_{r1,g}^d - x_{r2,g}^d) \quad (16)$$

here, an individual $x_{o,g}^d$ is randomly selected elite individual from the top 10% individuals, $x_{p,g}^d$ and $x_{q,g}^d$ are randomly selected individuals from the PS_{DE} , and $(x_{o,g}^d \neq x_{p,g}^d \neq x_{q,g}^d)$, then sort the three individuals by fitness values, the individuals with best, second, and worst is assigned to $x_{elite,g}^d$, $x_{r1,g}^d$, and $x_{r2,g}^d$, respectively. Eq. (16) is denoted as ‘current-to-pbest with sort’.

The archived promotion rates from the last L generations ($L = 15$) are stored and used to calculate the usage probabilities of the combined mutation operators. $Prob_{DE-MUT_w}$ is computed using Eqs. (17) to (22) for the mutation operator w , where w represents either ‘current-to-pbest with sort’ or ‘current-to-pbest with archive’.

$$M_{w,l} = M_{w,l+1}, \quad \text{for } w = 1, 2 \quad (17)$$

$$M_{w,L} = \frac{1}{PS_{DEw}} \sum_{k=1}^{PS_{DEw}} \frac{\max(0, f_k^{old} - f_k^{new})}{f_k^{old}}, \quad \text{and } w = 1, 2 \quad (18)$$

$$T_l = \exp((0.15 + 0.2 \cdot \frac{cf_e}{FES_{max}}) \cdot l), \quad \forall l \in \{1, \dots, L\}, \quad L = 15 \quad (19)$$

$$T'_l = \frac{T_l}{\sum_{l=1}^L T_l}, \quad L = 15 \quad (20)$$

$$I_{DE-MUT_w} = \frac{\sum_{l=1}^L T'_l M_{w,l}^2}{\sum_{l=1}^L T'_l M_{w,l}}, \quad \text{and } w = 1, 2 \quad (21)$$

$$Prob_{DE-MUT_w} = \max(0.1, \min(0.9, \frac{I_{DE-MUT_w}}{I_{DE-MUT_1} + I_{DE-MUT_2}})), \quad \text{and } w = 1, 2 \quad (22)$$

where $l = 1, \dots, L-1$ and $L = 15$; PS_{DEw} is the number of individuals using mutation operator w , and k denotes the k th individual using mutation operator w .

The motivation behind this design is that ‘current-to-pbest with sort’ excels in exploitation, while ‘current-to-pbest with archive’ excels in exploration. The switching condition is based on the historical success rates of the mutation operators, which effectively capture information from the landscape of the optimization problem.

3.3. Usage probability of EDA

In UMOEAs-IV, the usage probability of EDA is computed using Eq. (23). When $Prob_{EDA} > rand$, where $rand$ is a uniformly distributed random number in $[0,1]$, Eqs. (7) to (11) are applied.

$$Prob_{EDA} = \text{round}(\frac{5.6}{D^{0.8}}, 2) \quad (23)$$

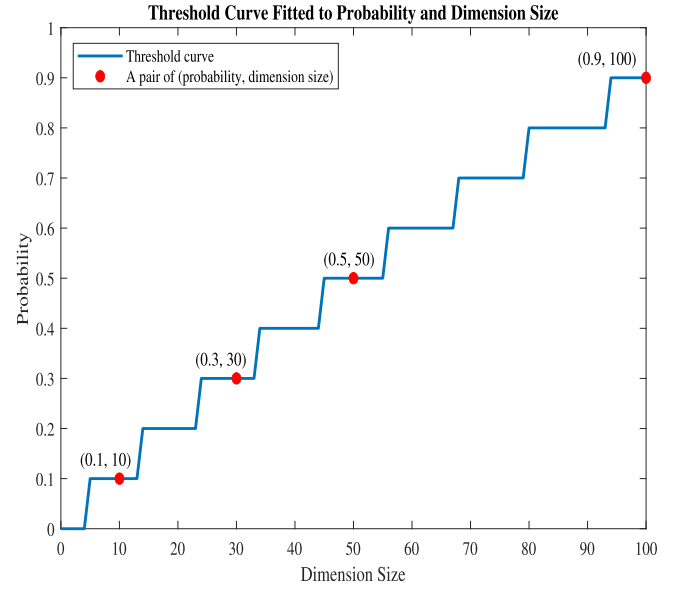
here, when D is set to 10, 30, 50, and 100, the corresponding values of $Prob_{EDA}$ are 0.89, 0.37, 0.24, and 0.14, respectively. EDA performs well in low-dimensional problems because the probabilistic models can accurately capture dependencies between variables, and convergence is often faster since the number of parameters in the model is manageable. In contrast, in high-dimensional problems, EDA can struggle due to the rapidly increasing complexity of the probabilistic model. The motivation behind this design is to apply EDA more frequently in low-dimensional problems and less frequently in high-dimensional ones.

Algorithm 1 Pseudocode of UMOEAs-IV

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1: Define  $PS = PS_{DE} + PS_{CMA}$ ,  $Prob_{DE} = Prob_{CMA} = 1$ ,
    $Prob_{DE\_MUT_1} = Prob_{DE\_MUT_2} = 0.5$ ,  $count_i = 0$ ,  $STE = \max(\lceil 97 \times \log(D) - 294 \rceil, 36)$ ,  $Threshold = \text{round}(\frac{4.5 \times D}{D+400}, 1)$ ,  $Prob_{EDA} = \text{round}(\frac{5.6}{D^{0.8}}, 2)$ ,  $Prob_{ls} = 0.1$ ,  $cy = 0$ , and all other parameters
   required.
2: for  $i : PS$  do
3:    $x_{i,g}^d \leftarrow$  uniformly distributed  $D$  random numbers.
4: end for
5: while termination condition is not satisfied do
6:    $cy = cy + 1$ 
7:   if  $cy == CS$  then
8:     Calculate  $Prob_{DE}$  and  $Prob_{CMA}$ .
9:   end if
10:  if  $cy == 2 \times CS$  then
11:    Share information.
12:     $Prob_{DE} = Prob_{CMA} = 1$ , and  $cy = 1$ .
13:  end if
14:  if  $\text{rand}(0, 1) \leq Prob_{DE}$  then
15:    Generate the scaling factor  $F$  according to Eqs. (14) and (15),
    and generate the Control Parameter (CR).
16:    for  $i : PS_{DE}$  do
17:      if  $\text{rand}(0, 1) \leq Prob_{DE\_MUT_1}$  then
18:        Calculate the mutant vector  $v_{i,g}^d$  according to Eq. (16).
19:      else
20:        Calculate the mutant vector  $v_{i,g}^d$  according to Eq. (5).
21:      end if
22:      Generate the trial vector  $u_{i,g}^d$ .
23:      if  $f(u_{i,g}^d) < f(x_{i,g}^d)$  then
24:         $x_{i,g+1}^d = u_{i,g}^d$ ,  $count_i = 0$ .
25:      else
26:         $x_{i,g+1}^d = x_{i,g}^d$ ,  $count_i = count_i + 1$ .
27:      end if
28:    end for
29:    Calculate the Failure Rate.
30:    Calculate the  $Prob_{DE\_MUT_w}$  using Eqs. (17) to (22).
31:    for  $i : PS_{DE}$  do
32:      if  $count_i > STE$  then
33:        if  $\frac{cfe}{FES_{max}} < Threshold$  then
34:          Update stagnant individual  $x_{i,g+1}^d$  using Eq. (26).
35:        else
36:          Update stagnant individual  $x_{i,g+1}^d$  using Eq. (27).
37:        end if
38:      end if
39:    end for
40:    if  $\text{rand}(0, 1) < Prob_{EDA}$  then
41:      Apply the Estimation-of-Distribution Algorithm (EDA) using
      Eqs. (7) to (11), update  $cfe$ , and sort  $PS_{DE}$ .
42:    end if
43:    Apply nonlinear population size reduction.
44:  end if
45:  if  $\text{rand}(0, 1) \leq Prob_{CMA}$  then
46:    Apply CMA-ES, update  $cfe$ , and sort  $PS_{CMA}$ .
47:  end if
48:  if  $cfe \geq 0.75 \times FES_{max}$  then
49:    if  $\text{rand}(0, 1) < Prob_{ls}$  then
50:      Apply Sequential Quadratic Programming (SQP) to the best
      individual to find a better solution.
51:    end if
52:  end if
53: end while

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**Fig. 2.** Threshold curve.**3.4. Stagnation strategy**

In UMOEAs-IV, a novel stagnation strategy is introduced that integrates both Gaussian learning (Eq. (26)) and individual learning (Eq. (27)). When the stagnation count of the i th individual reaches STE (Eq. (24)), the strategy checks whether $\frac{cfe}{FES_{max}} < Threshold$, where $Threshold$ is defined in Eq. (25). If the condition holds, Eq. (26) is applied; otherwise, Eq. (27) is used.

$$STE = \max(\lceil 97 \times \log(D) - 294 \rceil, 36) \quad (24)$$

$$Threshold = \text{round}(\frac{4.5 \cdot D}{D+400}, 1) \quad (25)$$

where $i = 1, \dots, PS_{DE}$, and D is the dimensionality.

$$x_{ss,g+1}^d = N(x_{r,g}^d, \sigma) + \text{rand} \cdot x_{ss,g}^d - \text{rand} \cdot x_{r,g}^d \quad (26)$$

where $x_{ss,g}^d$ represents the individual that stagnates STE times, $x_{r,g}^d$ is a randomly selected individual, which is better than $x_{ss,g}^d$, from DE population, N is the Gaussian distribution, and rand is a uniform distribution in $[0, 1)$. The motivation behind this design is that when an individual is in stagnation, the stagnation strategy either introduces perturbations to maintain diversity or enables learning from better individuals.

$$x_{ss,g+1}^d = \text{rand} \cdot x_{ss,g}^d + \text{rand} \cdot x_{r,g}^d \quad (27)$$

where $x_{ss,g}^d$ represents the stagnated individual, $x_{r,g}^d$ is a randomly selected individual from the DE population, and rand follows a uniform distribution in $[0, 1)$.

The principle for controlling the threshold is to balance exploration and exploitation across problems with different dimension sizes. When stagnation occurs, a smaller dimension size enhances the capability for exploitation (individual learning is employed more frequently), whereas a larger dimension size enhances the capability for exploration (Gaussian learning is employed more frequently). Fig. 2 demonstrates the relationship between probability and dimension size along the threshold curve.

4. Performance evaluation

A global optimization problem can be defined as a pair (X, f) , where $X \subset \mathbb{R}^D$ is a bounded subset of the D -dimensional space, and

Table 1
CEC2017 benchmark functions.

Types	f_n	Benchmark functions	Search range	$F(x^*)f_{bias}$
Unimodal functions	f_1	Shifted and Rotated Bent Cigar function	$[-100, 100]^D$	100
	f_2	Shifted and Rotated Sum Diff Pow function	$[-100, 100]^D$	200
	f_3	Shifted and Rotated Zakharov function	$[-100, 100]^D$	300
Simple Multimodal functions	f_4	Shifted and Rotated Rosenbrock's function	$[-100, 100]^D$	400
	f_5	Shifted and Rotated Rastrigin's function	$[-100, 100]^D$	500
	f_6	Shifted and Rotated Expanded Scaffer's F6 function	$[-100, 100]^D$	600
	f_7	Shifted and Rotated Lunacek Bi_Rastrigin function	$[-100, 100]^D$	700
	f_8	Shifted and Rotated Non-Continuous Rastrigin's function	$[-100, 100]^D$	800
	f_9	Shifted and Rotated Levy function	$[-100, 100]^D$	900
	f_{10}	Shifted and Rotated Schwefel's function	$[-100, 100]^D$	1000
Hybrid functions	f_{11}	Hybrid function 1 (N = 3)	$[-100, 100]^D$	1100
	f_{12}	Hybrid function 2 (N = 3)	$[-100, 100]^D$	1200
	f_{13}	Hybrid function 3 (N = 3)	$[-100, 100]^D$	1300
	f_{14}	Hybrid function 4 (N = 4)	$[-100, 100]^D$	1400
	f_{15}	Hybrid function 5 (N = 4)	$[-100, 100]^D$	1500
	f_{16}	Hybrid function 6 (N = 4)	$[-100, 100]^D$	1600
	f_{17}	Hybrid function 7 (N = 5)	$[-100, 100]^D$	1700
	f_{18}	Hybrid function 8 (N = 5)	$[-100, 100]^D$	1800
	f_{19}	Hybrid function 9 (N = 5)	$[-100, 100]^D$	1900
	f_{20}	Hybrid function 10 (N = 6)	$[-100, 100]^D$	2000
Composition functions	f_{21}	Composition function 1 (N = 3)	$[-100, 100]^D$	2100
	f_{22}	Composition function 2 (N = 3)	$[-100, 100]^D$	2200
	f_{23}	Composition function 3 (N = 4)	$[-100, 100]^D$	2300
	f_{24}	Composition function 4 (N = 4)	$[-100, 100]^D$	2400
	f_{25}	Composition function 5 (N = 5)	$[-100, 100]^D$	2500
	f_{26}	Composition function 6 (N = 5)	$[-100, 100]^D$	2600
	f_{27}	Composition function 7 (N = 6)	$[-100, 100]^D$	2700
	f_{28}	Composition function 8 (N = 6)	$[-100, 100]^D$	2800
	f_{29}	Composition function 9 (N = 3)	$[-100, 100]^D$	2900
	f_{30}	Composition function 10 (N = 3)	$[-100, 100]^D$	3000

$f : X \rightarrow \mathbb{R}$ is a real-valued function defined over X . This can be formalized as follows:

$$\forall x \in X, f(x_{\min}) \leq f(x). \quad (28)$$

The CEC2017 benchmark functions are listed in Table 1, where f_1 – f_3 , f_4 – f_{10} , f_{11} – f_{20} , and f_{21} – f_{30} are unimodal, simple multimodal, hybrid, and composition functions, respectively.

4.1. Parameter settings

The maximum number of function evaluations (FES_{max}) was defined as $10^4 \times D$ for all DE-based variants. Accordingly, FES_{max} was set to 10×10^4 , 30×10^4 , 50×10^4 , and 100×10^4 for problem dimensions of 10, 30, 50, and 100, respectively. The parameter settings for the proposed UMOEAs-IV and the compared DE-based variants are summarized in Table 2. For the comparison algorithms, default configurations provided by their developers were adopted, as these have been reported to yield their best performance.

4.2. Experimental results

In Table 3, the mean, minimum, and median values ranking first are counted. In Table 4, the Friedman test rankings are evaluated for all the compared algorithms. The tables display the means, minima, medians, standard deviations, and rankings for experiments across different dimensions: 10D, 30D, 50D, and 100D (see Tables 5, Table 6, Table 7, and Table 8). These statistical measures are based on 51 runs, with the best values and top rankings highlighted in bold. To better observe the experimental results, some of the data are retained with more digits. To evaluate performance, a two-sided Wilcoxon Rank Sum Test (WRST) was conducted at a significance level of 0.05, comparing UMOEAs-IV with other DE-based variants, including TSMS-DE, AuDE, NLPsr-jSO, mLSHADE, HAPI-DE, FDHD-DE, DDPSA-DE, DACDE, CELDE, ACD-DE, UMOEAs-III, IDE-EDA, LSHADE-SPACMA, LSHADE-cnEpSin, and EBOwithCMAR. In the tables, the

symbols “>”, “=”, and “<” indicate where UMOEAs-IV achieved significantly better results, showed no statistically significant difference, or performed significantly worse, respectively, when compared to the other DE-based variants. Figs. 3 and 4 show the evolutionary processes of UMOEAs-IV and the compared DE-based variants in 50-dimensional experiments, averaged over 51 runs.

4.3. Comparison and analysis

In Table 3, UMOEAs-IV achieves 8, 11, and 12 best mean values on the 30-, 50-, and 100-dimensional tests, respectively. It achieves 13, 13, and 12 best minimum values, and 13, 12, and 11 best median values on the 30-, 50-, and 100-dimensional tests, respectively. Finally, it achieves 34, 36, and 35 best total counts on the 30-, 50-, and 100-dimensional tests, respectively. These values rank first among all compared algorithms. In the 10-dimensional test, UMOEAs-IV achieves 10 (2nd), 17 (2nd), 11, and 38 (3rd) best mean, minimum, median, and total values, respectively. Overall, UMOEAs-IV achieves the best performance on the 30-, 50-, and 100-dimensional tests, while UMOEAs-III achieves the best performance on the 10-dimensional test. In Table 4, the averaged Friedman rank of UMOEAs-IV is 6.72, placing it 2nd in the 10-dimensional tests, second only to UMOEAs-III (6.45). The averaged Friedman ranks of UMOEAs-IV are 5.05, 3.63, and 3.30 on the 30-, 50-, and 100-dimensional tests, respectively, ranking 1st among all compared algorithms.

In Table 5, for the 10-dimensional experiments on unimodal functions, all compared algorithms show equivalent performance on f_1 ; For f_2 , all algorithms demonstrate equivalent top performance except that AuDE, mLSHADE, and LSHADE-SPACMA yield lower mean values; For f_3 , all algorithms illustrate equivalent top performance except DACDE, which has a lower mean value. For simple multimodal functions, all algorithms illustrate equivalent top performance on f_4 except DACDE; UMOEAs-IV achieves the best mean values on f_5 – f_6 and f_9 ; UMOEAs-III achieves the best mean values on f_7 – f_8 ; and CELDE achieves the best mean value on f_{10} . For hybrid functions, UMOEAs-IV achieves the

Table 2
Parameter settings for all compared algorithms.

Algorithm	Year	Default parameter settings
UMOEAs-IV	–	$PS_{DE} = 18 \times D$, $PS_{CMA} = 4 + \text{floor}(3 \times \ln(D))$, $CR_{init} = 0.5$, $T_{init} = 0.1$, $Arc_rate = 1.4$, the memory size $H = 6$. CS for 10, 30, 50, and 100-dimensional tests are set to 50, 100, 100, and 150, respectively.
TSMS-DE [45]	2025	$PS = 25 \times \log(D) \times \sqrt{D} \sim 4$, $\mu_F = 0.3$, $\mu_{CR} = 0.8$, $p = 0.25 \sim 0.11$, $r_{rac,A} = 1$, $r_{rac,B} = 4$, $\sigma = 0.3$, $\beta = 0.6$, $n = 2 \times D$, $\gamma_1 = 0.6$, $\gamma_2 = 0.3$.
AuDE [22]	2025	$PS = 18 \times D$, $\mu_F = 0.5$, $\mu_{CR} = 0.5$, $\epsilon = 0.5$, $\alpha = 0.9$, $\gamma = 0.95$, $C = 100$.
NLPSR-jSO [46]	2024	$PS = 75 \times D^{(2/3)} \sim 4$, $ A = PS$, $p_{min} = 0.0085$, $p_{max} = 0.17$, $\mu = 0.95$, $k = 3$, $k1 = 1.6$, $k2 = 2.0$, the memory size $H = 5$.
mLSHADE [17]	2024	$PS = 18 \times D$, $\mu_{F_r} = 0.5$, $\mu_{CR_r} = 0.5$, $\mu_{f_r} = 0.5$, $P_c = 0.4$, $P_s = 0.5$, $P_{LS} = 0.01$, the memory size $H = 5$.
HAPI-DE [47]	2024	$PS = 25 \times \log(D) \times \sqrt{D} \sim 4$, $\mu_F = 0.3$, $\mu_{CR} = 0.8$, the memory size $H = 4$.
FDHD-DE [48]	2024	$PS = 25 \times \log(D) \times \sqrt{D} \sim 4$, $F = 0.5$, $CR = 0.8$, $p = 0.11$, $r^{arc} = 1.4$, $k_{min} = 0.6$, $k_{max} = 6$, $c_o = 0$, $C_n = 0$, the memory size $H = 4$.
DDPSA-DE [49]	2024	$PS = 100$, $F = 0.7$, $CR = 0.5$, $c = 0.1$, $S_0 = 0.5$, $T = 20$, $p = 0.01$.
DACDE [50]	2024	$PS = 4 \times D$, $\mu_F = 0.8$, $\mu_{CR} = 0.5$, $GS = 1.5$, $\omega = 0.3$, the memory size $H = 2$.
CELDE [51]	2024	$PS = 25 \times \log(D) \times \sqrt{D} \sim 4$, $F_{init} = 0.5$, $CR_{init} = 0.5$, $Freq_{init} = 0.5$. G_{max} for 10, 30, 50, and 100-dimensional tests are set to 2163, 2745, 3022, and 3401, respectively. The memory size $H = 5$.
ACD-DE [52]	2024	$PS = 25 \times \ln(D) \times \sqrt{D} \sim 5$, $\mu_F = 0.5$, $\mu_{CR} = 0.9$, $F \sim C(\mu_F, 0.1)$, $CR \sim N(\mu_{CR}, 0.1)$, $p = 0.15 \sim 0.45$, $r^{arc} = 1.3$, the memory size $H = 6$.
UMOEAs-III [16]	2023	$PS_{DE} = 18 \times D$, $PS_{CMA} = 4 + \text{floor}(3 \times \ln(D))$, $F_{init} = 0.7$, $CR_{init} = 0.5$, $Freq_{init} = 0.5$, $T_{init} = 0.1$, the memory size $H = 6$. CS for 10, 30, 50, and 100-dimensional tests are set to 50, 100, 100, and 150, respectively.
IDE-EDA [25]	2023	$PS = 75 \times D^{(2/3)} \sim 4$, $k = 3$, $r^{arc} = 1$, $\tau = 0.9$, the memory size $H = 5$.
LSHADE-SPACMA [53]	2017	$PS = 18 \times D$, $P_{best} = 0.11$, $Arc_rate = 1.4$, $F_{cp} = 0.5$, $c = 0.8$, the memory size $H = 5$.
LSHADE-cnEpSin [26]	2017	$PS = 18 \times D$, $F_{init} = 0.5$, $CR_{init} = 0.5$, $Freq_{init} = 0.5$, the memory size $H = 5$.
EBOWithCMAR [54]	2017	For EBO, $PS_{1,max} = 18 \times D$, $PS_{1,min} = 4$, $PS_{2,max} = 46.8 \times D$, $PS_{2,min} = 10$, $H = 6$. For CMAR, $PS_3 = 4 + 3 \times \log(D)$, $\sigma = 0.3$. For local search, $prob_{ls} = 0.1$, $cfe_{ls} = 0.25 \times FES_{max}$.

best mean, minimum, and median values on both f_{11} and f_{14} ; TSMS-DE, FDHD-DE, CELDE, UMOEAs-III, IDE-EDA, LSHADE-SPACMA, and LSHADE-cnEpSin also achieve the best mean value on f_{11} ; ACD-DE and HAPI-DE achieve the best mean values on f_{12} and f_{13} , respectively; DDPSA-DE achieves the best mean values on f_{15} and f_{18} , while FDHD-DE achieves the best mean values on f_{16} , f_{17} , f_{19} , and f_{20} ; TSMS-DE also achieves the best mean value on f_{20} . For composition functions, UMOEAs-IV achieves the best mean, minimum, and median values on f_{26} ; LSHADE-SPACMA achieves the best mean values on f_{21} and f_{28} , while AuDE achieves the best mean value on f_{22} ; EBOWithCMAR, NLPSR-jSO, mLSHADE, and LSHADE-cnEpSin achieve the best mean values on f_{24} , f_{25} , f_{27} , and f_{29} , respectively; finally, UMOEAs-III achieves the best mean values on f_{23} and f_{30} . In summary, no algorithm dominates the performance in the low-dimensional tests. In summary, UMOEAs-IV attains a top-three performance on 15 functions overall: 3, 6, 2, and 4 on unimodal, simple multimodal, hybrid, and composition functions, respectively.

In Table 6, for the 30-dimensional experiments on unimodal functions, TSMS-DE, HAPI-DE, ACD-DE, IDE-EDA, LSHADE-SPACMA, and LSHADE-cnEpSin achieve the best mean values on f_1 ; NLPSR-jSO, HAPI-DE, DACDE, CELDE, ACD-DE, UMOEAs-III, IDE-EDA, and LSHADE-cnEpSin achieve the best mean values on f_2 ; and LSHADE-SPACMA and LSHADE-cnEpSin achieve the best mean values on f_3 . For simple multimodal functions, UMOEAs-IV achieves the best mean values on f_5 , f_8 , and f_{10} ; UMOEAs-III achieves the best mean values on f_4 and f_7 ; LSHADE-SPACMA achieves the best mean values on f_6 ; while all compared algorithms show equivalent performance on f_9 except AuDE, NLPSR-jSO, and mLSHADE. For hybrid functions, UMOEAs-IV achieves the best mean values on f_{16} ; IDE-EDA achieves the best mean values on f_{11} and f_{19} ; FDHD-DE achieves the best mean values on f_{12} and f_{15} ; LSHADE-SPACMA achieves the best mean values on f_{13} ; and DACDE achieves the best mean values on f_{14} , f_{17} – f_{18} , and f_{20} . For composition functions, UMOEAs-IV achieves the best mean values

on f_{26} – f_{27} and f_{30} ; UMOEAs-III, LSHADE-SPACMA, EBOWithCMAR, mLSHADE, and DACDE achieve the best mean values on f_{21} , f_{22} , f_{24} , f_{25} , and f_{29} , respectively; while FDHD-DE achieves the best mean values on f_{23} and f_{28} . In summary, UMOEAs-IV attains a top-three performance on 16 functions overall: 0, 5, 3, and 8 on unimodal, simple multimodal, hybrid, and composition functions, respectively.

In Table 7, for the 50-dimensional experiments on unimodal functions, LSHADE-SPACMA and LSHADE-cnEpSin achieve the best mean values on f_1 and f_3 , while CELDE and UMOEAs-III achieve the best mean values on f_2 . For simple multimodal functions, UMOEAs-IV achieves the best mean, minimum, and median values on f_5 , f_7 – f_8 , and f_{10} ; mLSHADE achieves the best mean values on f_4 ; and LSHADE-SPACMA achieves the best mean values on f_6 and f_9 . For hybrid functions, UMOEAs-IV achieves the best mean, minimum, and median values on f_{16} and f_{17} , and the best mean and median values on f_{14} and f_{20} . CELDE achieves the best mean values on f_{11} , while FDHD-DE achieves the best mean values on f_{12} – f_{13} , f_{15} , and f_{18} – f_{19} . For composition functions, UMOEAs-IV achieves the best mean values on f_{21} , f_{25} , and f_{30} ; TSMS-DE achieves the best mean values on f_{22} and f_{27} ; FDHD-DE, EBOWithCMAR, and UMOEAs-III achieve the best mean values on f_{23} , f_{26} , and f_{28} , respectively; and HAPI-DE achieves the best mean values on f_{24} and f_{29} . In summary, UMOEAs-IV attains a top-three performance on 16 functions overall: 0, 4, 5, and 7 on unimodal, simple multimodal, hybrid, and composition functions, respectively.

In Table 8, for the 100-dimensional experiments on unimodal functions, LSHADE-SPACMA achieves the best mean, minimum, and median values on f_1 and f_3 , while mLSHADE achieves the best mean value on f_2 . For simple multimodal functions, UMOEAs-IV achieves the best mean, minimum, and median values on f_5 , f_8 , and f_{10} ; mLSHADE achieves the best mean value on f_4 ; and LSHADE-SPACMA achieves the best mean values on f_6 – f_7 and f_9 . For hybrid functions, UMOEAs-IV achieves the best mean, minimum, and median values on f_{16} – f_{17} and f_{20} . FDHD-DE achieves the best mean values on f_{11} , f_{14} , and

Table 3

Ranking statistics based on the mean, minimum, and median values for all DE-based variants, evaluated on the CEC2017 benchmark functions across different dimensions.

<i>D</i>	Criteria	UMOEAs-IV	TSMS-DE	AuDE	NLPSR-jSO	mLSHADE	HAPI-DE	FDHD-DE	DDPSA-DE	DACDE	CELDE	ACD-DE	UMOEAs-III	IDE-EDA	LSHADE-SPACMA	LSHADE-cnEpSin	EBOwithCMAR
10 <i>D</i>	Mean [1st]	10	7	5	6	5	6	11	7	4	7	6	10	7	8	8	6
	Min [1st]	17	15	17	19	11	17	14	13	17	17	15	17	15	15	16	16
	Median [1st]	11	14	5	13	10	14	15	12	12	14	14	18	14	12	12	11
	TOTAL [1st]	38	36	27	38	26	37	40	32	33	38	35	45	36	35	36	33
	Mean (> = <)	/	8 19 3	22 6 2	7 17 6	16 12 2	9 17 4	9 14 7	13 14 3	15 11 4	12 15 3	11 10 9	4 21 5	10 16 4	15 12 3	11 15 4	13 11 6
30 <i>D</i>	Mean [1st]	8	2	0	1	1	3	5	1	7	2	3	5	5	6	4	2
	Min [1st]	13	7	2	3	3	8	6	8	7	5	5	5	6	9	5	6
	Median [1st]	13	6	0	2	2	5	6	4	7	4	4	5	4	7	6	2
	TOTAL [1st]	34	15	2	6	6	16	17	13	21	11	12	15	15	22	15	10
	Mean (> = <)	/	16 6 8	29 1 0	20 5 5	26 2 2	15 5 10	15 8 7	26 4 0	18 4 8	16 11 3	18 7 5	20 8 2	17 7 6	23 2 5	18 8 4	24 3 3
50 <i>D</i>	Mean [1st]	11	3	0	0	1	2	6	0	0	2	0	2	1	4	3	1
	Min [1st]	13	4	0	3	2	8	6	4	4	4	4	8	4	6	6	3
	Median [1st]	12	4	0	1	0	7	6	0	0	2	2	3	2	5	3	1
	TOTAL [1st]	36	11	0	4	3	17	18	4	4	8	6	13	7	15	12	5
	Mean (> = <)	/	17 4 9	30 0 0	23 4 3	28 0 2	20 5 5	18 3 9	29 1 0	26 4 0	18 5 7	23 4 3	22 4 4	21 4 5	21 4 5	22 2 6	26 3 1
100 <i>D</i>	Mean [1st]	12	0	0	0	3	1	4	0	1	3	0	0	0	5	0	1
	Min [1st]	12	3	0	1	1	2	1	1	0	6	0	4	1	4	0	1
	Median [1st]	11	0	0	0	4	1	4	0	2	3	0	0	0	5	0	0
	TOTAL [1st]	35	3	0	1	8	4	9	1	3	12	0	4	1	14	0	2
	Mean (> = <)	/	22 2 6	29 1 0	24 1 5	26 1 3	27 0 3	20 3 7	27 2 1	24 2 4	20 4 6	29 1 0	21 7 2	23 3 4	17 6 7	20 5 5	28 1 1

Table 4
The Friedman test rankings were conducted on the mean values for UMOEAs-IV, TSMS-DE, AuDE, NLPSR-jSO, mLSHADE, HAPI-DE, FDHD-DE, DDPSA-DE, DACDE, CELDE, ACD-DE, UMOEAs-III, IDE-EDA, LSHADE-SPACMA, LSHADE-cnEpSin, and EBOwithCMAR across CEC2017 benchmark functions.

<i>D</i>	Criteria	UMOEAs-IV	TSMS-DE	AuDE	NLPSR-jSO	mLSHADE	HAPI-DE	FDHD-DE	DDPSA-DE	DACDE	CELDE	ACD-DE	UMOEAs-III	IDE-EDA	LSHADE-SPACMA	LSHADE-cnEpSin	EBOwithCMAR
10 <i>D</i>	Averaged	6.72	7.33	12.60	7.72	10.53	8.35	7.63	9.08	11.50	7.82	8.13	6.45	7.35	9.20	8.68	6.90
	Friedman ranks																
30 <i>D</i>	Rank	2	4	16	7	14	10	6	12	15	8	9	1	5	13	11	3
	Averaged	5.05	6.22	14.97	7.55	11.77	6.73	6.50	13.40	9.05	7.63	7.83	6.82	7.08	9.05	8.28	8.07
50 <i>D</i>	Friedman ranks																
	Rank	1	2	16	7	14	4	3	15	12.5	8	9	5	6	12.5	11	10
100 <i>D</i>	Averaged	3.63	6.42	15.70	6.67	11.77	7.80	5.42	13.78	11.98	6.07	8.43	6.42	7.15	8.08	7.68	9.00
	Friedman ranks																
1000 <i>D</i>	Rank	1	4	16	6	13	9	2	15	14	3	11	4	7	10	8	12
	Averaged	3.30	7.63	15.37	6.30	10.27	8.03	7.03	12.27	12.30	4.90	10.83	5.77	9.20	6.10	7.47	9.23
10000 <i>D</i>	Friedman ranks																
	Rank	1	8	16	5	12	9	6	14	15	2	13	3	10	4	7	11

Table 5

Performance of DE Variants on 10D CEC2017 Benchmarks (51 Runs): Statistical Results and Wilcoxon Tests.

f_n	Criteria	UMOEa-IV	TSM-DE	AsDE	NLPsr-go	mLSHADE	HAP-DE	FORB-DE	DOPSA-DE	DADDE	CELDE	ACD-DE	UMOEa-III	IDE-EDA	LSHADE-SPACMA	LSHADE-collpin	EBOWithCMAR
f_1	Mean	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank(WRST)	1	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =
	Min	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
(Std Dev)		(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)
f_2	Mean	0.000000E+00	0.000000E+00	9.59588E-08	0.000000E+00	5.57288E-16	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	5.57288E-16	0.000000E+00	0.000000E+00
	Rank(WRST)	1	1 =	16 >	1 =	14 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	14 =	1 =	1 =
	Min	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
(Std Dev)		(0.000000E+00)	(0.000000E+00)	(6.852833E-07)	(0.000000E+00)	(3.979835E-15)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(3.979835E-15)	(0.000000E+00)	(0.000000E+00)
f_3	Mean	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank(WRST)	1	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =
	Min	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
(Std Dev)		(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)
f_4	Mean	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank(WRST)	1	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =	1 =
	Min	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
(Std Dev)		(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)
f_5	Mean	2.48556E-12	1.426922E+00	6.809476E+00	1.911882E+00	1.541211E+00	1.900108E+00	2.069338E+00	2.232527E+00	4.487070E+00	1.521706E+00	5.852706E+00	5.852706E-02	1.034592E+00	1.719178E+00	1.933651E+00	4.035314E-04
	Rank(WRST)	1	5 >	16 >	10 >	7 >	9 >	12 >	13 >	15 >	6 >	14 >	3 >	4 >	8 >	11 >	2 >
	Min	0.000000E+00	1.400883E-06	1.989918E+00	0.000000E+00	1.25055E-12	0.000000E+00	3.397588E-05	7.797954E-02	9.94591E-01	0.000000E+00	0.000000E+00	0.000000E+00	2.579367E-08	3.046481E-03	0.000000E+00	0.000000E+00
	Rank	1	10	16	1	8	1	11	13	14	1	15	1	1	9	12	1
	Median	1.648459E-12	1.003272E+00	5.969754E+00	1.989918E+00	1.989918E+00	1.989918E+00	1.989926E+00	1.26059E+00	3.979836E+00	1.989918E+00	3.006884E+00	2.273737E-13	9.949591E-01	1.990409E+00	1.990904E+00	1.307399E-12
	Rank	3	5	16	6	9	6	10	13	15	6	14	1	11	12	2	1
(Std Dev)		(2.96474E-12)	(7.254075E-01)	(4.188508E+00)	(9.510713E-01)	(6.93265E-01)	(1.131666E-01)	(1.10494E+00)	(9.168017E-01)	(1.993620E+00)	(8.195382E-01)	(1.460687E+00)	(2.364375E-01)	(8.209974E-01)	(6.320230E-01)	(7.011054E-01)	(3.681791E-03)
f_6	Mean	0.000000E+00	2.006238E-14	1.337492E-14	6.966281E-09	1.114577E-14	0.000000E+00	4.458307E-15	0.000000E+00	8.916615E-15	2.229154E-14	6.687461E-15	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	8.916615E-15
	Rank(WRST)	1	16 >	16 >	15 >	11 >	1 =	7 =	1 =	9 >	8 >	14 >	1 =	1 =	1 =	1 =	9 >
	Min	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	0.000000E+00	1.13686E-13	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
(Std Dev)		(0.000000E+00)	(4.377095E-14)	(3.396084E-07)	(3.699321E-14)	(4.974912E-08)	(3.414318E-14)	(0.000000E+00)	(2.22878E-14)	(0.000000E+00)	(3.086875E-14)	(4.558605E-14)	(2.701602E-14)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(3.086875E-14)
f_7	Mean	1.048878E+01	1.155534E+01	1.906456E+01	1.196154E+01	1.162205E+01	1.129655E+01	1.267358E+01	1.260361E+01	1.508946E+01	1.186737E+01	1.329187E+01	1.046636E+01	1.166597E+01	1.095228E+01	1.191560E+01	1.061656E+01
	Rank(WRST)	2	6 >	16 >	11 >	7 >	5 >	13 >	12 >	15 >	9 >	14 >	1 =	8 >	4 >	10 >	3 >
	Min	1.066922E+01	1.059803E+01	4.242854E+00	1.053031E+01	1.072898E+01	1.036704E+01	1.099870E+01	1.073047E+01	1.09248E+01	1.036692E+01	1.036692E+01	1.036692E+01	1.036692E+01	1.099832E+01	1.036692E+01	1.036692E+01
	Rank	2	9	1	8	11	6	13	16	14	10	12	3	5	7	15	4
	Median	1.043589E+01	1.156155E+01	1.593311E+01	1.198922E+01	1.165108E+01	1.124671E+01	1.215276E+01	1.268943E+01	1.469678E+01	1.184643E+01	1.302574E+01	1.043589E+01	1.165604E+01	1.091317E+01	1.184611E+01	1.050330E+01
	Rank	2	6	16	11	7	5	12	13	15	10	14	1	8	4	9	3
(Std Dev)		(1.428322E-01)	(4.776623E-01)	(7.714108E+00)	(6.418463E-01)	(4.695259E-01)	(5.312132E-01)	(1.604953E+00)	(2.329411E+00)	(3.77890E-01)	(1.529035E-01)	(1.385752E-01)	(5.773989E-01)	(3.562183E-01)	(4.79589E-01)	(2.439429E-01)	(2.439429E-01)
f_8	Mean	7.958079E-13	1.288542E+00	6.355955E+00	2.048445E+00	1.424157E+00	1.550749E+00	2.342033E+00	2.370274E+00	4.955285E+00	1.755815E+00	3.316556E+00	3.187690E-13	1.249930E+00	9.37047E-01	2.031035E+00	9.754501E-02
	Rank(WRST)	2	6 >	16 >	11 >	7 >	8 >	12 >	13 >	15 >	9 >	14 >	1 <	5 >	3 >	3 >	3 >
	Min	0.000000E+00	0.000000E+00	1.638456E+00	0.000000E+00	4.547474E-13	0.000000E+00	9.94591E-01	7.370134E-05	1.989918E+00	0.000000E+00	9.94591E-01	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank	1	1	15	1	10	1	13	11	16	1	1	1	1	1	12	1
	Median																

Table 5 (continued).

f_n	Criteria	UMOEAs-IV	TSMS-DE	AdDE	NLPSS-JSO	mLSHADE	HAPI-DE	FDDH-DE	DPSA-DE	DACDE	CELDE	ACD-DE	UMOEAs-III	IDE-EDA	LSHADE-SPACMA	LSHADE-cnEgSin	EBOWithCMAR
f_{25}	Mean	3.003402E+02	3.003114E+02	3.022114E+02	3.012348E+02	3.005994E+02	3.020899E+02	3.003698E+02	3.021258E+02	3.061492E+02	3.005705E+02	3.021735E+02	3.002896E+02	3.007122E+02	3.023727E+02	3.019945E+02	3.003835E+02
	Rank[WRST]	3	2	14	9	11	7	11	12	6	16	15	1	8	10	15	5
	Min	3.000000000000000E+02	3.000000000000000E+02	5.734364094678313E-10	3.000000000000000E+02	3.000000000000000E+02	3.000000000000000E+02	3.000000000000000E+02	3.0000000000011564E+02	3.000000000000000E+02	3.000000000000000E+02	3.000000000000000E+02	3.000000000000000E+02	3.000007499549838E+02	3.000000000000000E+02	3.000000000000000E+02	3.000000000180480E+02
	Rank	2	2	1	2	14	2	2	13	2	2	2	12	2	16	2	15
	Std Dev	3.0000000000226455E+02	3.000000000000009E+02	3.0078519634703780E+02	3.000000000000005E+02	3.000000000000000E+02	3.000000000000000E+02	3.000000000000000E+02	3.028816322924390E+02	3.06176056090860E+02	3.000000000000005E+02	3.028816322924390E+02	3.0000000001209930E+02	3.000000000000000E+02	3.029168639071377E+02	3.027421489824137E+02	3.0000000003378182E+02
f_{25}	Mean	2.082277E+02	2.765393E+02	2.648205E+02	2.984700E+02	2.819150E+02	2.788735E+02	3.166121E+02	2.717716E+02	3.209798E+02	2.986955E+02	2.597246E+02	1.917565E+02	2.889927E+02	2.477332E+02	2.868187E+02	1.312642E+02
	Rank[WRST]	3	8	6	13	10	9	15	7	16	14	5	2	12	4	11	1
	Min	1.000000E+02	1.000000E+02	1.000000E+02	1.000000E+02	1.000000E+02	1.000000E+02	1.000000E+02	1.000000E+02	1.000000E+02	0.000000E+00	1.000000E+02	1.000000E+02	1.000000E+02	0.000000E+00	0.000000E+00	1.893608E+01
	Rank	5	5	5	5	5	5	5	5	5	1	5	5	5	1	1	4
	Std Dev	1.110952E+02	9.253313E+01	1.083607E+02	7.996861E+01	9.082852E+01	9.482702E+01	4.490204E+01	9.894160E+01	5.588067E+01	8.068822E+01	1.043739E+02	1.034602E+02	8.837569E+01	1.099204E+02	9.350252E+01	7.762802E+01
f_{25}	Mean	4.140219E+02	4.148915E+02	4.163976E+02	4.059684E+02	4.223130E+02	4.211554E+02	4.166201E+02	4.169440E+02	4.162836E+02	4.166170E+02	4.095312E+02	4.104334E+02	4.122074E+02	4.203669E+02	4.229759E+02	4.213377E+02
	Rank[WRST]	5	6	8	1	15	13	10	7	11	9	2	3	12	3	16	14
	Min	3.977429E+02	3.977429E+02	3.977429E+02	3.977429E+02	3.977429E+02	3.977429E+02	3.977429E+02	3.977429E+02	3.977429E+02	3.977429E+02	3.977429E+02	3.977429E+02	3.977429E+02	3.977429E+02	3.977429E+02	3.977429E+02
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Std Dev	2.228286E+01	2.228281E+01	2.284731E+01	1.747832E+01	2.266050E+01	2.287719E+01	2.281794E+01	2.292557E+01	2.285851E+01	2.285851E+01	2.008970E+01	2.045025E+01	2.306887E+01	2.288731E+01	2.287381E+01	2.267233E+01
f_{25}	Mean	2.607843E+02	3.000000E+02	2.980392E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	2.686275E+02	3.000000E+02	3.000000E+02	3.000000E+02	2.901961E+02
	Rank[WRST]	1	5	4	5	5	5	5	5	5	5	5	2	5	5	5	3
	Min	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02
	Rank	1	5	5	5	5	5	5	5	5	5	5	5	5	5	5	1
	Std Dev	4.930895E+01	0.000000E+00	1.400280E+01	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	4.686233E+01	0.000000E+00	0.000000E+00	0.000000E+00	3.003266E+01
f_{27}	Mean	3.822898E+02	3.930933E+02	3.930585E+02	3.893974E+02	3.792648E+02	3.934836E+02	3.934850E+02	3.896041E+02	3.910275E+02	3.892547E+02	3.908465E+02	3.894973E+02	3.894677E+02	3.894714E+02	3.889246E+02	3.919140E+02
	Rank[WRST]	2	14	13	5	1	15	16	9	11	4	8	6	6	7	3	12
	Min	3.714013E+02	3.859023E+02	3.873236E+02	3.740355E+02	3.893081E+02	3.895180E+02	3.873236E+02	3.868892E+02	3.884666E+02	3.873236E+02	3.878799E+02	3.899055E+02	3.899055E+02	3.884065E+02	3.895180E+02	3.907519E+02
	Rank	1	4	8	11	14	15	6	5	10	6	3	11	11	9	15	1
	Std Dev	6.544608E+00	1.894525E+00	7.188630E+00	2.195412E-01	3.186310E+00	1.396241E+00	1.364006E+00	1.740856E+00	1.740856E+00	2.205434E+00	1.871185E+00	1.539065E-01	1.573376E-01	6.832904E-01	2.665857E+00	
f_{25}	Mean	3.113258E+02	3.179875E+02	3.817222E+02	3.113588E+02	3.240575E+02	3.322903E+02	3.222545E+02	3.811929E+02	3.341637E+02	3.835593E+02	3.111273E+02	3.111114E+02	3.122283E+02	3.022692E+02	3.507219E+02	3.097814E+02
	Rank[WRST]	5	8	15	6	10	11	12	14	12	16	4	3	7	1	13	2
	Min	3.000000E+02	3.000000E+02	0.000000E+00	0.000000E+00	3.000000E+02	3.000000E+02	3.000000E+02	0.000000E+00	0.000000E+00	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02
	Rank	3	3	3	1	3	3	3	3	1	3	3	3	3	3	3	3
	Std Dev	4.134795E+01	6.287158E+01	1.344463E+02	8.266426E+01	6.665116E+01	9.012252E+01	7.704375E+01	1.335103E+02	1.120254E+02	1.274957E+02	5.562519E+01	4.063201E+01	6.112930E+01	1.620515E+01	1.033613E+02	4.449828E+01
f_{25}	Mean	2.356641E+02	2.364542E+02	2.553110E+02	2.330344E+02	2.304385E+02	2.376740E+02	2.320580E+02	2.391211E+02	2.382094E+02	2.319208E+02	2.323626E+02	2.303257E+02	2.340882E+02	2.312181E+02	2.290101E+02	2.330503E+02
	Rank[WRST]	12	11	16	8	13	13	6	15	14	5	7	2	10	9	1	9
	Min	2.279653E+02	2.268124E+02	2.332292E+02	2.272277E+02	2.269690E+02	2.272614E+02	2.278198E+02	2.277691E+02	2.278198E+02	2.277691E+02	2.264994E+02	2.276472E+02	2.264655E+02	2.264655E+02	2.261464E+02	2.2811970E+02
	Rank	13	4	16	7	5	8	13	15	11	3	6	10	2	1	14	
	Std Dev	2.351392E+02	2.348127E+02	2.498563E+02	2.330127E+02	2.298468E+02	2.373715E+02	2.319682E+02	2.383664E+02	2.358813E+02	2.318508E+02	2.321710E+02	2.300364E+02	2.341604E+02	2.310657E+02	2.286223E+02	2.331392E+02
f_{25}	Mean	3.945080E+02	3.946575E+02	4.089965E+02	3.945142E+02	4.314690E+02	3.945142E+02	3.945080E+02	4.074356E+02	4.185557E+02	3.945062E+02	3.945080E+02	3.937787E+02	3.945080E+02	4.426598E+02	3.945131E+02	4.074356E+02
	Rank	6	10	13	8	15	9	3	12	14	5	3	1	2	16	7	11
	Min	2.005847E+01	1.802958E+00	2.218704E+05	5.739721E-02	9.572885E+02	2.032071E-01	1.144255E+05	1.601916E+05	4.952873E+04	6.795748E-03	4.081873E+01	3.997189E-02	1.941847E+05	1.097213E+05	1.307490E+01	

Table 6 (continued).

	Criteria	UMOEAs-IV	TSMS-DE	AuDE	NLPSR-BO	mLSHADE	HAPI-DE	FDHD-DE	DOPSA-DE	DACDE	CEDE	ACD-DE	UMOEAs-III	IDE-EDA	LSHADE-SPACMA	LSHADE-collgSim	EBOwithCMAR	
f_{11}	Mean	3.59259E+00	4.613012E+00	3.681989E+01	3.717569E+00	4.316641E+01	1.518557E+01	4.426671E+00	2.049763E+01	1.093834E+01	7.044184E+00	6.387438E+00	5.733354E+00	1.628814E+00	1.909478E+01	1.800660E+01	8.749727E+00	
	Rank WRST	2	5 >	15 >	3 >	16 >	11 >	4 >	10 >	8 >	6 >	13 >	12 >	9 >	12 >	9 >	9 >	
	Min	0.000000E+00	4.547474E-13	9.495948E+00	9.692940E-10	2.487933E+01	1.574466E+00	8.835793E-01	1.989918E+00	2.984877E+00	4.547474E-13	6.36463E-12	2.517027E-10	0.000000E+00	1.364290E+00	1.955414E-11	1.955414E-11	
	Rank	1	4	15	9	16	12	14	10	13	14	4	6	8	1	7	1	7
	Median	9.949591E-01	2.984877E+00	2.785879E+01	1.990318E+00	3.98051E+01	8.954626E+00	2.394985E+00	1.194632E+00	8.954632E+00	1.989918E+00	2.984877E+00	3.979836E+00	9.949591E-01	5.975539E+00	4.409534E+00	2.984877E+00	
	(Std Dev)	(8.336868E+00)	(8.772519E+00)	(2.780964E+01)	(8.343576E+00)	(1.454782E+01)	(8.506187E+00)	(2.114932E+01)	(1.173040E+01)	(1.452796E+01)	(1.208871E+01)	(9.427726E+00)	(1.871472E+00)	(2.601501E+01)	(2.299363E+01)	(1.810388E+01)		
f_{12}	Mean	3.114679E+02	9.459603E+01	1.183857E+03	6.970734E+01	1.246468E+03	4.666752E+02	1.667638E+01	1.183196E+03	1.923020E+02	1.412450E+02	4.669630E+02	4.348432E+02	1.391699E+02	5.138575E+02	3.659626E+02	4.776311E+02	
	Rank WRST	7	3 <	15 >	2 <	16 >	10 >	1 <	14 >	6 <	5 <	11 >	9 >	4 <	13 >	8 >	12 >	
	Min	1.124432E+01	4.163614E-01	4.760671E+02	6.354192E-01	5.612224E+02	7.449948E+00	2.086245E-01	4.702344E+02	3.935786E+00	3.794370E+00	1.496942E+01	7.775221E+00	4.141333E+00	7.866308E+00	8.1372E+01	1.876366E+01	
	Rank	10	2	15	3	7	1	14	5	4	11	8	9	4	11	12	12	
	Median	3.074898E+02	1.228047E+02	1.151295E+03	1.701446E+01	1.275746E+03	4.381345E+02	4.141485E+00	1.109253E+03	1.315925E+02	1.297575E+02	4.744019E+02	4.184782E+02	1.317209E+02	5.458609E+02	3.539878E+02	4.663047E+02	
	(Std Dev)	(1.503979E+02)	(6.750319E+01)	(4.227730E+02)	(7.786998E+01)	(4.077238E+02)	(2.758210E+02)	(3.622236E+01)	(3.923423E+02)	(2.446421E+02)	(1.017964E+02)	(2.353448E+02)	(2.465649E+02)	(8.932518E+01)	(2.124537E+02)	(1.833439E+02)	(2.423921E+02)	
f_{13}	Mean	1.604012E+01	1.150139E+01	3.843727E+01	1.461816E+01	8.575941E+02	1.144730E+01	1.210134E+01	3.595875E+01	1.932197E+01	1.803849E+01	1.377113E+01	1.473962E+01	1.393696E+01	1.140195E+01	1.305282E+01	1.495917E+01	
	Rank WRST	1	3 <	15 >	8 <	16 >	2 <	4 <	13 >	12 >	6 >	9 >	7 >	1 <	5 <	10 >	11 >	
	Min	3.559627E-01	1.364242E-12	1.347484E+01	1.811607E+00	2.994992E+01	2.273737E-13	1.679209E-04	3.979836E+00	3.979836E+00	2.033719E+00	1.006371E+00	1.728215E-04	9.949591E-01	6.91270E-03	2.064722E+00	2.064722E+00	
	Rank	6	2	15	10	2	1	3	13	14	11	8	4	7	5	12	12	
	Median	1.659377E-01	1.534693E+01	3.494397E+01	1.601236E+01	2.109256E+02	1.488295E+01	1.489951E+01	1.654538E+01	2.076104E+01	1.892938E+01	1.654538E+01	1.550980E+01	1.517932E+01	1.459621E+01	1.296655E+01	1.296655E+01	
	(Std Dev)	(5.978976E+00)	(6.005994E+00)	(2.515682E+01)	(4.662825E+00)	(1.017453E+03)	(6.366610E+00)	(5.897931E+00)	(1.373254E+01)	(6.530837E+00)	(3.300539E+00)	(6.819929E+00)	(1.016571E+01)	(7.159182E+00)	(6.855734E+00)	(7.776078E+00)	(5.427197E+00)	
f_{14}	Mean	1.876275E+01	2.047010E+01	5.804228E+01	2.138870E+01	7.865131E+01	2.517128E+01	2.012249E+01	2.569137E+01	8.937498E+00	1.890981E+01	1.996564E+01	1.975797E+01	2.079650E+01	2.308102E+01	2.098623E+01	2.246437E+01	
	Rank WRST	2	7 >	15 >	10 >	16 >	13 >	6 >	14 >	1 <	3 >	5 >	4 >	8 >	12 >	9 >	11 >	
	Min	2.273737E-13	2.521497E-02	2.994958E+01	1.996575E+00	5.880551E+01	1.61347E-06	2.984877E+00	2.984877E+00	6.509077E-03	1.32760E+00	2.001005E+01	3.05781E+00	2.001005E+01	3.629465E+00	9.899340E+00	9.899340E+00	
	Rank	1	4	15	7	16	10	2	8	14	3	6	13	5	14	13	13	
	Median	2.002381E+01	2.307202E+01	5.194154E+01	2.216395E+01	7.771713E+01	2.208065E+01	2.201641E+01	2.602003E+01	6.967131E+00	2.109655E+01	2.298776E+01	2.299736E+01	2.117576E+01	2.301999E+01	2.207499E+01	2.298205E+01	
	(Std Dev)	(5.255051E+00)	(7.372499E+00)	(2.312272E+01)	(4.067276E+00)	(1.097697E+01)	(1.661277E+01)	(6.199804E+00)	(7.279608E+00)	(6.671400E+00)	(6.457328E+00)	(7.550138E+00)	(7.195703E+00)	(2.974169E+00)	(1.412073E+00)	(4.413045E+00)	(2.200732E+00)	
f_{15}	Mean	1.332665E+00	1.153407E+00	4.243027E+01	1.121695E+00	1.555241E+02	1.743081E+00	7.729039E-01	1.548733E+01	5.127040E+00	1.159325E+00	1.906972E+00	2.797080E+00	8.867695E-01	4.843480E+00	2.962050E+00	3.598005E+00	
	Rank WRST	6	4 >	15 >	3 <	16 >	16 >	7 >	14 >	15 >	5 >	8 >	9 >	2 >	12 >	10 >	11 >	
	Min	1.584167E-01	2.850925E-01	5.580650E+00	4.553470E-01	5.224958E+00	2.227566E-01	3.747365E-01	3.298723E+00	1.457884E+00	3.899885E-01	3.342768E-01	2.519255E-01	3.878905E-01	2.015198E-01	1.891718E-01	4.225194E-01	
	Rank	1	6	15	12	16	4	9	14	13	7	8	10	3	11	5	2	
	Median	5.000002E-01	1.338900E+00	2.933125E+01	7.526269E-01	1.526389E+02	1.444442E+00	5.471843E-01	1.228951E+00	4.472162E+00	6.093868E-01	1.700529E+00	2.660874E+00	5.594294E-01	4.387941E+00	2.596273E+00	3.362090E+00	
	(Std Dev)	(1.997724E+00)	(6.332058E-01)	(3.525023E+01)	(6.625478E-01)	(4.234865E+01)	(1.011631E+00)	(4.825388E-01)	(1.162072E+01)	(2.603832E+00)	(1.107921E+00)	(1.024430E+00)	(1.795250E+00)	(4.949749E-01)	(2.400606E+00)	(2.026691E+00)	(2.314745E+00)	
f_{16}	Mean	1.017856E+01	1.538597E+02	7.493828E+02	3.231192E+01	1.413412E+02	7.915588E+01	2.591474E+02	3.115604E+02	3.002826E+01	1.670081E+02	5.761790E+01	2.382078E+01	5.564701E+01	1.855959E+01	7.645004E+01	7.645004E+01	
	Rank WRST	1	12 >	16 >	5 >	11 >	10 >	9 >	14 >	15 >	4 >	13 >	3 >	6 >	2 >	8 >	8 >	
	Min	2.492696E+00	1.713607E+01	1.993296E+01	3.070338E+00	3.065872E+00	3.011878E+00	3.1241037E+00	3.521603E+00	1.66677E+01	3.503538E+00	1.920496E+01	3.139538E+00	5.264062E+00	4.435728E+00	4.436726E+00	4.435728E+00	
	Rank	3	14	16	6	5	4	12	9	14	13	8	15	7	11	2	1	
	Median	1.020326E+01	1.584922E+02	5.898267E+02	1.705425E+01	1.325093E+02	1.221850E+02	3.917962E+01	2.719605E+02	2.787649E+02	1.392988E+01	1.711299E+02	2.191329E+01	1.944059E+01	1.489658E+01	2.733000E+01	2.733000E+01	
	(Std Dev)	(3.412236E+00)	(9.756966E+01)	(4.324811E+02)	(4.848326E+01)	(1.115757E+02)	(1.045609E+02)	(7.759335E+01)	(1.341102E+02)	(1.380808E+02)	(4.214202E+01)	(1.043417E+02)	(6.682429E+01)	(3.592978E+01)	(7.522888E+01)	(1.843980E+01)	(8.096419E+01)	
f_{17}	Mean	2.936465E+01	2.777443E+01	1.490872E+02	2.943800E+01	3.821850E+01	2.110847E+01	2.886745E+01	1.116576E+01	1.729045E+01	3.085146E+01	3.384195E+01	3.564689E+01	3.687667E+01	2.921042E+01	2.802054E+01	3.144078E+01	
	Rank WRST	7	3 >	16 >	8 >	14 >	2 <	5 >	15 >	1 <	9 >	12 >	12 >	6 >	10 >	6 >	11 >	
	Min	1.463376E+01	1.250895E+01	3.707047E+01	8.781290E+00	2.142344E+01	7.488124E+00	1.611251E+01	1.418448E+01	3.464355E+00	8.985798E+00	1.135182E+01	1.363448E+01	1.092477E+01	1.624374E+01	8.893932E+00	8.893932E+00	
	Rank	11	8	16	3	12	10	12	13	5	7	9	15	6	13	4	4	
	Median	2.966575E+01	2.872831E+01	1.378036E+02	3.136554E+01	3.753109E+01	2.110971E+01	3.040923E+01	3.978957E+01	1.255249E+01	3.151693E+01	3.618877E+01	3.614671E+01	2.844661E+01	2.844661E+01	3.265378E+01	3.265378E+01	
	(Std Dev)	(5.261450E+00)	(7.301939E+00)	(8.557402E+01)	(8.384986E+00)	(7.309451E+00)	(8.025067E+00)	(6.149902E+00)	(1.110558E+01)	(1.164050E+01)	(6.461664E+00)	(7.018546E+00)	(5.780626E+00)	(8.892253E+00)	(5.751906E+00)	(6.157315E+00)	(6.157315E+00)	
f_{18}	Mean	2.095195E+01	1.987825E+01	5.942958E+01	2.081054E+01	1.327539E+02	2.053147E+01	2.057100E+01	7.922604E+01	1.601943E+01	1.952268E+01	2.064958E+01	2.185051E+01	2.073802E+01	2.386548E+01	2.107573E+01	2.243012E+01	
	Rank WRST	9	3 <	14 >	8 >	16 >	4 <	5 <	15 ></									

Table 6 (continued).

		UMOEAs-IV	TSMS-DE	AuDE	NLPSS-JSO	mLSHADE	HAPI-DE	FDHD-DE	DOPSA-DE	DACDE	CELDE	ACD-DE	UMOEAs-III	IDE-EDA	LSHADE-SPACMA	LSHADE-cnffsin	EBOwithCMAR
f ₁	Mean	1.995789E+02	2.112468E+02	2.562499E+02	2.082090E+02	2.145885E+02	2.063149E+02	2.080818E+02	2.152088E+02	2.329843E+02	2.081092E+02	2.074638E+02	1.960017E+02	2.062898E+02	2.069755E+02	2.129357E+02	2.030192E+02
	Rank(WRST)	2	11	16	10	13	10	10	10	9	8	10	1	10	12	4	12
	Min	1.000000E+02	2.058881E+02	2.216204E+02	2.018585E+02	2.074916E+02	2.000356E+02	2.053822E+02	2.067658E+02	2.161528E+02	2.039121E+02	2.035325E+02	1.000000E+02	2.000078E+02	2.000000E+02	2.057329E+02	2.000000E+02
	Max	1	12	16	7	14	6	10	15	15	9	8	2	5	3	11	4
	Median	2.017386E+02	2.117914E+02	2.449953E+02	2.076516E+02	2.143025E+02	2.058641E+02	2.079072E+02	2.148885E+02	2.328896E+02	2.079055E+02	2.072245E+02	2.018793E+02	2.057706E+02	2.061894E+02	2.134718E+02	2.029664E+02
	Rank	1	11	16	8	13	5	10	14	15	9	7	2	4	6	12	3
	(Std Dev)	(1.428324E+01)	(2.353671E+00)	(3.534656E+01)	(3.318735E+00)	(3.024450E+00)	(3.285479E+00)	(1.456132E+00)	(2.892874E+00)	(7.517850E+00)	(1.717936E+00)	(1.482705E+00)	(2.428192E+01)	(2.785934E+00)	(3.916077E+00)	(2.407288E+00)	(1.521209E+00)
f ₂	Mean	1.0000000000000005E+02	1.0000000000000005E+02	1.000971426835287E+02	1.0000000000000008E+02	1.0000000000000005E+02	1.000963358385014E+02	1.0000000000000005E+02	1.381361977206217E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000004E+02	1.0000000000000001E+02	1.0000000000000005E+02	1.0000000000000005E+02
	Rank(WRST)	11	3	15	13	14	14	16	3	3	3	3	11	2	1	3	1
	Min	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000000E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000000E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02
	Max	4	4	4	4	4	4	4	1	1	1	4	4	1	1	4	4
	Median	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000009E+02	1.0000000000000009E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000000E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000000E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02
	Rank	3	3	15	15	3	3	3	1	3	3	3	3	3	1	3	3
	(Std Dev)	(1.236753E-13)	(1.435226E-14)	(4.856395E-01)	(2.195198E-13)	(8.922335E-14)	(4.815833E-01)	(1.435226E-14)	(2.723469E+02)	(1.435226E-14)	(6.390146E-14)	(1.435226E-14)	(1.483793E-13)	(6.390146E-14)	(1.670244E-13)	(1.435226E-14)	(1.435226E-14)
f ₃	Mean	3.451587E+02	3.458528E+02	4.022600E+02	3.529243E+02	3.600166E+02	3.474969E+02	3.444022E+02	3.622775E+02	3.786369E+02	3.492529E+02	3.454789E+02	3.480450E+02	3.507599E+02	3.550981E+02	3.541484E+02	3.516605E+02
	Rank(WRST)	2	16	16	13	13	13	1	14	15	15	3	6	12	11	11	9
	Min	3.359359E+02	3.341498E+02	3.642035E+02	3.448642E+02	3.487739E+02	3.350671E+02	3.372593E+02	3.494309E+02	3.603857E+02	3.449796E+02	3.360129E+02	3.391635E+02	3.425361E+02	3.414236E+02	3.468866E+02	3.433274E+02
	Max	3	1	16	10	10	2	5	14	15	11	4	8	7	12	9	10
	Median	3.454965E+02	3.457970E+02	3.985278E+02	3.522770E+02	3.603276E+02	3.480646E+02	3.448312E+02	3.608673E+02	3.787148E+02	3.492061E+02	3.452823E+02	3.480101E+02	3.501445E+02	3.552933E+02	3.541867E+02	3.516166E+02
	Rank	3	4	16	10	13	5	1	14	15	7	2	6	8	12	11	9
	(Std Dev)	(4.181433E+00)	(4.357177E+00)	(2.690277E+01)	(4.259646E+00)	(5.403883E+00)	(7.685759E+00)	(2.888883E+00)	(6.306531E+00)	(8.937854E+00)	(2.457521E+00)	(4.573918E+00)	(3.176047E+00)	(3.846514E+00)	(3.521266E+00)	(3.718818E+00)	(3.319014E+00)
f ₄	Mean	4.194450E+02	4.203151E+02	4.787190E+02	4.279516E+02	4.261428E+02	4.190358E+02	4.209488E+02	4.333387E+02	4.534655E+02	4.250215E+02	4.230031E+02	4.189896E+02	4.261649E+02	4.292998E+02	4.287495E+02	4.161087E+02
	Rank(WRST)	4	5	16	11	11	11	11	3	14	10	10	10	10	10	10	1
	Min	2.000000E+02	4.027668E+02	4.235733E+02	4.216338E+02	3.995822E+02	4.216335E+02	4.217648E+02	4.246335E+02	4.335740E+02	4.167623E+02	4.212019E+02	4.160841E+02	4.214048E+02	4.240813E+02	4.230114E+02	4.230114E+02
	Max	1	5	15	11	6	4	7	14	16	9	8	3	10	13	12	2
	Median	4.233712E+02	4.209950E+02	4.636800E+02	4.273847E+02	4.265097E+02	4.231077E+02	4.331077E+02	4.248495E+02	4.399566E+02	4.232434E+02	4.257358E+02	4.294344E+02	4.283828E+02	4.249315E+02	4.249315E+02	4.249315E+02
	Rank	5	1	16	11	10	2	3	14	15	7	6	4	9	13	12	8
	(Std Dev)	(3.142072E+01)	(4.682203E+00)	(4.895771E+01)	(4.027032E+00)	(4.172427E+00)	(8.918113E+00)	(2.354443E+00)	(4.082548E+00)	(7.921812E+00)	(1.675026E+00)	(2.873830E+00)	(3.123066E+01)	(2.657873E+00)	(2.932700E+00)	(2.705942E+00)	(4.412095E+01)
f ₅	Mean	3.826708E+02	3.861044E+02	3.865766E+02	3.866967E+02	3.824656E+02	3.875678E+02	3.866958E+02	3.869519E+02	3.867315E+02	3.866825E+02	3.867306E+02	3.853118E+02	3.866945E+02	3.867011E+02	3.866774E+02	3.864962E+02
	Rank(WRST)	2	6	6	11	1	10	16	10	15	8	13	3	9	12	7	5
	Min	3.786273E+02	3.822992E+02	3.834368E+02	3.866878E+02	3.791616E+02	3.834325E+02	3.866878E+02	3.867129E+02	3.866913E+02	3.866601E+02	3.866913E+02	3.812092E+02	3.866913E+02	3.866878E+02	3.866611E+02	3.835336E+02
	Max	1	4	6	10	2	5	16	16	14	14	14	10	10	9	7	10
	Median	3.818616E+02	3.866913E+02	3.868966E+02	3.869434E+02	3.813993E+02	3.867267E+02	3.866930E+02	3.867973E+02	3.867267E+02	3.866813E+02	3.867327E+02	3.862192E+02	3.866928E+02	3.866989E+02	3.866774E+02	3.867372E+02
	Rank	2	6	15	9	1	11	8	16	11	5	13	7	10	4	13	13
	(Std Dev)	(3.319886E+00)	(1.113755E+00)	(1.228562E+00)	(6.645300E-03)	(2.804607E+00)	(2.835224E+00)	(7.344592E-03)	(1.399629E-01)	(2.402908E-02)	(1.363812E-02)	(1.709380E-02)	(1.594876E+00)	(4.975633E-03)	(9.580117E-03)	(8.690137E-03)	(8.326968E-01)
f ₆	Mean	5.340853E+02	8.573460E+02	1.347657E+03	9.286974E+02	1.030296E+03	7.442904E+02	8.540374E+02	1.083717E+03	1.282566E+03	8.887745E+02	8.819074E+02	6.337619E+02	8.922444E+02	9.483455E+02	9.492566E+02	6.255960E+02
	Rank(WRST)	1	6	16	10	15	14	14	15	15	10	10	10	10	11	12	14
	Min	2.000000E+02	6.522431E+02	2.000000E+02	2.000000E+02	2.000000E+02	7.195973E+02	8.537617E+02	1.088102E+03	1.088102E+03	8.058991E+02	2.000000E+02	8.193208E+02	8.613777E+02	8.613777E+02	2.000000E+02	2.000000E+02
	Max	3	6	2	12	15	7	16	8	16	8	9	3	11	14	10	5
	Median	3.000000E+02	8.564280E+02	1.458899E+03	9.276115E+02	1.028394E+03	8.993551E+02	1.078996E+03	8.863513E+02	1.276209E+03	8.863513E+02	8.700641E+02	8.902333E+02	9.570726E+02	9.570726E+02	9.570726E+02	9.570726E+02
	Rank	1	4	16	10	13	9	3	14	15	7	6	2	8	11	12	8
	(Std Dev)	(3.227346E+02)	(5.598272E+01)	(4.341885E+02)	(4.634462E+01)	(6.646043E+01)	(3.171230E+02)	(4.591522E+01)	(8.002318E+01)	(9.292044E+01)	(3.343494E+01)	(3.164876E+01)	(3.164876E+02)	(3.565231E+01)	(4.465478E+01)	(3.178055E+02)	(3.178055E+02)
f ₇	Mean	4.776061E+02	4.850042E+02	5.082076E+02	4.903099E+02	4.946292E+02	4.939536E+02	4.882202E+02	5.035298E+02	4.901478E+02	5.026493E+02	4.971972E+02	4.992404E+02	4.943492E+02	5.054322E+02	5.032490E+02	5.049213E+02
	Rank(WRST)	1	2	16	5	8	10	13	13	13	11	9	10	10	15	12	14
	Min	4.432255E+02	4.712951E+02	4.761055E+02	4.676310E+02	4.507808E+02	4.755771E+02	4.733150E+02	4.652504E+02	4.652504E+02	4.798928E+02	4.869435E+02	4.724599E+02	4.913931E+02	4.946232E+02	4.932344E+02	4.932344E+02
	Max	1	5	8	4	2	9	7	12	3	13	10	11	6	14	16	15
	Median	4.787306E+02	4.854582E+02	5.097970E+02	4.911686E+02	5.000070E+02	4.93										

Table 7

Performance of DE Variants on 50D CEC2017 Benchmarks (51 Runs): Statistical Results and Wilcoxon Tests.

	Criteria	UMOEa-IV	TSM-DE	AuDE	NLPSP-BO	mLSHADE	HAP-DE	FOND-DE	DDPSA-DE	DACDE	CEDE	ACD-DE	UMOEa-III	IDE-EDA	LSHADE-SPACMA	LSHADE-cdfpdm	EBOWithCMAR
f_1	Mean	1.448950E-14	1.47681E-14	5.632708E-07	1.311606E-11	1.231141E-09	1.532543E-14	1.839052E-14	2.312747E-14	4.794124E-09	1.309628E-14	1.560408E-14	1.421085E-14	1.309628E-14	0.000000E+00	0.000000E+00	7.995977E-12
	Rank WRST	6	7	16	13	14	8	10	11	15	3	9	5	3	1	1	12
	Min	1.421085E-14	1.421085E-14	6.706117E-09	5.684327E-14	1.64279E-14	1.421085E-14	1.421085E-14	1.421085E-14	3.647926E-11	0.000000E+00	1.421085E-14	1.421085E-14	0.000000E+00	0.000000E+00	0.000000E+00	7.105427E-14
	Rank	5	5	16	13	12	5	5	5	15	1	5	5	1	1	1	13
	Median	1.421085E-14	1.421085E-14	1.295577E-07	2.827960E-12	1.738130E-10	1.421085E-14	1.421085E-14	2.842171E-14	9.863612E-10	1.421085E-14	1.421085E-14	1.421085E-14	0.000000E+00	0.000000E+00	0.000000E+00	2.145839E-12
	Rank	3	3	16	13	14	3	3	11	15	3	3	3	3	1	1	12
	(Std Dev)	(1.98991E-15)	(2.75885E-15)	(1.01765E-06)	(3.19641E-11)	(2.666271E-09)	(3.85859E-15)	(6.539537E-15)	(8.507916E-15)	(1.03301E-08)	(3.85859E-15)	(4.267998E-15)	(0.000000E+00)	(4.792356E-15)	(0.000000E+00)	(0.000000E+00)	(1.84955E-11)
	Mean	7.356207E-14	6.681223E-14	6.27231E+36	5.294118E-01	3.284610E-08	2.745098E-01	1.839052E-14	1.011765E+01	3.779155E+05	0.000000E+00	3.416055E-13	0.000000E+00	1.839052E-14	3.343731E-14	1.313725E+00	5.82875E-10
f_2	Rank WRST	7	6	12	12	10	11	3	14	15	1	8	1	3	13	9	14
	Min	0.000000E+00	0.000000E+00	2.382690E-04	0.000000E+00	0.694822E-13	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	3.416055E-13
	Rank	1	1	16	1	15	1	1	1	1	1	1	1	1	1	1	14
	Median	5.68432E-14	2.842171E-14	6.930531E+04	0.000000E+00	1.091109E-10	0.000000E+00	2.842171E-14	1.000000E+00	1.360000E+02	0.000000E+00	2.842171E-14	0.000000E+00	0.000000E+00	2.842171E-14	1.000000E+00	3.174705E-11
	Rank	10	6	16	1	12	1	6	1	15	1	6	1	6	13	1	14
	(Std Dev)	(5.689355E-14)	(1.106366E-13)	(3.420051E+37)	(9.241849E-01)	(2.029291E-07)	(5.321064E-01)	(1.589943E-14)	(3.780934E+01)	(2.665531E+06)	(0.000000E+00)	(2.121075E-12)	(0.000000E+00)	(4.646703E-14)	(2.458495E-14)	(1.618520E+00)	(2.29840E-09)
	Mean	6.464546E-14	1.415513E-13	9.574107E+04	1.375388E-12	2.835272E-10	1.170306E-13	1.237180E-13	1.237180E-13	9.160552E-05	1.159160E-13	5.68432E-14	8.801575E-14	0.000000E+00	0.000000E+00	2.28535E-09	1.37372E-09
	f_3	Rank WRST	4	11	16	12	13	8	9	9	15	7	6	3	1	1	1
Min		5.68432E-14	5.68432E-14	7.129423E-05	6.252776E-13	4.376943E-12	5.68432E-14	5.68432E-14	5.68432E-14	9.678081E-08	5.68432E-14	0.000000E+00	5.68432E-14	0.000000E+00	0.000000E+00	0.000000E+00	3.009291E-10
Rank		4	4	16	12	13	4	4	4	15	4	4	1	4	1	1	14
Median		5.68432E-14	1.13668E-13	1.205176E+05	1.193721E-12	7.088374E-11	1.13668E-13	1.13668E-13	2.534521E-05	1.13668E-13	5.68432E-14	5.68432E-14	1.13668E-13	0.000000E+00	0.000000E+00	1.672447E-09	1.672447E-09
Rank		3	5	16	12	13	5	5	5	15	5	3	5	1	1	1	14
(Std Dev)		(1.975538E-14)	(4.457416E-14)	(5.738628E+04)	(6.056446E-13)	(6.441988E-10)	(3.297264E-14)	(4.052403E-14)	(4.646703E-14)	(1.575572E-04)	(2.32126E-14)	(2.843285E-14)	(1.13668E-14)	(3.074536E-14)	(0.000000E+00)	(0.000000E+00)	(1.536779E-09)
Mean		3.443187E+01	4.582001E+01	6.939339E+01	4.367013E+01	7.468657E+00	8.850237E+00	3.754761E+01	4.312838E+01	3.782963E+01	4.487229E+01	6.103411E+01	4.383924E+01	5.975933E+01	3.338679E+01	4.883827E+01	4.243879E+01
f_4		Rank WRST	4	12	16	10	1	2	10	7	7	14	5	5	3	14	8
	Min	5.68432E-14	3.669696E+00	2.108295E-02	8.021118E+00	1.13668E-13	5.68432E-14	1.92145E-03	5.68432E-14	4.738500E-02	7.933753E-04	5.329401E-03	5.68432E-14	3.876969E+00	1.731349E-02	7.854795E+00	3.012701E-12
	Rank	1	13	11	16	5	1	8	12	7	9	1	10	14	10	6	6
	Median	2.773459E+01	2.851271E+01	7.005018E+01	6.195127E+01	3.609619E-12	1.705303E-13	2.851271E+01	2.851271E+01	2.851271E+01	2.604938E+01	2.096550E+01	2.851271E+01	2.851271E+01	2.598851E+01	2.851271E+01	2.851271E+01
	Rank	6	10	16	14	2	10	7	15	5	10	3	7	4	7	4	6
	(Std Dev)	(2.903400E+01)	(3.482037E+01)	(4.374866E+01)	(3.837108E+01)	(2.635400E+01)	(2.891167E+01)	(2.955812E+01)	(4.636969E+01)	(2.691863E+01)	(4.433107E+01)	(5.079180E+01)	(3.012148E+01)	(4.681470E+01)	(4.106428E+01)	(1.844977E+01)	(3.054338E+01)
	Mean	2.809296E+00	2.549979E+01	9.049979E+01	2.509930E+01	2.610732E+01	1.780656E+01	1.474088E+01	3.49167E+01	6.763222E+01	1.517257E+01	1.262917E+01	6.268019E+00	5.755155E+00	1.616774E+01	2.670136E+01	8.174027E+00
	f_5	Rank WRST	1	11	16	5	12	10	7	14	15	8	6	3	9	10	13
Min		3.410605E-12	1.469895E+01	3.283363E+01	4.974796E+00	1.392943E+01	1.216091E+01	9.392715E+00	3.880454E+01	8.954669E+00	7.572697E+00	2.984877E+00	6.266906E+00	1.989918E+00	1.169461E+01	1.989918E+00	1.989918E+00
Rank		1	13	15	5	12	11	9	14	16	8	7	4	6	2	10	3
Median		1.989918E+00	2.607243E+01	8.656130E+01	1.193951E+01	2.586849E+01	1.720241E+01	1.497207E+01	3.482356E+01	7.064200E+01	1.493335E+01	1.267136E+01	7.959672E+00	1.584392E+01	4.974795E+00	2.964200E+01	7.959672E+00
Rank		1	12	16	5	11	10	8	15	7	6	4	9	2	13	3	3
(Std Dev)		(1.524671E+00)	(4.370052E+00)	(4.297009E+01)	(3.871220E+00)	(4.769827E+00)	(3.479312E+00)	(2.535869E+00)	(4.716193E+00)	(1.141786E+01)	(3.081175E+00)	(1.899110E+00)	(2.203934E+00)	(3.733742E+00)	(2.227586E+00)	(7.515235E+00)	(2.708895E+00)
Mean		7.141879E-08	9.401924E-10	4.384436E-02	9.764362E-07	1.153634E-02	2.721366E-08	4.500837E-04	4.605081E-05	1.248936E-06	8.460594E-09	8.754123E-09	2.870899E-07	1.114577E-13	6.360357E-07	9.560784E-08	9.560784E-08
f_6		Rank WRST	7	2	16	11	15	6	10	12	12	3	10	4	1	10	8
	Min	2.273737E-13	0.000000E+00	5.218935E-03	0.000000E+00	4.794276E-08	0.000000E+00	1.559215E-09	1.917712E-07	5.286876E-07	1.13668E-13	0.000000E+00	2.273737E-13	1.13668E-13	0.000000E+00	0.000000E+00	0.000000E+00
	Rank	10	1	16	1	13	1	12	14	15	1	10	8	1	1	1	1
	Median	4.547474E-13	1.13668E-13	2.677338E-02	4.314852E-07	8.521000E-03	1.13668E-13	2.560456E-04	1.732869E-05	9.071276E-07	2.273737E-13	1.13668E-13	2.842171E-12	1.145344E-07	1.36868E-13	4.314852E-07	4.794276E-08
	Rank	6	1	16	10	15	1	14	13	12	5	1	7	9	11	8	1
	(Std Dev)	(2.084932E-07)	(6.711301E-09)	(3.777404E-02)	(1.731010E-06)	(1.055734E-02)	(7.610750E-08)	(5.466297E-04)	(1.395129E-04)	(8.893173E-07)	(1.665183E-08)	(2.987241E-08)	(2.081758E-08)	(3.822991E-07)	(1.591934E-14)	(5.585771E-07)	(1.415608E-07)
	Mean	5.609122E+01	7.053408E+01	1.384259E+02	6.138387E+01	7.589846E+01	6.494145E+01	8.038458E+01	1.160587E+02	6.745198E+01	6.342505E+01	5.688217E+01	6.966940E+01	5.698617E+01	7.643652E+01	5.813618E+01	5.813618E+01
	f_7	Rank WRST	1	11	16	5	12	7	8	15	10	6	2	10	3	13	4
Min		5.41172E+01	6.269300E+01	8.708190E+01	6.732059E+01	5.945870E+01	6.046826E+01	7.079543E+01	8.317245E+01	6.126040E+01	6.020725E+01	5.511462E+01	6.284519E+01	6.284519E+01	6.759472E+01	5.526783E+01	5.526783E+01
Rank		1	10	16	5	12	6	8	14	15	9	7	3	11	2	13	4
Median		5.61004E+01	7.127219E+01	1.200007E+02	6.110144E+01	7.362103E+01	6.406764E+01	6.517517E+01	1.164602E+02	6.682700E+01	6.327910E+01	5.690197E+01	6.945423E+01	5.686179E+01	7.399900E+01	5.816335E+01	5.816335E+01
Rank		1	11	16	5	12	7	8	14	15	9	6	3	10	2	13	4
(Std Dev)		(1.002291E+00)	(4.023517E+00)	(6.98682E+01)	(2.794988E+00)	(7.359875E+00)	(2.726755E+00)	(1.910999E+00)	(5.866352E+00)	(1.265784E+01)	(3.338305E+00)	(2.036277E+00)	(8.374368E-01)	(3.466146E+00)	(1.243754E+00)	(6.055426E+00)	(

Table 7 (continued).

f_n	Criteria	UMOEAs-IV	TSMs-DE	AuDE	NLPsr- β SO	mLSHADE	HAPI-DE	FDHD-DE	DOPSA-DE	DACDE	CXDE	ACD-DE	UMOEAs-III	IDE-EDA	LSHADE-SPACMA	LSHADE-csflgpin	EBOwithCMAR
f_{12}	Mean	1.621733E+03	1.160232E+03	3.874637E+04	1.167988E+03	2.798873E+03	1.542897E+03	3.840425E+02	4.561945E+03	5.952050E+03	7.853336E+02	1.992997E+03	1.839664E+03	1.437609E+03	1.607862E+03	1.292722E+03	2.152114E+03
	Rank\WRST	9	3 <	16 >	4 <	13 >	7 =	1 <	14 >	15 >	2 <	11 >	10 >	6 <	8 =	5 <	12 >
	Min	8.895511E+02	3.849683E+02	4.299258E+03	4.848846E+02	5.137516E+02	3.816676E+02	1.404602E+02	1.650239E+03	8.899793E+02	2.628315E+02	5.888183E+02	5.429034E+02	8.031624E+02	5.957674E+02	9.258913E+02	9.258913E+02
	Rank	12	4	16	5	6	3	1	15	13	2	11	9	7	10	8	14
	Median	1.586410E+03	1.159506E+03	2.872502E+04	1.225216E+03	2.076088E+03	1.514398E+03	3.768603E+02	4.085983E+03	3.868220E+03	8.132541E+02	2.043569E+03	1.781051E+03	1.360770E+03	1.600239E+03	1.282923E+03	1.966613E+03
	Rank	8	3	16	4	13	7	1	15	14	2	12	10	6	9	5	11
	(Std Dev)	(3.759362E+02)	(3.331556E+02)	(2.979651E+04)	(3.381837E+02)	(2.667592E+03)	(5.449356E+02)	(1.932313E+02)	(3.223107E+03)	(5.518341E+03)	(2.676749E+02)	(5.035735E+02)	(5.440466E+02)	(4.457496E+02)	(4.230204E+02)	(3.613247E+02)	(8.089452E+02)
f_{13}	Mean	2.713304E+01	1.017292E+01	7.107987E+02	2.228348E+01	3.931731E+02	3.328818E+01	7.705859E+00	2.398066E+02	2.491003E+01	3.330444E+01	4.644927E+01	3.716403E+01	3.367727E+01	3.299988E+01	7.016506E+01	4.163187E+01
	Rank\WRST	5	2 <	16 >	3 =	15 >	7 >	1 <	4 >	8 =	12 >	10 >	9 >	6 >	13 >	11 >	11 >
	Min	9.958176E-01	2.305019E+00	8.532670E+01	6.420564E+00	3.880568E+01	4.974807E+00	2.147809E+00	4.895068E+01	1.065689E+01	2.801690E+00	6.003927E+00	3.170392E+00	4.974795E+00	5.442976E+00	6.998542E+00	6.998542E+00
	Rank	1	3	16	10	14	7	2	15	12	4	9	5	6	8	11	11
	Median	3.378421E+01	9.718285E+00	2.077540E+02	1.177811E+01	2.746465E+02	4.104888E+01	6.846637E+00	1.878201E+02	2.184466E+01	3.854246E+01	4.609290E+01	4.276589E+01	3.847822E+01	3.871260E+01	6.536676E+01	4.614034E+01
	Rank	5	2	15	3	16	9	14	4	7	11	10	6	8	13	12	12
	(Std Dev)	(1.887756E+01)	(6.266817E+00)	(1.221373E+03)	(1.780004E+01)	(4.156057E+02)	(1.890092E+01)	(5.777284E+00)	(1.933834E+02)	(1.051376E+01)	(2.189795E+01)	(2.811936E+01)	(2.204075E+01)	(2.553690E+01)	(1.856441E+01)	(3.589566E+01)	(1.935700E+01)
f_{14}	Mean	2.326610E+01	2.928532E+01	2.137036E+02	2.530002E+01	1.678260E+02	5.240401E+01	2.486795E+01	1.968546E+02	3.482372E+01	2.360157E+01	2.633436E+01	2.864161E+01	2.381225E+01	2.878860E+01	2.627415E+01	3.149620E+01
	Rank\WRST	1	10 >	16 >	5 >	14 >	13 >	4 >	15 >	12 >	2 >	7 >	8 >	3 >	9 >	6 >	11 >
	Min	1.999804E+01	2.435188E+01	7.679912E+01	2.217035E+01	1.274845E+02	2.313498E+01	2.146571E+01	1.013200E+01	2.122616E+01	2.143335E+01	2.036871E+01	3.025396E+01	2.300257E+01	2.273820E+01	2.147765E+01	2.147765E+01
	Rank	2	12	14	8	16	11	7	15	1	5	6	4	3	10	9	13
	Median	2.299670E+01	2.810531E+01	2.061982E+02	2.468106E+01	1.702379E+02	2.799092E+01	2.477627E+01	3.612485E+01	3.21726E+01	2.798029E+01	2.798029E+01	2.364155E+01	2.800824E+01	2.594526E+01	2.997524E+01	2.997524E+01
	Rank	1	11	16	4	14	9	5	15	13	2	7	8	3	10	6	12
	(Std Dev)	(1.691083E+00)	(2.54567E+00)	(7.592421E+01)	(2.497090E+00)	(1.675258E+01)	(5.137079E+01)	(1.934293E+00)	(5.942514E+01)	(7.747161E+00)	(1.527593E+00)	(2.92603E+00)	(3.730212E+00)	(1.935615E+00)	(3.417551E+00)	(2.437742E+00)	(5.048594E+00)
f_{15}	Mean	2.183355E+01	2.241432E+01	1.666798E+02	2.087485E+01	3.935866E+02	2.697331E+01	1.847100E+01	2.724744E+02	2.943910E+02	1.882971E+01	2.724628E+01	2.953373E+01	2.186512E+01	3.060424E+01	2.538360E+01	3.038722E+01
	Rank\WRST	4	6 =	14 >	3 >	16 >	8 >	1 <	10 >	13 >	2 <	9 >	11 >	5 >	7 >	11 >	11 >
	Min	1.825554E+01	1.911322E+01	3.975535E+01	1.718526E+01	1.925862E+02	2.105836E+01	1.706971E+01	7.227934E+01	2.321578E+01	1.708378E+01	2.193674E+01	2.064611E+01	1.810632E+01	2.105020E+01	1.973860E+01	2.228478E+01
	Rank	5	6	14	3	16	10	1	13	2	11	8	4	9	7	12	12
	Median	2.139899E+01	2.203748E+01	1.516245E+02	2.063274E+01	3.900895E+02	2.626900E+01	1.816075E+01	2.906140E+02	2.919551E+01	1.905146E+01	2.631938E+01	2.850289E+01	2.143694E+01	3.033283E+01	2.486716E+01	2.871655E+01
	Rank	4	6	14	3	16	8	1	15	12	2	9	10	5	13	7	11
	(Std Dev)	(1.964073E+00)	(2.670037E+00)	(9.904818E+01)	(1.732783E+00)	(6.432027E+01)	(4.449842E+00)	(9.042724E+01)	(1.219809E+02)	(4.231524E+00)	(1.105538E+00)	(3.786926E+00)	(8.285713E+00)	(2.022046E+00)	(5.229939E+00)	(3.224762E+00)	(5.200239E+00)
f_{16}	Mean	1.372411E+02	3.248465E+02	1.607310E+03	3.365006E+02	3.655754E+02	2.958167E+02	3.278314E+02	6.590915E+02	6.527276E+02	3.195345E+02	3.597437E+02	3.513056E+02	2.989799E+02	4.224256E+02	2.760698E+02	4.066641E+02
	Rank\WRST	1	6 >	16 >	8 >	11 >	3 >	7 >	15 >	14 >	5 >	10 >	9 >	4 >	13 >	2 >	12 >
	Min	1.229585E+02	1.334508E+02	4.417087E+02	1.300386E+02	1.308124E+02	1.274496E+02	1.397177E+02	1.387101E+02	1.284736E+02	1.280177E+02	1.399989E+02	1.280192E+02	1.277075E+02	1.370992E+02	1.262478E+02	1.262478E+02
	Rank	1	11	16	8	10	5	15	12	9	7	14	6	4	3	13	2
	Median	1.280542E+02	3.144886E+02	1.266792E+03	3.475290E+02	3.656551E+02	2.642546E+02	3.287713E+02	7.077558E+02	6.167489E+02	3.473457E+02	3.648177E+02	3.579950E+02	2.688094E+02	4.014131E+02	2.676409E+02	3.671384E+02
	Rank	1	5	16	8	11	2	6	15	14	7	10	9	4	13	3	12
	(Std Dev)	(3.200835E+01)	(1.274563E+02)	(8.137284E+02)	(1.392574E+02)	(7.614750E+01)	(8.756523E+01)	(1.249754E+02)	(1.923550E+02)	(2.200424E+02)	(1.178038E+02)	(1.181329E+02)	(1.192786E+02)	(1.439348E+02)	(1.978343E+02)	(1.161709E+02)	(1.434911E+02)
f_{17}	Mean	1.676398E+02	3.224481E+02	1.226173E+03	2.273537E+02	3.911579E+02	3.067458E+02	2.542680E+02	5.101464E+02	4.598982E+02	2.367753E+02	2.918878E+02	2.985298E+02	2.923675E+02	2.662556E+02	2.149265E+02	3.024849E+02
	Rank\WRST	1	12 >	16 >	3 >	13 >	11 >	10 >	14 >	4 >	7 >	9 >	9 >	8 >	2 >	10 >	10 >
	Min	4.822723E+01	1.825800E+02	4.846511E+02	5.322207E+01	2.047227E+02	1.892275E+02	8.283900E+01	2.150971E+02	1.103636E+02	9.512665E+01	1.048756E+02	8.135234E+01	9.010528E+01	6.729880E+01	9.332247E+01	1.066808E+02
	Rank	1	12	16	2	14	13	5	15	11	8	9	4	6	3	7	10
	Median	1.842699E+02	3.035339E+02	1.108677E+03	2.286304E+02	3.867978E+02	2.803287E+02	2.529566E+02	4.975129E+02	4.663729E+02	2.303298E+02	2.759335E+02	2.738261E+02	2.588349E+02	2.377314E+02	2.377090E+02	2.788738E+02
	Rank	1	12	16	2	13	11	6	15	14	3	9	8	7	5	4	10
	(Std Dev)	(4.907653E+01)	(7.549502E+01)	(5.138626E+02)	(9.110394E+01)	(1.211219E+02)	(8.564217E+01)	(8.075967E+01)	(1.372722E+02)	(1.362805E+02)	(7.665456E+01)	(7.122462E+01)	(8.136036E+01)	(1.018548E+02)	(9.917010E+01)	(6.684888E+01)	(8.967798E+01)
f_{18}	Mean	2.599503E+01	2.229665E+01	1.076140E+02	2.228778E+01	1.390370E+02	2.482885E+01	2.085299E+01	1.101569E+02	3.145475E+01	2.189212E+01	2.770555E+01	2.799834E+01	2.325427E+01	3.244030E+01	2.438381E+01	3.322346E+01
	Rank\WRST	8	4 <	14 >	3 <	16 >	7 =	1 <	15 >	11 >	2 <	9 >	10 >	5 <	12 >	6 <	13 >
	Min	2.042563E+01	2.052929E+01	3.874454E+01	2.052333E+01	2.735812E+01	2.055708E+01	2.052235E+01	3.587992E+01	2.209102E+01	2.054878E+01	2.199464E+01	2.038696E+01	2.060482E+01	2.214715E+01	2.142464E+01	2.429981E+01
	Rank	2	5	16	4	14	7	3	15	11	6	10	1	12	9	13	13
	Median	2.550539E+01	2.204027E+01	9.130162E+01	2.219768E+01	1.261213E+02	2.449451E+01	2.069438E+01	8.071581E+01	2.860031E+01	2.177547E+01	2.650125E+01	2.826534E+01	2.293994E+01	3.053550E+01	2.445331E+01	3.223799E+01
	Rank	8	3	15	4	16	7	1	14	11	2	9	10	5	12	6	13
	(Std Dev)	(3.113726E+00)	(1.193692E+00)														

Table 7 (continued).

		UMOEAs-IV	TSMs-DE	AutDE	NLPSS-JSO	mLSHADE	HAPI-DE	FHDD-DE	DOPSA-DE	DACDE	CELDE	ACD-DE	UMOEAs-III	IDE-EDA	LSHADE-SPACMA	LSHADE-cnfgsin	EBOwithCMAR
f_{21}	Mean	2.046774E+02	2.268524E+02	2.844048E+02	2.125284E+02	2.269897E+02	2.195756E+02	2.167081E+02	2.350285E+02	2.715648E+02	2.169707E+02	2.142213E+02	2.104749E+02	2.167069E+02	2.154260E+02	2.264910E+02	2.127808E+02
	Rank(WRST)	1	12	1	8	1	8	1	14	1	14	1	2	1	1	1	1
	Min	2.000000E+02	2.113465E+02	2.421472E+02	2.030282E+02	2.184955E+02	2.115858E+02	2.110125E+02	2.216560E+02	2.482397E+02	2.060945E+02	2.068755E+02	2.042276E+02	2.078448E+02	2.029405E+02	2.122795E+02	2.050593E+02
	Max	10	10	15	3	3	11	9	14	16	6	7	4	8	2	12	5
	Median	2.049388E+02	2.268827E+02	2.818574E+02	2.127265E+02	2.266337E+02	2.185908E+02	2.168819E+02	2.249100E+02	2.715756E+02	2.170091E+02	2.141001E+02	2.097585E+02	2.169035E+02	2.093168E+02	2.268953E+02	2.124934E+02
	Std Dev	(2.287828E+00)	(6.161506E+00)	(2.398044E+01)	(3.754868E+00)	(3.484461E+00)	(4.308225E+00)	(2.645351E+00)	(5.159209E+00)	(1.247529E+01)	(3.333881E+00)	(2.379667E+00)	(3.365461E+00)	(3.949484E+00)	(1.050258E+01)	(7.070330E+00)	(4.486508E+00)
f_{22}	Mean	6.975856E+02	1.000000E+02	7.884433E+03	1.951154E+03	1.006014E+02	1.000000E+02	3.117769E+02	2.830672E+03	2.851409E+03	1.864626E+03	1.761143E+02	2.491535E+02	1.033541E+03	1.779710E+03	1.103159E+03	3.540778E+02
	Rank(WRST)	8	1	16	13	1	1	6	14	1	15	1	5	1	1	1	7
	Min	1.0000000000000009E+02	1.0000000000000005E+02	1.00000000000000177E+02	1.0000000000000009E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02	1.0000000000000005E+02
	Max	13	1	16	13	1	1	15	1	1	15	1	1	1	1	1	1
	Median	1.0000000000000014E+02	1.0000000000000005E+02	8.895571249542762E+03	2.653142705831165E+03	1.0000000000000009E+02	1.0000000000000005E+02	3.817738419767047E+03	3.605610320662439E+03	1.02956995425386E+03	1.0000000000000009E+02	1.0000000000000009E+02	1.0000000000000009E+02	1.0000000000000009E+02	1.0000000000000009E+02	1.0000000000000009E+02	1.0000000000000009E+02
	Std Dev	(9.698635E+02)	(1.889028E-13)	(3.594740E+03)	(1.730637E+03)	(1.339659E+00)	(8.532874E-07)	(7.807495E+02)	(1.816065E+03)	(1.785674E+03)	(1.842300E+03)	(4.839708E+02)	(7.526256E+02)	(1.801499E+03)	(1.998491E+03)	(1.568920E+03)	(8.834092E+02)
f_{23}	Mean	4.255519E+02	4.286330E+02	5.086686E+02	4.388129E+02	4.413818E+02	4.271783E+02	4.198069E+02	4.580789E+02	4.980553E+02	4.305474E+02	4.278042E+02	4.310076E+02	4.334032E+02	4.408116E+02	4.389088E+02	4.373202E+02
	Rank(WRST)	2	5	16	10	1	3	1	1	1	14	6	7	1	12	1	9
	Min	4.079194E+02	4.126502E+02	4.645363E+02	4.203732E+02	4.211247E+02	4.033141E+02	4.055305E+02	4.383527E+02	4.65681E+02	4.128632E+02	4.11725E+02	4.145031E+02	4.180668E+02	4.286995E+02	4.208912E+02	4.186025E+02
	Max	3	5	15	10	12	1	2	14	16	6	4	7	8	13	11	9
	Median	4.267748E+02	4.299420E+02	5.069153E+02	4.371741E+02	4.427929E+02	4.25261E+02	4.197468E+02	4.575259E+02	4.953737E+02	4.308107E+02	4.260357E+02	4.316541E+02	4.335902E+02	4.450501E+02	4.392888E+02	4.372129E+02
	Std Dev	(7.692094E+00)	(6.548956E+00)	(2.187850E+01)	(1.003456E+01)	(7.272816E+00)	(1.200295E+01)	(6.351109E+00)	(8.857756E+00)	(1.585837E+01)	(5.972821E+00)	(1.146078E+01)	(5.866892E+00)	(7.190833E+00)	(6.285906E+00)	(7.021152E+00)	(6.367471E+00)
f_{24}	Mean	5.016679E+02	4.993677E+02	6.044515E+02	5.070478E+02	5.191083E+02	4.911719E+02	4.998639E+02	5.297335E+02	5.743433E+02	5.059047E+02	5.024317E+02	5.034319E+02	5.071026E+02	5.134371E+02	5.127178E+02	5.067829E+02
	Rank(WRST)	4	2	13	9	1	3	1	1	1	13	5	6	10	11	1	1
	Min	4.993300E+02	4.860206E+02	5.380769E+02	4.982467E+02	5.003596E+02	4.686892E+02	4.926388E+02	5.130278E+02	5.97746E+02	4.980662E+02	4.889231E+02	4.960667E+02	4.99498E+02	5.002507E+02	5.021655E+02	5.002771E+02
	Max	5	2	15	7	12	1	4	16	9	16	3	3	8	11	13	10
	Median	5.010489E+02	4.989126E+02	5.756822E+02	5.071077E+02	5.184315E+02	4.906110E+02	5.282046E+02	5.543209E+02	5.743209E+02	5.027508E+02	5.029114E+02	5.117156E+02	5.067393E+02	5.117156E+02	5.066416E+02	5.066416E+02
	Std Dev	(3.144555E+00)	(6.259742E+00)	(9.057137E+01)	(4.695180E+00)	(9.558238E+00)	(3.674609E+00)	(6.527239E+00)	(9.172782E+00)	(1.535097E+01)	(3.509706E+00)	(6.211201E+00)	(4.589569E+00)	(7.528929E+00)	(6.523637E+00)	(2.521152E+00)	(2.521152E+00)
f_{25}	Mean	4.790390E+02	4.809905E+02	5.199838E+02	4.802383E+02	5.159662E+02	5.202280E+02	4.804707E+02	5.299156E+02	4.883455E+02	4.814529E+02	4.814263E+02	4.801808E+02	4.811566E+02	4.811107E+02	4.802628E+02	4.882382E+02
	Rank(WRST)	1	6	1	14	3	1	15	1	1	10	9	1	1	7	4	11
	Min	4.586002E+02	4.601000E+02	4.560387E+02	4.602307E+02	4.603735E+02	4.281261E+02	4.602307E+02	4.607012E+02	4.552095E+02	4.801724E+02	4.802250E+02	4.598740E+02	4.802307E+02	4.773573E+02	4.773218E+02	4.591265E+02
	Max	4	11	3	15	7	1	13	2	12	16	16	6	10	9	5	5
	Median	4.801561E+02	4.824350E+02	5.234920E+02	4.802350E+02	5.200319E+02	5.282569E+02	4.802350E+02	5.298268E+02	4.802734E+02	4.802088E+02	4.802812E+02	4.794621E+02	4.802435E+02	4.804256E+02	4.801840E+02	4.803123E+02
	Std Dev	(4.360133E+00)	(5.366676E+00)	(3.566669E+01)	(8.014665E-03)	(2.363566E+01)	(4.529643E+01)	(1.622630E+00)	(3.590317E+01)	(2.235061E+01)	(3.163818E+00)	(3.482478E+00)	(8.416542E+00)	(3.146722E+00)	(3.185721E+00)	(1.459008E+00)	(2.300136E+01)
f_{26}	Mean	8.291530E+02	1.083869E+03	1.967994E+03	1.112467E+03	1.465357E+03	1.103252E+03	9.982821E+02	1.439678E+03	1.835366E+03	1.073517E+03	1.054248E+03	8.200560E+02	1.086494E+03	1.130140E+03	1.223522E+03	6.600555E+02
	Rank(WRST)	3	7	1	10	1	1	1	1	1	15	1	8	1	12	1	1
	Min	3.000000E+02	9.291791E+02	1.483757E+03	9.750998E+02	1.255006E+03	8.581916E+02	4.976375E+02	1.191152E+03	1.504274E+03	6.7468223E+02	9.088995E+02	3.000000E+02	9.880870E+02	1.026326E+03	5.950511E+02	3.000000E+02
	Max	1	7	15	10	14	5	4	16	10	11	6	8	12	9	3	3
	Median	9.848215E+02	1.095643E+03	1.992254E+03	1.112632E+03	1.095245E+03	1.009088E+03	1.441368E+03	1.837427E+03	1.064333E+03	1.073475E+03	1.062276E+03	1.007266E+03	1.117920E+03	1.247281E+03	1.07281E+03	3.000001E+02
	Std Dev	(3.168878E+02)	(6.847803E+01)	(2.457680E+02)	(5.589796E+01)	(1.171455E+02)	(9.525082E+01)	(6.940263E+01)	(1.146736E+02)	(1.529247E+02)	(4.200221E+01)	(7.604514E+01)	(3.427873E+02)	(6.185527E+01)	(4.349466E+01)	(6.39175E+01)	(4.024750E+02)
f_{27}	Mean	5.008677E+02	4.950380E+02	5.675918E+02	4.977280E+02	6.463989E+02	5.175462E+02	5.013062E+02	5.388483E+02	5.043118E+02	5.190105E+02	5.285309E+02	5.173002E+02	5.098919E+02	5.338666E+02	5.271882E+02	5.254333E+02
	Rank(WRST)	3	1	15	1	16	8	4	4	5	9	12	7	6	13	1	1
	Min	4.774977E+02	4.752474E+02	5.183440E+02	4.786020E+02	5.041126E+02	4.923447E+02	4.785627E+02	5.168317E+02	4.637130E+02	5.022323E+02	5.123438E+02	4.993083E+02	4.938966E+02	5.147655E+02	5.076118E+02	5.090525E+02
	Max	3	2	16	5	10	11	4	15	1	13	8	7	7	14	11	12
	Median	5.012338E+02	4.941437E+02	5.547540E+02	4.974608E+02	6.565132E+02	5.173236E+02	5.025742E+02	5.362724E+02	5.190515E+02	5.278036E+02	5.167451E+02	5.103958E+02	5.326703E+02	5.264074E+02	5.240995E+02	5.240995E+02
	Std Dev	(8.998809E+00)	(1.382155E+01)	(4.202499E+01)	(1.013175E+01)	(4.658387E+01)	(1.106274E+01)	(8.631690E+00)	(1.413660E+01)	(2.132007E+01)	(1.013189E+01)	(8.242749E+00)	(8.669795E+00)	(7.450222E+00)	(1.165728E+01)	(1.385349E+01)	(9.591251E+00)
f_{28}	Mean	4.584167E+02	4.588152E+02	4.884526E+02	4.588487E+02	4.872866E+02	4.880454E+02	4.588487E+02	4.919525E+02	4.596444E+02	4.607163E+02	4.703419E+02	4.573225E+02	4.617202E+02	4.626798E+02	4.575119E+02	4.625177E+02
	Rank(WRST)	3	4	1	15	6	1										

Table 8
Performance of DE Variants on 100D CEC2017 Benchmarks (51 Runs): Statistical Results and Wilcoxon Tests.

	Criteria	UMOEa-IV	TSMs-DE	AdE	NLPSR-JSO	mLSHADE	HAPF-DE	FHWD-DE	DDPSA-DE	DACDE	CElDE	ACD-DE	UMOEa-III	IDE-EDA	LSHADE-SPACMA	LSHADE-csfgpma	EBOWithCMAR
f_1	Mean	2.814307E-14	4.389716E-10	3.325676E-03	1.275901E-04	9.296022E-06	1.312414E-13	4.182188E-10	5.684342E-14	8.038377E+00	2.786442E-14	6.264193E-07	3.789561E-14	3.879981E-11	6.687461E-15	4.161691E-11	1.851389E-08
	Rank/WRST	3	10 >	15 >	13 >	13 >	6 >	9 >	5 >	16 >	2 >	4 >	7 >	1 >	1 >	11 >	11 >
	Min	1.421085E-14	8.810730E-13	6.439516E-05	4.870415E-08	1.789289E-10	5.684342E-14	1.705303E-13	2.842171E-14	1.009754E+00	1.421085E-14	8.364935E-10	1.421085E-14	3.126388E-13	0.000000E+00	2.273737E-13	1.570299E-11
	Rank	2	10	15	14	12	6	7	5	16	2	3	2	9	1	8	11
	Median	2.842171E-14	1.859775E-10	2.362072E-03	5.734701E-05	1.867499E-07	1.136868E-13	1.047624E-10	4.263256E-14	6.697588E+00	2.842171E-14	2.460428E-07	2.842171E-14	8.952838E-12	0.000000E+00	6.792789E-12	6.100521E-09
	Rank	2	10	15	14	12	6	9	5	16	2	3	2	8	1	7	11
	(Std Dev)	(4.485047E-15)	(6.387677E-10)	(3.561628E-03)	(2.409990E-04)	(3.516356E-05)	(6.735335E-14)	(8.615208E-10)	(6.405928E-14)	(5.575286E+00)	(6.939080E-15)	(1.251486E-06)	(5.183879E-14)	(8.205119E-11)	(7.163704E-15)	(1.002913E-10)	(3.988414E-08)
f_2	Mean	5.438922E-08	1.434679E+05	9.694334E+103	5.602825E+20	3.108494E-09	7.407948E+09	2.236360E+03	7.478964E+29	4.386226E+06	7.196427E+06	1.009412E+02	2.444955E+02	2.158083E+00	4.767853E+04	9.288640E-08	9.288640E-08
	Rank/WRST	2	9 >	16 >	13 >	12 >	12 >	7 >	15 >	10 >	11 >	5 >	4 >	4 >	8 >	3 >	3 >
	Min	1.330136E-11	3.349373E-08	1.687933E+31	2.000000E+01	3.467449E-12	0.000000E+00	9.089263E-11	0.000000E+00	1.137877E+24	0.000000E+00	2.621684E-06	7.389644E-13	4.263256E-12	8.526513E-14	6.558309E-10	6.558309E-10
	Rank	9	12	16	14	7	1	10	1	15	1	13	1	6	5	11	11
	Median	1.805347E-09	1.073078E+02	7.167848E+67	2.239510E+05	3.976766E-10	2.000000E+00	8.756948E-05	1.170000E+02	1.000000E+30	1.280000E+02	6.975333E+03	5.409598E-07	5.000000E+01	4.278098E-06	3.423071E-08	3.423071E-08
	Rank	3	10	16	14	1	8	7	11	15	12	13	5	2	6	4	4
	(Std Dev)	(3.134435E-07)	(9.127717E+05)	(6.923139E+104)	(4.001022E+21)	(8.700121E-09)	(5.290285E+10)	(1.406971E+04)	(1.960378E+29)	(3.002198E+07)	(2.945635E+07)	(2.978643E+02)	(1.133343E+03)	(1.161706E+01)	(3.389353E+05)	(1.563371E-07)	(1.563371E-07)
f_3	Mean	1.312191E-11	8.429026E-05	1.594732E+05	3.091648E-05	1.338699E-06	5.580045E-08	3.754106E-08	2.999049E-05	3.212309E+02	1.057024E-07	1.563489E-03	1.702697E-09	4.237930E-06	1.114597E-15	8.218332E-11	2.074272E-05
	Rank/WRST	2	13 >	16 >	12 >	8 >	6 >	11 >	15 >	7 >	7 >	4 >	9 >	1 >	1 >	10 >	10 >
	Min	1.138686E-13	6.479333E-06	4.286623E+01	1.817182E-06	1.791037E-07	3.555215E-09	1.619526E-09	2.270056E-07	1.894791E+01	8.504514E-09	2.179689E-04	1.705303E-13	3.317101E-07	0.000000E+00	1.404322E-11	6.777905E-06
	Rank	2	12	16	11	8	6	5	9	15	7	14	3	10	1	4	13
	Median	1.273293E-11	1.650527E-05	6.871515E+03	1.945637E-05	1.131131E-06	5.044882E-08	3.004146E-08	6.404388E-06	2.904033E+02	6.978985E-08	1.452170E-03	2.330580E-11	2.246121E-06	0.000000E+00	5.650236E-11	1.946996E-05
	Rank	2	13	16	11	8	6	5	10	15	7	14	3	9	1	4	12
	(Std Dev)	(9.797790E-12)	(1.020255E-04)	(1.782203E+05)	(3.342698E-05)	(8.045989E-07)	(4.251754E-08)	(3.368340E-08)	(6.628735E-05)	(2.062827E+02)	(1.318052E-07)	(1.143928E-03)	(4.077513E-09)	(4.550453E-06)	(7.959671E-15)	(7.895539E-11)	(8.442298E-06)
f_4	Mean	1.617375E+02	1.885868E+02	1.965904E+02	2.007251E+02	4.806387E+00	6.028297E+00	2.240887E+02	2.209818E+02	1.959679E+02	1.966343E+02	1.712605E+02	1.877716E+02	2.011424E+02	1.988094E+02	1.971684E+02	1.971684E+02
	Rank/WRST	4	7 =	9 =	14 =	4 <	2 <	11 <	15 <	8 <	10 <	5 <	6 <	16 <	13 <	12 <	12 <
	Min	1.136686E-13	8.313898E+01	9.769146E+01	1.855079E+02	7.158079E-13	1.257374E-07	8.416851E+01	2.183356E-10	1.760351E+02	1.737824E+02	1.698434E+02	8.359111E+01	1.846718E+02	1.136686E-13	3.041123E-11	3.041123E-11
	Rank	1	7	10	16	3	6	9	5	13	12	11	1	8	15	14	4
	Median	1.918186E+02	1.973586E+02	1.954889E+02	1.973588E+02	3.986624E+00	3.986624E+00	1.973588E+02	1.973588E+02	1.973588E+02	1.973588E+02	1.890382E+02	1.920687E+02	1.920687E+02	1.920687E+02	1.973588E+02	1.973588E+02
	Rank	6	10	9	13	1	2	13	3	15	5	10	4	7	8	16	10
	(Std Dev)	(8.113121E+01)	(2.326721E+01)	(3.209618E+01)	(1.008912E+01)	(1.394132E+01)	(1.657950E+01)	(1.923887E+01)	(4.378647E+01)	(1.102787E+01)	(1.087679E+01)	(6.192320E+00)	(4.853727E+01)	(2.229091E+01)	(1.027255E+01)	(8.599737E+00)	(3.111328E+01)
f_5	Mean	9.695974E+00	6.359890E+01	2.275665E+02	2.265168E+01	7.649475E+01	4.151804E+01	4.386812E+01	1.179916E+02	2.154514E+02	3.123857E+01	2.795818E+01	2.110874E+01	4.856367E+01	1.227116E+01	5.452456E+01	2.892055E+01
	Rank/WRST	1	12 >	16 >	13 >	13 >	8 >	14 >	15 >	7 >	7 >	3 >	3 >	2 >	11 >	6 >	6 >
	Min	4.974795E+00	4.240159E+01	1.253647E+02	1.591934E+01	5.074290E+01	3.013518E+01	2.288406E+01	8.755637E+01	3.141081E+02	2.268399E+01	1.691430E+01	9.949591E+00	1.890422E+01	5.969754E+00	4.235930E+01	1.691430E+01
	Rank	1	12	15	4	13	10	9	14	16	8	6	3	7	2	11	5
	Median	8.954632E+00	6.059391E+01	2.019762E+02	2.188910E+01	7.561684E+01	4.091180E+01	4.416024E+01	1.184000E+02	2.093032E+02	3.033789E+01	2.832866E+01	2.089414E+01	5.139966E+01	1.193951E+01	5.450131E+01	2.885381E+01
	Rank	1	12	15	4	13	8	9	14	16	7	5	3	10	2	11	6
	(Std Dev)	(2.741780E+00)	(1.125951E+01)	(1.303314E+02)	(4.355950E+00)	(1.200671E+01)	(5.123543E+00)	(5.829739E+00)	(1.589239E+01)	(4.412649E+01)	(5.204420E+00)	(5.519707E+00)	(4.806028E+00)	(1.341549E+01)	(3.496070E+00)	(5.092670E+00)	(5.157779E+00)
f_6	Mean	1.950896E-07	6.726142E-07	1.787272E+00	2.716906E-05	7.449722E-01	1.638587E-01	1.019325E-01	1.076323E-01	1.711505E-03	5.171166E-07	2.187610E-04	4.482183E-06	1.265943E-04	1.136686E-13	6.193563E-05	1.592274E-05
	Rank/WRST	2	4 >	16 >	15 >	11 >	11 >	14 >	13 >	12 >	3 >	3 >	5 >	9 >	1 <	8 >	6 >
	Min	3.706191E-11	1.136686E-13	4.504528E-06	3.721192E-01	2.440503E-07	2.440503E-07	1.540502E-02	1.121137E-03	2.643151E-08	3.983590E-07	7.841550E-08	1.042042E+00	1.136686E-13	2.823879E-05	3.761982E-06	3.761982E-06
	Rank	3	1	16	8	15	7	14	13	12	5	6	4	10	1	11	9
	Median	1.128043E-07	2.407206E-07	1.668518E+00	2.045077E-05	7.320374E-01	2.801816E-04	1.038383E-01	9.898157E-02	1.201636E-03	4.697221E-07	5.851981E-06	3.994774E-06	2.231162E-05	1.136686E-13	5.826945E-05	1.567854E-05
	Rank	2	3	16	8	15	11	14	13	12	4	5	6	9	1	10	7
	(Std Dev)	(2.536444E-07)	(9.279064E-07)	(8.409866E-01)	(2.487359E-05)	(2.116822E-01)	(2.956374E-03)	(5.460856E-02)	(7.682275E-02)	(3.385695E-04)	(4.072876E-07)	(3.798477E-04)	(3.049714E-06)	(4.097478E-04)	(0.000000E+00)	(2.434043E-05)	(7.336332E-06)
f_7	Mean	1.120176E+02	1.404465E+02	3.347920E+02	1.201505E+02	1.637345E+02	1.399858E+02	1.340501E+02	2.231876E+02	3.499320E+02	1.316360E+02	1.318391E+02	1.140675E+02	1.638926E+02	1.119167E+02	1.609305E+02	1.213521E+02
	Rank/WRST	2	10 >	15 >	12 >	12 >	9 >	16 >	14 >	6 >	6 >	3 >	3 >	13 >	5 >	11 >	11 >
	Min	1.091491E+02	1.251780E+02	2.394643E+02	1.111532E+02	1.326497E+02	1.283122E+02	1.225801E+02	1.938157E+02	2.246788E+02	1.200990E+02	1.216295E+02	1.102820E+02	1.426896E+02	1.094624E+02	1.456248E+02	1.153402E+02
	Rank	1	9	16	4	11	10	8	14	15	6	7	3	12	2	13	5
	Median	1.120276E+02	1.366030E+02	3.034805E+02	1.203375E+02	1.635396E+02	1.409404E+02	1.338547E+02	2.212489E+02	3.382901E+02	1.318960E+02	1.324422E+02	1.145026E+02	1.650256E+02	1.119499E+02	1.610027E+02	1.211337E+02
	Rank	2	9	15	4</												

Table 8 (continued).

f_n	Criteria	UMOEAs-IV	TSMS-DE	AuDE	NLPSR-BO	mLSHADE	HAPs-DE	FDHD-DE	DDPSA-DE	DACDE	CILDE	ACD-DE	UMOEAs-III	IDE-EDA	LSHADE-SPACMA	LSHADE-collgSim	EBOwithCMAR
f_1	Mean	4.305039E+01	5.598296E+01	6.401219E+01	6.500599E+01	7.252642E+02	2.098260E+02	2.407442E+01	1.025625E+03	2.116976E+02	2.939836E+01	2.390883E+02	5.599950E+01	9.020279E+01	5.114896E+01	6.940422E+01	6.625294E+01
	Rank/Worst	3	5 >	16 >	7 >	14 >	11 >	12 >	15 >	12 >	2 <	13 >	6 >	10 >	4 >	8 >	9 >
	Min	1.492438E+01	2.840113E+01	7.112399E+02	2.664305E+01	5.540808E+02	6.560720E+01	1.460053E+01	5.241032E+02	1.152548E+02	1.293480E+01	5.251971E+02	1.691430E+01	2.089414E+01	1.951782E+01	3.084371E+01	3.084371E+01
	Rank	3	8	1	2	11	2	14	13	13	1	12	4	6	5	9	5
	Median	4.254642E+01	5.251452E+01	1.060977E+03	5.150743E+01	7.093921E+02	1.741239E+02	2.265335E+01	1.004601E+03	2.118954E+02	2.462119E+01	2.218832E+02	4.974787E+01	4.155755E+01	3.762578E+01	5.870249E+01	5.870249E+01
	Rank	5	8	16	7	14	11	1	15	12	2	13	6	10	4	3	9
	(Std Dev)	(1.291916E+01)	(2.109295E+01)	(1.854864E+04)	(3.086372E+01)	(9.836025E+01)	(1.172704E+02)	(1.076031E+01)	(5.161572E+01)	(1.818409E+01)	(2.365673E+01)	(7.236071E+01)	(3.057071E+01)	(2.598564E+01)	(5.672195E+01)	(2.726219E+01)	(2.726219E+01)
f_2	Mean	9.002845E+03	1.223139E+04	2.461931E+05	1.802876E+04	4.379632E+03	1.761364E+04	5.181336E+03	2.588750E+04	1.132909E+05	5.955109E+03	3.670182E+04	1.011929E+04	1.553826E+04	4.768739E+03	4.838626E+03	4.169111E+03
	Rank/Worst	7	9 >	16 >	12 >	2 <	11 >	5 >	13 >	15 >	6 >	14 >	10 >	10 >	3 >	4 >	1 <
	Min	2.780469E+03	4.641700E+03	7.151881E+04	7.835024E+03	2.878315E+03	7.993429E+03	3.226345E+03	1.740956E+03	2.863838E+04	3.111698E+03	1.376321E+04	3.000222E+03	4.270660E+03	3.319193E+03	3.165452E+03	2.851503E+03
	Rank	1	10	16	12	3	7	15	15	15	5	14	4	9	8	6	2
	Median	5.473092E+03	1.069512E+04	2.525648E+05	1.625454E+04	4.112914E+03	1.490507E+04	4.938833E+03	2.431167E+04	9.007186E+04	5.498213E+03	3.193324E+04	9.477827E+03	1.416337E+04	4.850040E+03	4.753293E+03	4.215346E+03
	Rank	6	9	16	12	1	11	5	13	15	7	14	8	10	4	3	2
	(Std Dev)	(6.525932E+03)	(6.312973E+03)	(9.969436E+04)	(8.886248E+03)	(1.829116E+03)	(7.974992E+03)	(9.567864E+02)	(1.155145E+04)	(6.271740E+04)	(1.799279E+03)	(1.539153E+04)	(6.282793E+03)	(6.279341E+03)	(6.292139E+02)	(7.043895E+02)	(6.002374E+02)
f_3	Mean	2.335343E+02	2.282813E+02	5.228997E+02	1.410298E+02	1.962748E+03	1.131762E+02	7.540281E+01	3.711889E+02	1.559835E+02	3.471703E+01	4.363536E+02	2.040927E+02	1.366100E+02	1.339031E+02	1.107339E+02	2.603490E+02
	Rank/Worst	11	10 =	16 >	7 <	15 >	4 <	2 <	13 =	8 <	14 >	9 <	6 <	5 <	3 <	12 =	12 =
	Min	1.199829E+02	1.389391E+02	2.626195E+02	7.202824E+01	4.610804E+02	6.654848E+01	2.008645E+01	6.227746E+01	5.894558E+01	1.138200E+01	2.222866E+02	7.172084E+01	6.250509E+01	8.687298E+01	5.552979E+01	1.070847E+02
	Rank	12	13	15	9	7	16	2	5	4	1	14	8	6	10	3	11
	Median	2.236891E+02	2.275282E+02	3.018409E+02	1.406129E+02	1.898498E+02	2.102134E+02	7.725290E+01	1.475882E+02	2.906704E+01	9.478377E+01	1.911024E+02	1.334954E+02	1.309568E+02	2.937582E+02	2.497340E+02	2.497340E+02
	Rank	10	11	16	7	15	3	2	12	8	1	14	9	6	5	4	13
	(Std Dev)	(7.476919E+01)	(4.686036E+01)	(6.418516E+03)	(3.684964E+01)	(8.607837E+02)	(5.302375E+01)	(1.768871E+01)	(5.566217E+02)	(5.331379E+01)	(1.738494E+01)	(1.515513E+02)	(7.526611E+01)	(3.906125E+01)	(2.870670E+01)	(3.249828E+01)	(9.948342E+01)
f_4	Mean	6.611462E+01	4.336303E+01	4.782271E+02	3.786166E+01	2.755944E+02	8.218382E+01	2.896023E+01	4.303884E+02	9.187893E+01	3.020342E+01	2.134518E+02	6.942426E+01	4.826414E+01	7.614839E+01	5.020042E+01	1.462153E+02
	Rank/Worst	7	4 <	16 >	3 <	14 >	10 >	1 <	15 >	11 >	2 <	13 >	8 >	5 <	9 >	6 <	12 >
	Min	4.289837E+01	3.634918E+01	2.206733E+02	2.614782E+01	2.008655E+02	4.488664E+01	2.403554E+01	1.560266E+02	5.919131E+01	2.355402E+01	1.035855E+02	4.485994E+01	5.708842E+01	3.944372E+01	3.268777E+01	3.268777E+01
	Rank	7	5	16	3	15	8	2	14	11	1	13	9	6	10	7	11
	Median	6.182632E+01	4.413166E+01	4.667823E+02	3.724164E+01	2.657569E+02	7.274596E+01	2.911953E+01	4.372319E+02	9.197180E+01	2.920720E+01	2.199666E+02	6.882378E+01	4.791478E+01	7.572102E+01	4.999058E+01	1.443702E+02
	Rank	7	4	16	3	14	9	1	15	11	2	13	8	5	10	6	12
	(Std Dev)	(1.806423E+01)	(4.170405E+00)	(1.454414E+02)	(4.503743E+00)	(4.533733E+01)	(4.029354E+01)	(2.182306E+00)	(1.451531E+02)	(1.572790E+01)	(3.381159E+00)	(3.410945E+01)	(1.208788E+01)	(6.776670E+00)	(1.134622E+01)	(6.189197E+00)	(3.208222E+01)
f_5	Mean	1.410858E+02	7.778516E+01	3.578622E+03	9.551662E+01	2.417617E+02	1.930548E+02	7.758593E+01	5.562315E+02	1.228876E+02	4.010742E+01	2.660155E+02	1.611414E+02	1.238459E+02	1.106344E+02	8.426847E+01	1.681060E+02
	Rank/Worst	9	16 >	3 <	16 >	5 <	12 >	13 >	7 <	14 >	6 <	14 >	6 <	6 <	4 <	11 >	11 >
	Min	6.792341E+01	3.995434E+01	1.358962E+02	4.392939E+01	1.493237E+02	1.008191E+02	2.239316E+01	1.869043E+02	4.446919E+01	1.474237E+01	1.460208E+02	6.419933E+01	6.423317E+01	4.534939E+01	9.268777E+01	9.268777E+01
	Rank	9	3	16	9	12	2	16	5	16	1	10	7	11	10	6	7
	Median	1.395397E+02	6.741237E+01	2.130822E+03	8.288678E+01	2.233825E+02	1.954260E+02	9.205531E+01	2.495847E+02	1.24984E+02	2.618594E+01	1.620533E+02	1.193008E+02	1.013594E+02	7.957393E+01	1.674711E+02	1.674711E+02
	Rank	9	2	16	4	13	12	5	15	14	10	7	6	10	3	11	11
	(Std Dev)	(4.653615E+01)	(3.238346E+01)	(3.871860E+03)	(3.295972E+01)	(7.278183E+01)	(3.864410E+01)	(3.060304E+01)	(4.603000E+02)	(8.601840E+01)	(2.762556E+01)	(5.812766E+01)	(4.161546E+01)	(2.801491E+01)	(3.405745E+01)	(2.839768E+01)	(4.038366E+01)
f_6	Mean	2.236204E+02	1.855821E+03	3.683448E+03	1.522158E+03	7.570314E+02	9.245804E+02	1.786059E+03	2.321121E+03	2.507391E+03	1.403213E+03	1.477478E+03	1.208735E+03	1.685990E+03	1.415007E+03	1.197982E+03	1.591029E+03
	Rank/Worst	1	13 >	16 >	9 >	2 >	3 >	12 >	14 >	15 >	6 >	14 >	5 >	11 >	7 >	4 >	10 >
	Min	1.310423E+02	9.614283E+02	1.382440E+03	9.940935E+02	4.746573E+02	1.401874E+02	9.138417E+02	1.200406E+03	1.556371E+03	6.217663E+02	1.006250E+03	6.414987E+02	7.311041E+02	6.498036E+02	6.980536E+02	6.980536E+02
	Rank	1	12	16	10	4	12	14	15	14	7	10	8	9	7	8	8
	Median	1.382111E+02	1.900240E+03	3.119396E+03	1.488738E+03	7.689363E+02	8.878433E+02	1.826374E+03	2.350617E+03	2.488182E+03	1.380733E+03	1.448440E+03	1.181168E+03	1.660174E+03	1.307461E+03	1.210912E+03	1.625346E+03
	Rank	1	13	16	9	2	3	12	14	15	7	8	4	11	6	5	10
	(Std Dev)	(1.583722E+02)	(2.915356E+02)	(1.500545E+03)	(3.407182E+02)	(2.035891E+02)	(3.880293E+02)	(2.328800E+02)	(3.418447E+02)	(4.577957E+02)	(2.860535E+02)	(2.461792E+02)	(2.736739E+02)	(3.550065E+02)	(4.851539E+02)	(2.482592E+02)	(3.988278E+02)
f_7	Mean	2.432501E+02	1.401862E+03	4.230578E+03	1.086292E+03	1.112034E+03	1.114659E+03	1.227828E+03	1.799156E+03	1.548468E+03	1.012909E+03	1.071784E+03	1.059038E+03	1.203915E+03	1.024176E+03	9.328193E+02	1.208486E+03
	Rank/Worst	1	16 >	7 >	7 >	8 >	9 >	12 >	15 >	14 >	3 >	6 >	5 >	10 >	2 >	11 >	11 >
	Min	8.287858E+01	9.459948E+02	1.805145E+03	2.035771E+02	7.792613E+02	3.764325E+02	3.764325E+02	1.075125E+03	6.668947E+02	5.075557E+01	2.857211E+01	1.100491E+02	6.519398E+02	2.495323E+02	5.482437E+02	5.482616E+02
	Rank	1	14	16	2	4	12	4	15	15	7	6	12	9	3	8	8
	Median	2.377913E+02	1.947985E+03	1.109236E+03	1.066401E+03	1.151211E+03	1.212433E+03	1.852196E+03	1.549872E+03	1.025398E+03	1.046958E+03	1.100377E+03	1.242566E+03	1.083148E+03	9.383654E+02	1.163499E+03	1.163499E+03
	Rank	1	13	16	8	5	9										

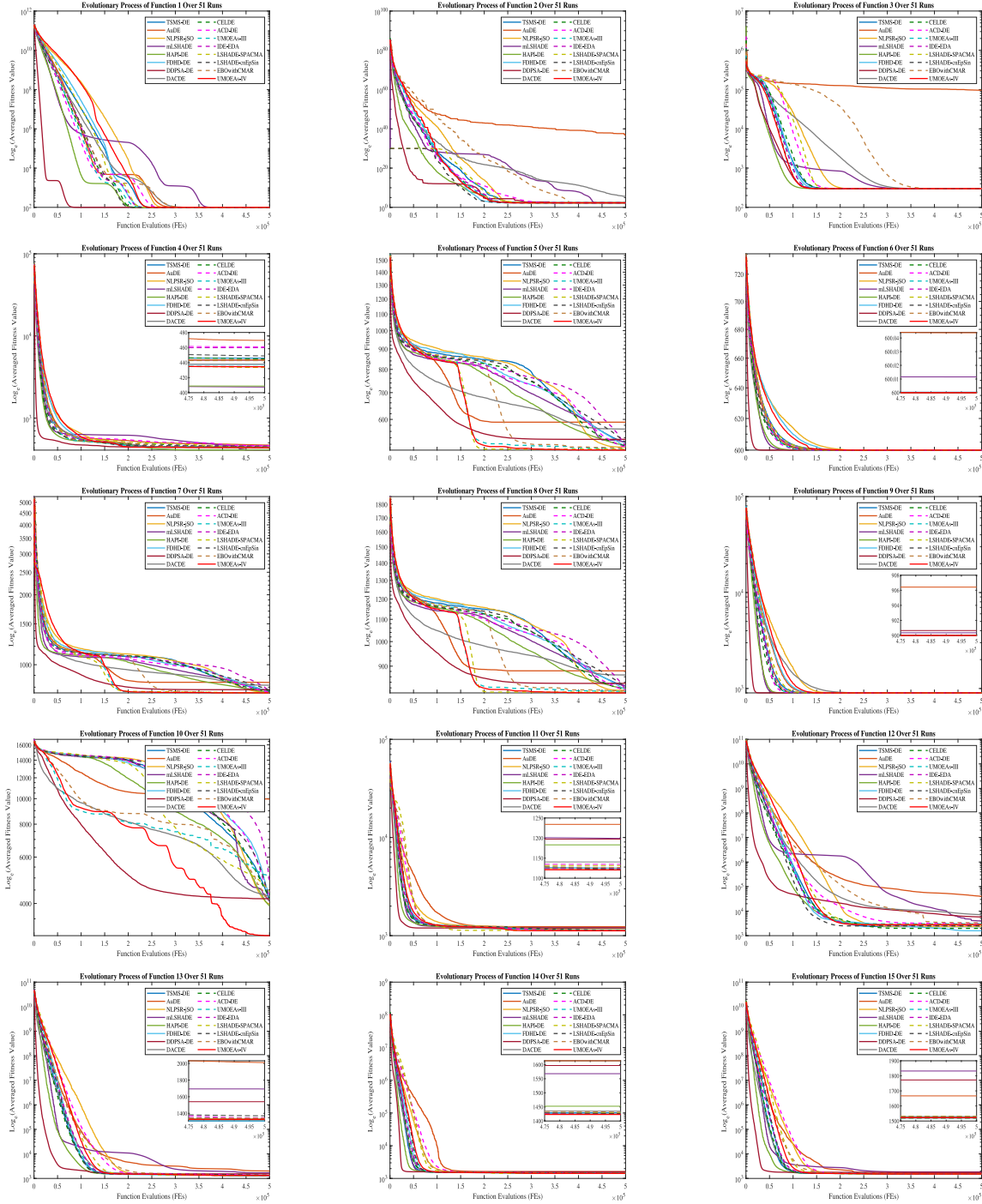


Fig. 3. The proposed UMOEAs-IV and all compared DE-based variants were subjected to evolutionary processes in 50-dimensional experiments. These processes were averaged over 51 runs and evaluated on the $f_1 - f_{15}$ functions of the CEC2017 benchmark. The fitness values and function evaluations (FEs) are depicted on the Y- and X-axes, respectively.

f_{18} ; EBOwithCMAR achieves the best mean value on f_{12} ; and CELDE achieves the best mean values on f_{13} , f_{15} , and f_{19} . For composition functions, UMOEAs-IV achieves the best mean values on $f_{21}-f_{23}$, f_{26} , and $f_{29}-f_{30}$. HAPI-DE, FDHD-DE, DACDE, and mLSHADE achieve the best mean values on f_{24} , f_{25} , f_{27} , and f_{28} , respectively. In summary, UMOEAs-IV attains a top-three performance on 21 functions overall: 3, 5, 4, and 9 on unimodal, simple multimodal, hybrid, and composition functions, respectively.

From Tables 5 to Table 8, it is evident that as the dimensionality increases, UMOEAs-IV shows progressively better performance on composition functions and demonstrates stable performance on both simple multimodal and hybrid functions. For unimodal functions, it also achieves excellent results in both the 10- and 100-dimensional tests. In Figs. 3 and 4, it can be observed that UMOEAs-IV exhibits an excellent convergence rate in both the early and later stages of evolution across various benchmark functions.

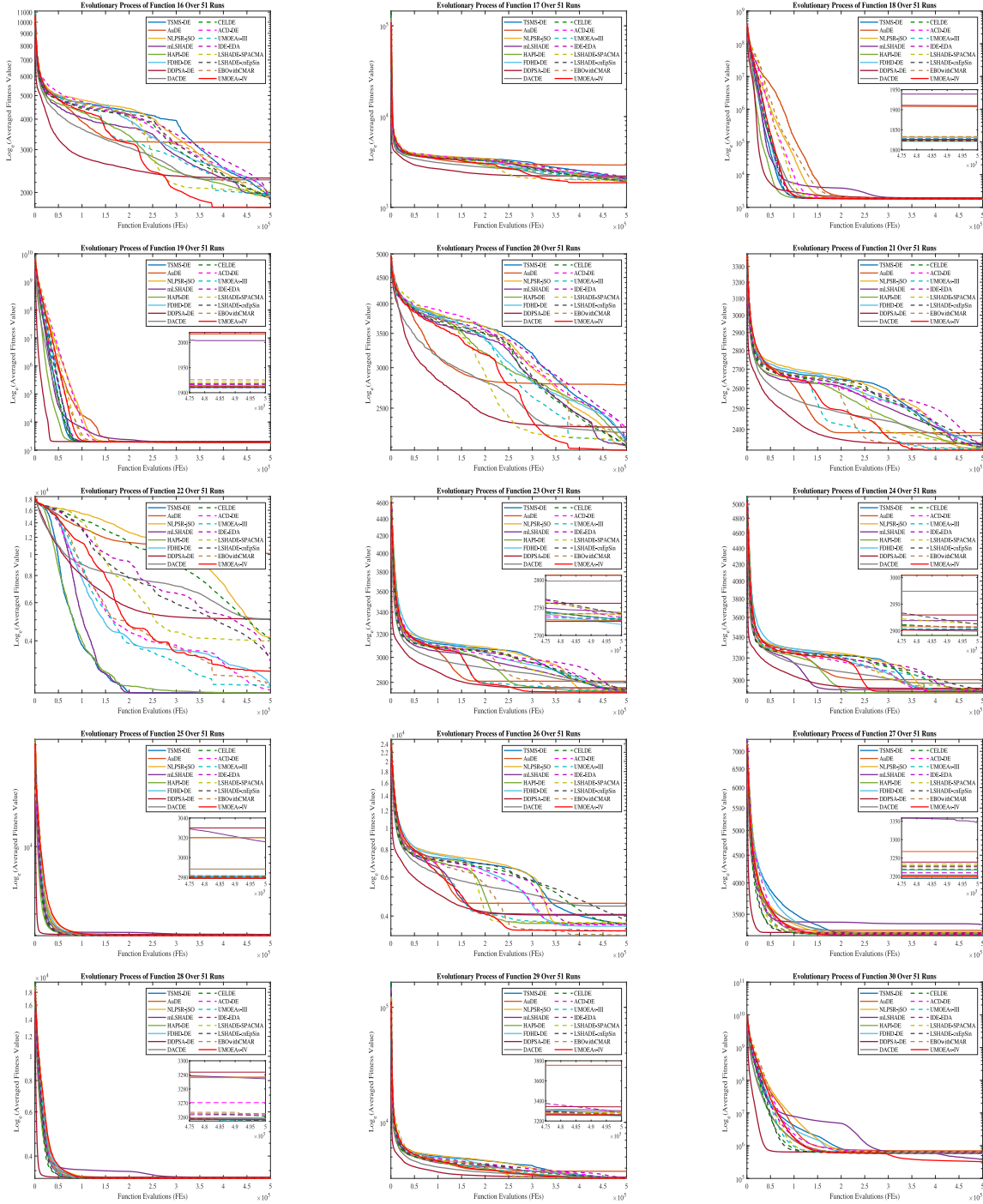


Fig. 4. The proposed UMOEAs-IV and all compared DE-based variants were subjected to evolutionary processes in 50-dimensional experiments. These processes were averaged over 51 runs and evaluated on the $f_{16} - f_{30}$ functions of the CEC2017 benchmark. The fitness values and Function Evaluations (FEs) are depicted on the Y- and X-axes, respectively.

4.4. Ablation test

To better evaluate the performance and effectiveness of the integrated mechanisms, ablation tests are conducted. Table 9 presents the results of these experiments, highlighting the contribution of each integrated mechanism in UMOEAs-IV. These mechanisms include the following:

- UMOEAs-IV vs. UMOEAs-IV using ‘current-to-pbest with sort’.
- UMOEAs-IV vs. UMOEAs-IV using ‘current-to-pbest with archive’.

- UMOEAs-IV vs. UMOEAs-IV using scaling factor F from UMOEAs-III.
- UMOEAs-IV vs. UMOEAs-IV using mutation strategy from UMOEAs-III.
- UMOEAs-IV vs. UMOEAs-IV without using EDA.
- UMOEAs-IV vs. UMOEAs-IV using Gaussian learning.
- UMOEAs-IV vs. UMOEAs-IV using individual learning.
- UMOEAs-IV vs. UMOEAs-IV without using stagnation strategy.
- UMOEAs-IV vs. UMOEAs-IV using original stagnation strategy from [17].

In the table, UMOEAs-IV achieves the best average Friedman rank of 4.57 among all variants. The average Friedman ranks of UMOEAs-IV using ‘current-to-pbest with sort’, using ‘current-to-pbest with an archive’, using the scaling factor F from UMOEAs-III, using the mutation strategy from UMOEAs-III, without using EDA, using Gaussian learning, using individual learning, without using the stagnation strategy, and using the original stagnation strategy are 5.43, 6.00, 5.63, 6.73, 5.07, 6.00, 5.57, 5.23, and 4.77, respectively, all worse than 4.57. These results demonstrate that the proposed UMOEAs-IV is well-tuned and highlight the effectiveness of its integrated mechanisms.

Figs. 5 and 6 present scatter plots of the 2-dimensional individuals on f_{10} and f_{28} at generations 0, 5, 10, and 20. The results are shown for UMOEAs-IV under different settings: using ‘current-to-pbest with sort’, using ‘current-to-pbest with an archive’, adopting the scaling factor F from UMOEAs-III, applying the mutation strategy from UMOEAs-III, without EDA, with Gaussian learning, with individual learning, without the stagnation strategy, and with the original stagnation strategy from [17]. The orange star represents the global optimum. The results show that the extent of scattering and clustering is strongly influenced by the specified mechanisms.

4.5. Search pattern of UMOEAs-IV

To better analyze the sources of the best fitness values (G_{best}) and the contributions of individual mechanisms, the portions of G_{best} contributed by ‘current-to-pbest with sort’, ‘current-to-pbest with archive’, Gaussian learning, individual learning, EDA, CMA-ES, and the SQP method are recorded and traced. Figs. 7 and 8 show the evolution of G_{best} by source for different functions in single 50-dimensional runs. The search patterns can be summarized as follows:

- Both mutation operators and EDA alternately contribute to the G_{best} in the very early stage of evolution. Then, CMA-ES contributes to the G_{best} , followed by another alternating contribution from the mutation operators and EDA. Finally, CMA-ES contributes to the G_{best} until the end of the evolution.
- Both mutation operators and CMA-ES contribute to the G_{best} in the early stage of evolution, and then CMA-ES contributes to the G_{best} until the end of the evolution.
- Both the mutation operators and CMA-ES alternately contribute to the G_{best} in the early stage of evolution. Then, CMA-ES contributes to the G_{best} , followed by alternating contributions from both the mutation operators and CMA-ES until the end.
- Both the mutation operators and EDA alternately contribute to the G_{best} in the early stage of evolution, after which the mutation operators continue to contribute alternately to the G_{best} until the end.
- Both the mutation operators and EDA alternately contribute to the G_{best} in the early stage of evolution, CMA-ES contributes in the middle stage, and finally, the mutation operators contribute to the G_{best} .
- EDA initially contributes to the G_{best} , followed by CMA-ES near the middle stage of evolution. Then, the mutation operators, CMA-ES, and SQP alternately contribute to the G_{best} . Finally, CMA-ES contributes to the G_{best} until the end of the evolution.
- EDA, the mutation operators, and CMA-ES alternately contribute to the G_{best} , and individual learning contributes to the G_{best} suddenly, followed by the mutation operators and SQP. Finally, CMA-ES contributes to the G_{best} until the end of the evolution.
- Both the mutation operators and EDA alternately contribute to the G_{best} in the early stage of evolution, and then the mutation operator ‘current-to-pbest with archive’ contributes to the G_{best} until the end.
- Both the mutation operators and EDA alternately contribute to the G_{best} in the early stage of evolution, and then the mutation operator ‘current-to-pbest with sort’ contributes to the G_{best} until the end.
- Both the mutation operators, CMA-ES, and EDA alternately contribute to the G_{best} in the early stage of evolution, and then the mutation

operator ‘current-to-pbest’ with archive contributes to the G_{best} until the end.

- Both the mutation operators and CMA-ES alternately contribute to the G_{best} throughout the evolution, while EDA makes a sudden, short contribution in the middle and then contributes to the G_{best} in the late stage until the end.

From the above search pattern, it is evident that when UMOEAs-IV optimizes a particular problem, an appropriate search pattern is automatically generated to better fit the problem landscape and guide individuals to evolve more efficiently and effectively. It is worth noting that in the stagnation strategy, Gaussian learning does not contribute to G_{best} directly, and individual learning rarely contributes to G_{best} (only for f_{20} and f_{30}), but their disturbances can help individuals find better solutions under other mechanisms.

Figs. 9 and 10 show the box plots of all compared algorithms in 50-dimensional tests. From the figures, it can be observed that the proposed UMOEAs-IV demonstrates stable and robust performance in most cases. The boxes in the plots are generally narrow, and UMOEAs-IV exhibits smaller minimum, median, and maximum values across the tests.

5. Real-world applications: Livestock feed ration optimization

In livestock production, feed represents a major expense, accounting for a large share of product cost depending on the animal’s stage and breed [55]. Thus, it is essential to provide an optimal diet at the lowest cost to reduce operational expenses and increase profit. Feed mix formulation aims to identify the appropriate ingredients and their costs while satisfying nutrient requirements [56]. In this work, two Livestock Feed Ration Optimization (LFRO) problems are considered as applications of UMOEAs-IV: the beef cattle case and the dairy cattle case [57,58].

5.1. Beef cattle case

For beef cattle, the objective is to allocate the selected ingredients (Y_i) with quantities (x_i) and their respective costs (C_i). The aim is to determine the x_i values that minimize the total cost in Eq. (29) while satisfying the constraints [57].

Minimize:

$$f(\bar{x}) = \sum_{i=1}^n x_i C_i \quad (29)$$

Subject to:

$$\begin{aligned} h_1(\bar{x}) &= \sum_{i=1}^n x_i DMI_i - a = 0, \quad g_1(\bar{x}) = b - \sum_{i=1}^n x_i CP_i \leq 0, \\ g_2(\bar{x}) &= \sum_{i=1}^n x_i CP_i - c \leq 0, \quad g_3(\bar{x}) = d - \sum_{i=1}^n x_i TDN_i \leq 0, \\ g_4(\bar{x}) &= \sum_{i=1}^n x_i TDN_i - e \leq 0, \quad g_5(\bar{x}) = f - \sum_{i=0}^n x_i CA_i \leq 0, \\ g_6(\bar{x}) &= \sum_{i=0}^n x_i CA_i - g \leq 0, \quad g_7(\bar{x}) = h - \sum_{i=0}^n x_i P_i \leq 0, \\ g_8(\bar{x}) &= \sum_{i=0}^n x_i P_i - j \leq 0, \quad g_9(\bar{x}) = k - \sum_{i=0}^n x_i Rhage_i \leq 0, \\ g_{10}(\bar{x}) &= \sum_{i=0}^n x_i Rhage_i - l \leq 0, \quad g_{11}(\bar{x}) = m - \sum_{i=0}^n x_i MC_i \leq 0, \\ g_{12}(\bar{x}) &= \sum_{i=0}^n x_i MC_i - o \leq 0, \quad g_{13}(\bar{x}) = p - \sum_{i=1}^n x_i Conc_i \leq 0, \\ g_{14}(\bar{x}) &= \sum_{i=1}^n x_i Conc_i - q \leq 0. \end{aligned}$$

where DMI, CP, TDN, Ca, P, and Rhage denote dry matter intake, crude protein, total digestible nutrients, calcium, phosphorus, and roughages, respectively, measured in kilograms. Based on this formulation, four different cases are defined in this paper.

5.2. Dairy cattle case

For dairy cattle, n denotes the number of ingredients under consideration (64 variables). Let x_i be the weight in kilograms of ingredient Y_i , and C_i be its cost. The objective function is the total cost, formulated as the sum of the weight x_i of each selected ingredient multiplied by its respective cost C_i . The goal is to find the values of x_i that minimize the overall cost in Eq. (30) while satisfying the constraints [57].

Table 9 (continued).

f_n	Criteria	UMOEAs-IV	UMOEAs-IV								
			using current-to-pbest with sort	using current-to-pbest with archive	scaling factor F from UMOEAs-III	mutation strategy from UMOEAs-III	without EDA	using Gaussian learning	using individual learning	without stagnation strategy	using original stagnation strategy
f_{19}	Mean	1.120802E+01	1.165126E+01	1.251695E+01	1.712903E+01	1.862630E+01	1.365525E+01	1.100103E+01	1.109556E+01	1.272045E+01	1.213238E+01
	(Std Dev)	(2.236191e+00)	(2.180867e+00)	(3.091705e+00)	(2.976231e+00)	(3.313307e+00)	(2.740439e+00)	(2.102226e+00)	(2.274835e+00)	(1.986084e+00)	(2.461526e+00)
	Friedman Rank	3	4	6	9	10	8	1	2	7	5
f_{20}	Mean	6.989713E+01	7.385810E+01	6.849791E+01	9.205398E+01	9.252620E+01	6.843042E+01	7.209463E+01	7.577876E+01	1.032486E+02	7.526815E+01
	(Std Dev)	(9.041067e+00)	(1.075287e+01)	(1.006633e+01)	(2.072067e+01)	(2.109537e+01)	(9.670723e+00)	(1.287242e+01)	(1.163572e+01)	(3.251624e+01)	(1.063093e+01)
	Friedman Rank	3	5	2	8	9	1	4	7	10	6
f_{21}	Mean	2.046774E+02	2.060299E+02	2.043456E+02	2.059506E+02	2.059774E+02	2.042560E+02	2.059783E+02	2.038619E+02	2.059471E+02	2.049720E+02
	(Std Dev)	(2.287828e+00)	(2.290488e+00)	(2.352379e+00)	(3.070512e+00)	(2.468304e+00)	(2.139199e+00)	(2.435832e+00)	(2.112273e+00)	(3.115966e+00)	(2.825106e+00)
	Friedman Rank	4	10	3	7	8	2	9	1	6	5
f_{22}	Mean	6.975856E+02	8.676381E+02	7.617402E+02	1.140800E+03	4.325960E+02	7.050247E+02	2.721116E+02	6.058402E+02	1.228352E+03	7.163312E+02
	(Std Dev)	(9.698635e+02)	(9.846531e+02)	(1.067122e+03)	(1.334981e+03)	(9.452114e+02)	(1.018463e+03)	(4.225431e+02)	(8.135206e+02)	(1.696720e+03)	(8.422053e+02)
	Friedman Rank	4	8	7	9	2	5	1	3	10	6
f_{23}	Mean	4.255519E+02	4.235606E+02	4.248245E+02	4.239534E+02	4.272946E+02	4.244066E+02	4.270519E+02	4.233183E+02	4.224198E+02	4.206598E+02
	(Std Dev)	(7.692094e+00)	(6.835103e+00)	(8.476232e+00)	(7.163899e+00)	(6.818000e+00)	(7.368647e+00)	(6.288384e+00)	(6.101282e+00)	(5.989968e+00)	(6.727662e+00)
	Friedman Rank	8	4	7	5	10	6	9	3	2	1
f_{24}	Mean	5.016679E+02	5.006477E+02	5.036591E+02	5.029719E+02	5.044565E+02	5.023520E+02	5.023602E+02	5.019030E+02	4.988714E+02	5.006869E+02
	(Std Dev)	(3.144555e+00)	(3.547731e+00)	(3.152068e+00)	(3.224802e+00)	(4.135057e+00)	(3.945674e+00)	(3.449783e+00)	(3.773782e+00)	(2.592453e+00)	(3.465960e+00)
	Friedman Rank	4	2	9	8	10	6	7	5	1	3
f_{25}	Mean	4.790390E+02	4.775467E+02	4.801516E+02	4.785415E+02	4.801856E+02	4.791407E+02	4.779232E+02	4.789974E+02	4.784577E+02	4.786543E+02
	(Std Dev)	(4.360133e+00)	(6.110328e+00)	(2.493140e+00)	(3.527422e+00)	(1.284968e+01)	(4.086055e+00)	(6.011270e+00)	(4.067884e+00)	(4.415563e+00)	(4.435111e+00)
	Friedman Rank	7	1	9	4	10	8	2	6	3	5
f_{26}	Mean	8.291530E+02	7.699261E+02	8.423802E+02	8.317140E+02	7.823515E+02	8.564815E+02	7.701549E+02	8.800326E+02	7.997438E+02	7.981450E+02
	(Std Dev)	(3.168878e+02)	(3.392248e+02)	(3.247736e+02)	(3.024086e+02)	(3.343687e+02)	(3.007991e+02)	(3.533188e+02)	(2.925101e+02)	(2.986429e+02)	(3.001590e+02)
	Friedman Rank	6	1	8	7	3	9	2	10	5	4
f_{27}	Mean	5.008677E+02	4.979298E+02	5.057891E+02	5.191118E+02	5.174209E+02	5.007956E+02	4.996483E+02	5.025404E+02	5.006618E+02	5.032726E+02
	(Std Dev)	(8.998809e+00)	(1.181852e+01)	(7.587776e+00)	(9.234550e+00)	(1.075468e+01)	(1.067850e+01)	(1.014020e+01)	(9.770209e+00)	(1.160485e+01)	(1.041639e+01)
	Friedman Rank	5	1	8	10	9	4	2	6	3	7
f_{28}	Mean	4.584167E+02	4.582553E+02	4.585316E+02	4.537711E+02	4.555000E+02	4.582665E+02	4.582394E+02	4.583858E+02	4.585074E+02	4.583084E+02
	(Std Dev)	(4.866012e−01)	(7.832090e−01)	(4.383190e−01)	(1.211870e+01)	(1.257275e+01)	(5.934354e−01)	(8.412778e−01)	(4.519639e−01)	(3.703203e−01)	(4.330883e−01)
	Friedman Rank	8	4	10	1	2	5	3	7	9	6
f_{29}	Mean	3.598263E+02	3.661430E+02	3.530522E+02	3.519417E+02	3.488063E+02	3.531349E+02	3.674324E+02	3.660998E+02	3.522415E+02	3.647400E+02
	(Std Dev)	(1.416799e+01)	(1.431566e+01)	(1.422747e+01)	(1.972600e+01)	(1.614215e+01)	(1.360649e+01)	(1.528255e+01)	(2.194246e+01)	(1.400938e+01)	(1.801316e+01)
	Friedman Rank	6	9	4	2	1	5	10	8	3	7
f_{30}	Mean	3.232140E+05	3.235478E+05	3.138635E+05	2.797790E+04	4.129904E+04	2.997036E+05	5.881748E+05	4.920970E+05	5.880007E+05	5.166813E+05
	(Std Dev)	(8.310948e+04)	(7.311665e+04)	(7.032627e+04)	(3.936646e+04)	(3.413686e+04)	(9.206153e+04)	(2.946856e+04)	(8.133048e+04)	(2.341229e+04)	(7.367036e+04)
	Friedman Rank	5	6	4	1	2	3	10	7	9	8
Averaged Friedman ranks		4.57	5.43	6.00	5.63	6.73	5.07	6.00	5.57	5.23	4.77

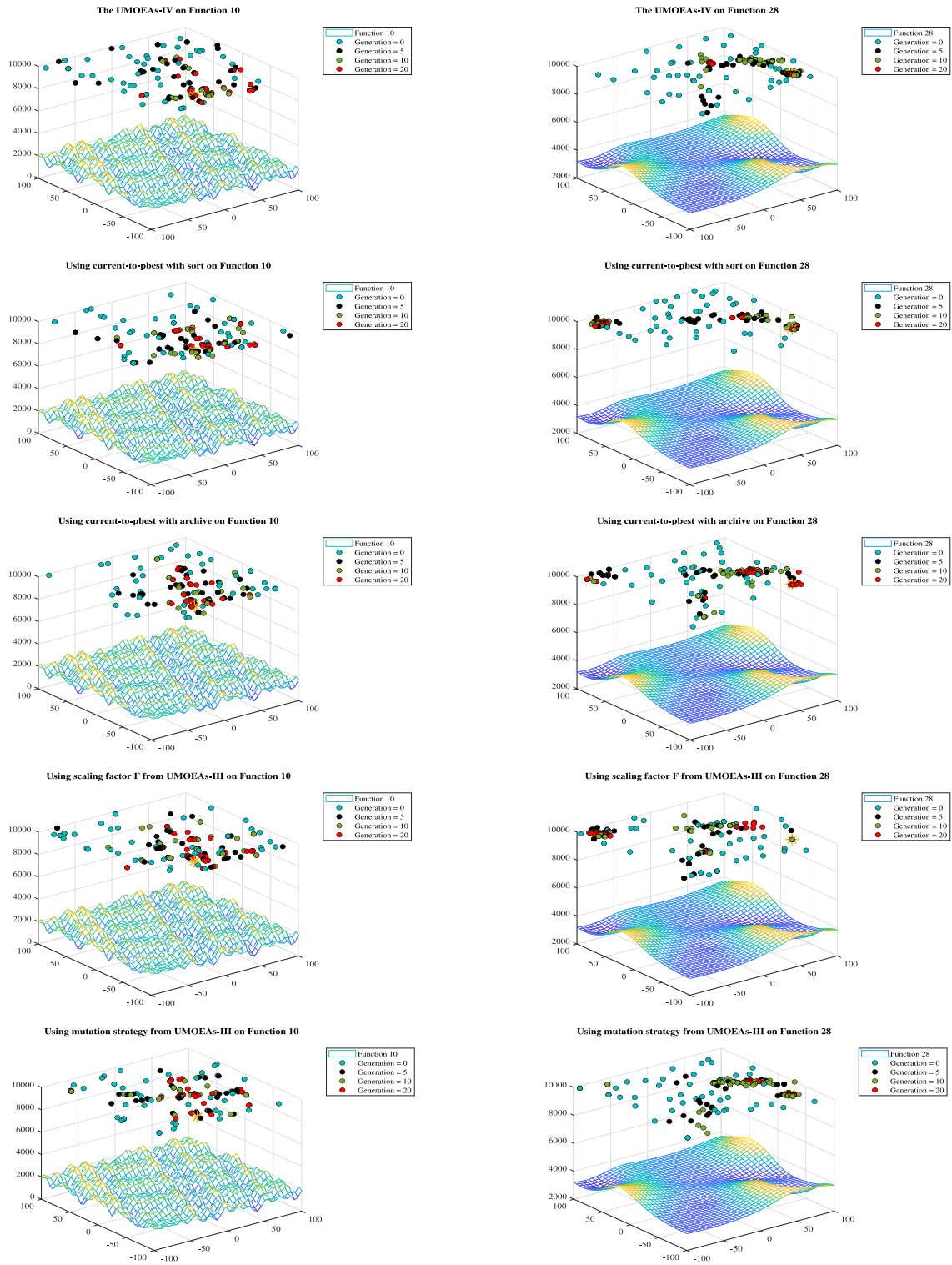


Fig. 5. Plots illustrating the 2-dimensional evolution of individuals across different generations (0, 5, 10, and 20) with and without various mechanisms on functions 10 and 28.

$$\begin{aligned}
 &\text{Minimize:} \\
 &f(\bar{x}) = \sum_{i=1}^n x_i C_i \\
 &\text{Subject to:} \\
 &h_1(\bar{x}) = \sum_{i=1}^n x_i M P_i - r = 0, \quad h_2(\bar{x}) = \sum_{i=1}^n x_i L y s_i - s = 0,
 \end{aligned} \tag{30}$$

$h_3(\bar{x}) = \sum_{i=1}^n x_i C a_i - t = 0$, $h_4(\bar{x}) = \sum_{i=1}^n x_i P_i - u = 0$,
 $h_5(\bar{x}) = \sum_{i=1}^n x_i M E_i - v = 0$, $h_6(\bar{x}) = \sum_{i=1}^n x_i M e t_i - z = 0$.
 where MP, Lys, Ca, P, ME, and Met denote metabolizable protein, lysine, calcium, phosphorous, metabolizable energy, and methionine, respectively. All nutrients are measured in kilograms. This paper defines three different cases based on the aforementioned problem.

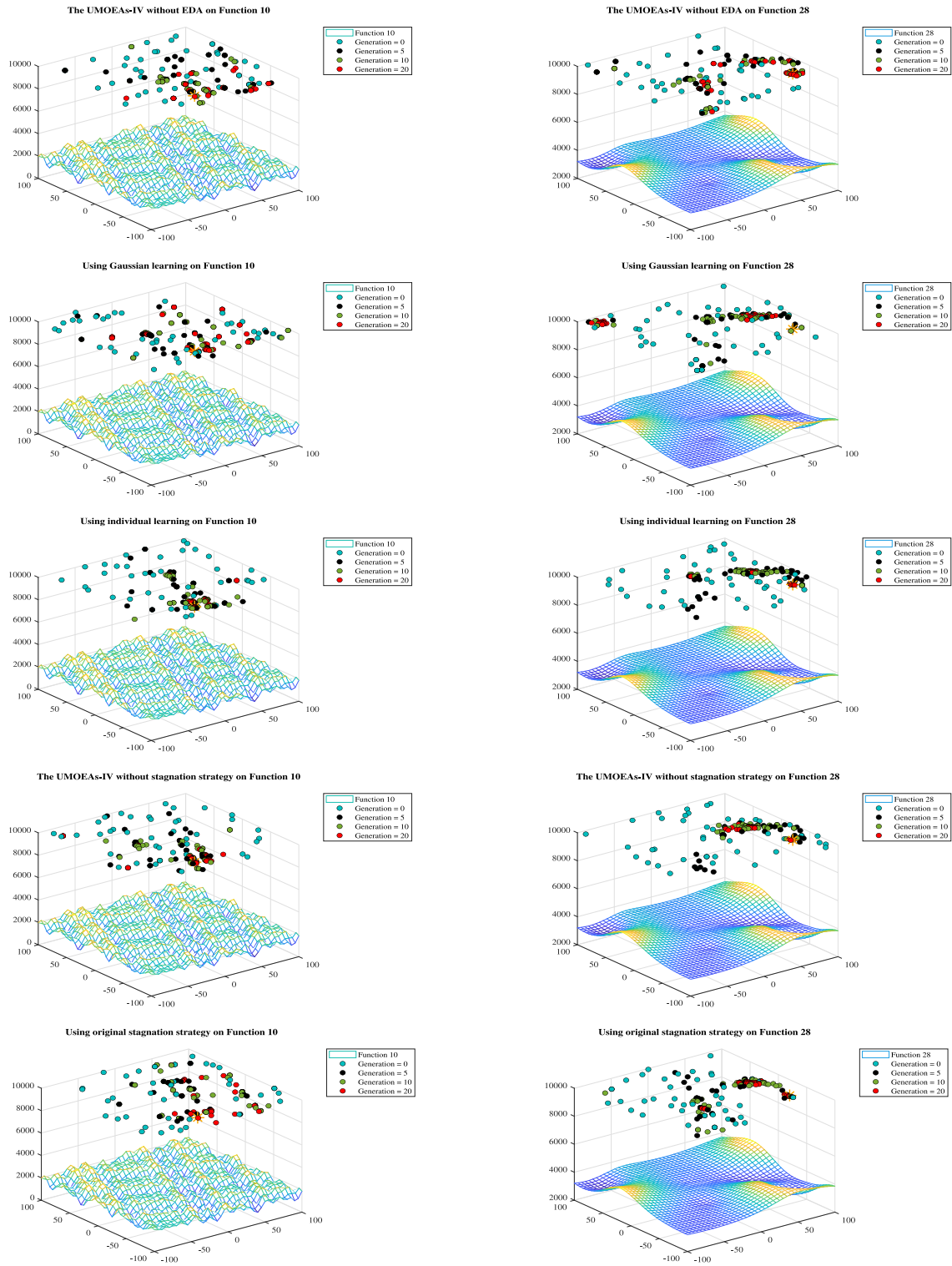


Fig. 6. Plots illustrating the 2-dimensional evolution of individuals across different generations (0, 5, 10, and 20) with and without various mechanisms on functions 10 and 28.

Tables 10 and 11 present the experimental results of all compared DE-based variants applied to beef cattle and dairy cattle under different cases. In Table 10, the best, median, mean, worst, standard deviation (Std Dev), mean constraint violation (MCV), feasibility rate (FR), and success rate (SR) are reported; for detailed definitions of MCV, FR,

and SR, please refer to [58]. UMOEAs-IV achieves an FR of 100% in cases 2, 3, and 4 for beef cattle, and in cases 1 and 3 for dairy cattle, indicating that it consistently finds feasible solutions in these scenarios. The SRs show that the two LFRO problems are difficult to solve, as all algorithms yield an SR of 0. Table 11 reports the performance

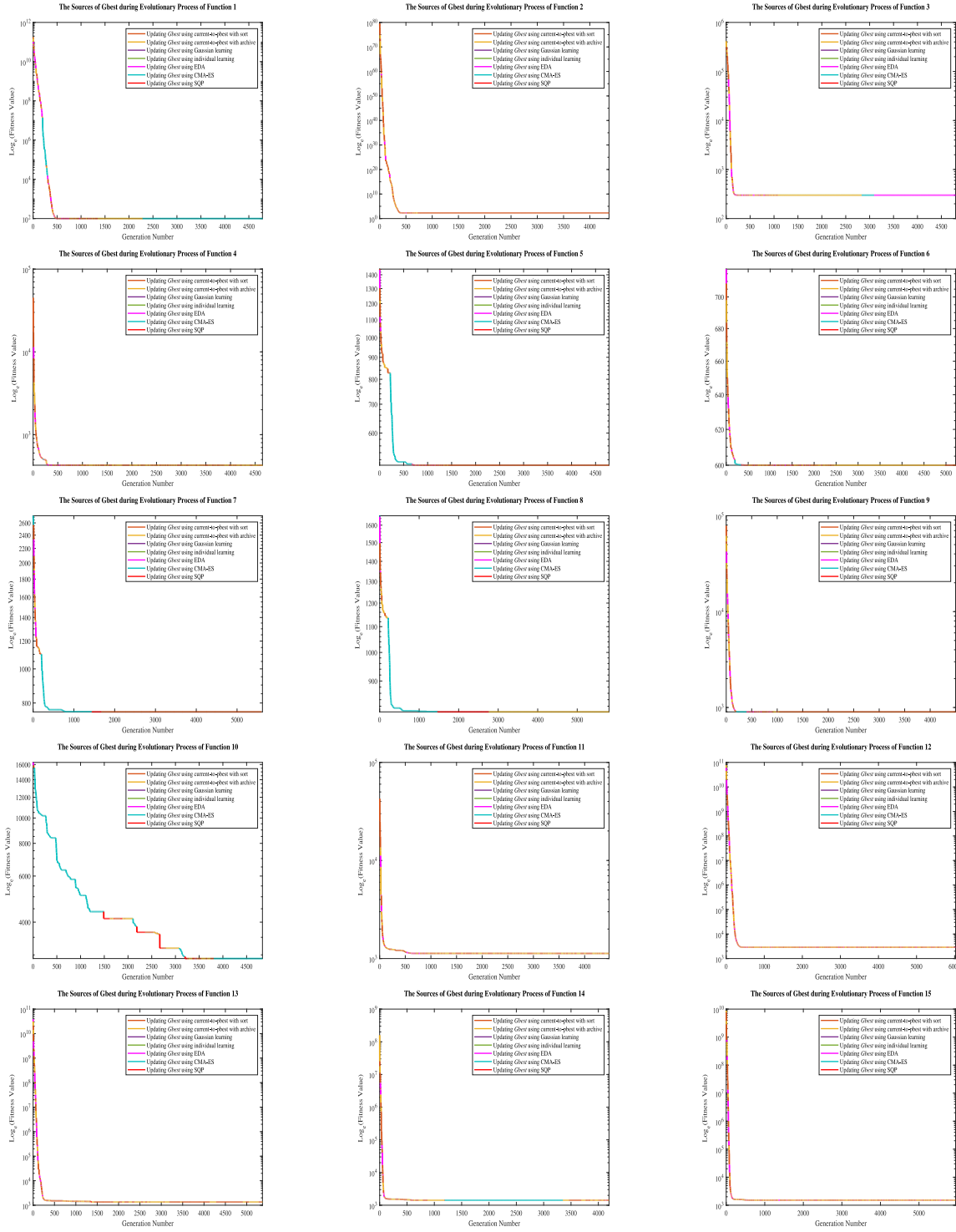


Fig. 7. Sources of the best particles in 50-dimensional experiments over the evolutionary process of an individual run on the $f_1 - f_{15}$ functions.

measure (PM), which follows the guidelines from [58,59]. A smaller weighted value indicates better performance, and UMOEAs-IV achieves the smallest weighted value ($3.36\text{E}-02$), ranking first.

6. Computational complexity of algorithms

The computational complexity of UMOEAs-IV was evaluated according to the criteria defined in the CEC2017 benchmark competition [60]. All experiments were performed under the following system configuration: • CPU: AMD Ryzen 5 3600, 6-Core Processor @ 3.60 GHz; • RAM:

32 GB; • OS: Windows 10 Professional; • Software: MATLAB 2021a. The computational complexity of UMOEAs-IV across different dimensions is summarized in Table 12, where T_0 denotes the time required to execute the following specified statements:

$x = 0.55$;

for $i = 1 : 1000000$

$x = x + x$; $x = x/2$; $x = x * x$; $x = \text{sqrt}(x)$; $x = \log(x)$; $x = \exp(x)$; $x = x/(x + 2)$;

end

Table 11
Performance measure value of all DE-based variants on livestock feed ration optimization problems.

Criteria	UMOEAs-IV	TSMS-DE	AuDE	NLPSR-jSO	mLSHADE	HAPI-DE	FDHD-DE	DDPSA-DE	DACDE	CELDE	ACD-DE	UMOEAs-III	IDE-EDA	LSHADE-SPACMA	LSHADE-cnEpSin	EBOwithCMAR
Best	6.65E-02	1.01E-01	1.35E-01	1.18E-01	6.80E-02	8.31E-02	7.36E-02	2.22E-01	1.52E-01	9.72E-02	3.37E-02	6.79E-02	1.35E-01	1.56E-01	1.34E-01	6.66E-02
Mean	1.06E-03	7.53E-02	1.52E-01	1.58E-01	3.40E-02	7.08E-02	7.05E-02	2.24E-01	1.60E-01	6.99E-02	7.20E-02	3.19E-03	1.55E-01	1.66E-01	1.66E-01	4.07E-02
Median	1.53E-04	7.47E-02	1.28E-01	1.57E-01	2.65E-02	7.00E-02	4.72E-02	2.24E-01	1.54E-01	6.99E-02	5.43E-02	6.29E-03	1.50E-01	1.65E-01	1.61E-01	3.24E-02
Weighted	3.36E-02	8.79E-02	1.39E-01	1.38E-01	4.95E-02	7.68E-02	6.74E-02	2.23E-01	1.55E-01	8.36E-02	4.93E-02	3.62E-02	1.44E-01	1.61E-01	1.49E-01	5.20E-02
Rank	1	9	11	10	4	7	6	16	14	8	3	2	12	15	13	5

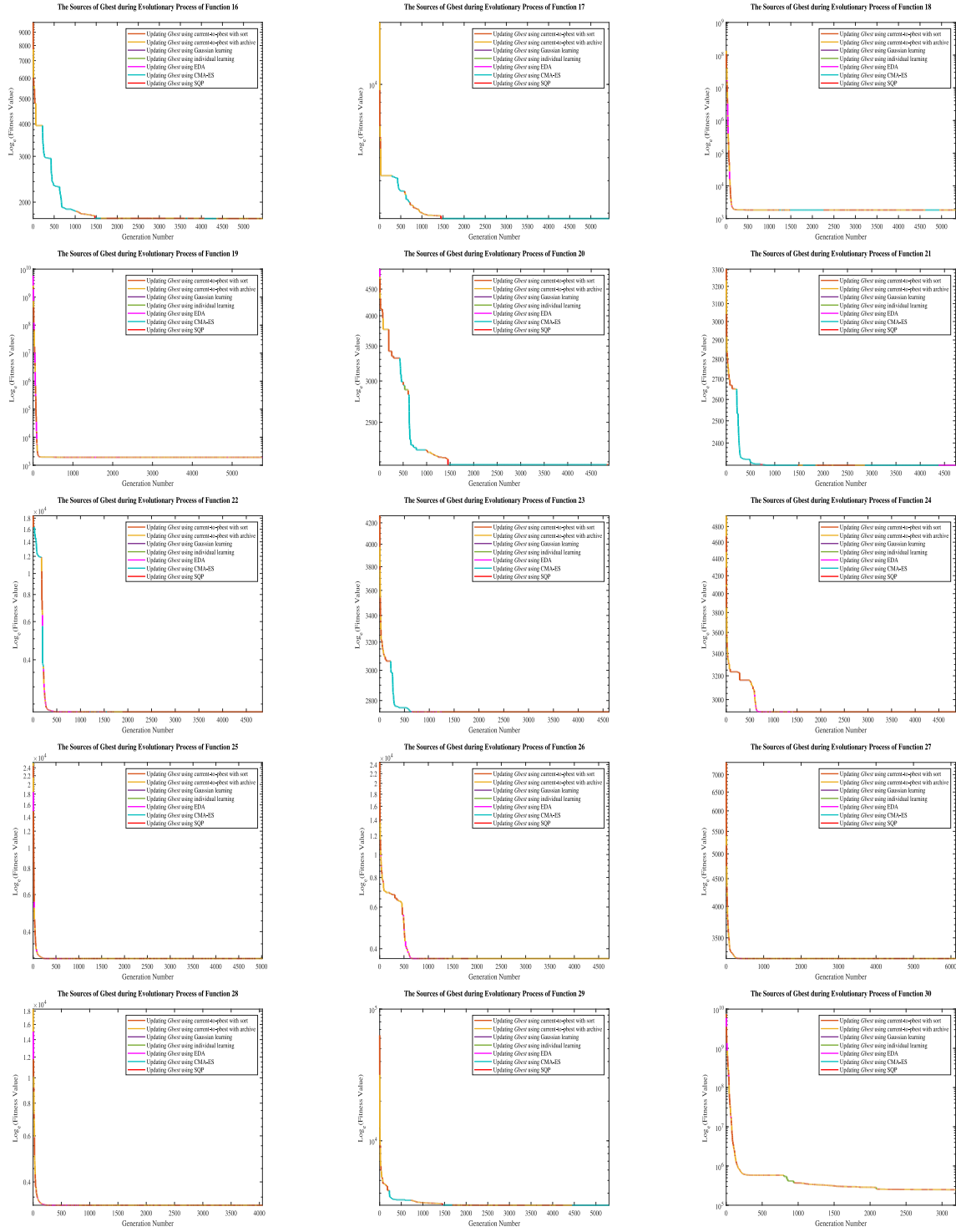


Fig. 8. Sources of the best particles in 50-dimensional experiments over the evolutionary process of an individual run on the $f_{16} - f_{30}$ functions.

Table 12
Computational complexity for UMOEAs-IV.

Dimension	T_0	T_1	$\hat{T}_2$	$(\hat{T}_2 - T_1)/T_0$
10D	0.0601	0.5106	2.4882	32.9051
30D	0.0601	0.6686	3.2046	42.1968
50D	0.0601	0.9271	3.7560	47.0703
100D	0.0601	2.1813	7.8991	95.1382

Here, T_1 denotes the time required to execute the benchmark function f_{18} independently (without using UMOEAs-IV) with 2×10^5 evaluations in D dimensions. The term T_2 represents the runtime of the UMOEAs-IV algorithm on function f_{18} under the same number of evaluations and dimensionality. The averaged runtime over five runs is denoted as $\hat{T}_2$. The units of T_0 , T_1 , and $\hat{T}_2$ are seconds (s). As shown in Table 12, T_1 , $\hat{T}_2$, and $(\hat{T}_2 - T_1)/T_0$ scale linearly with the number of dimensions.

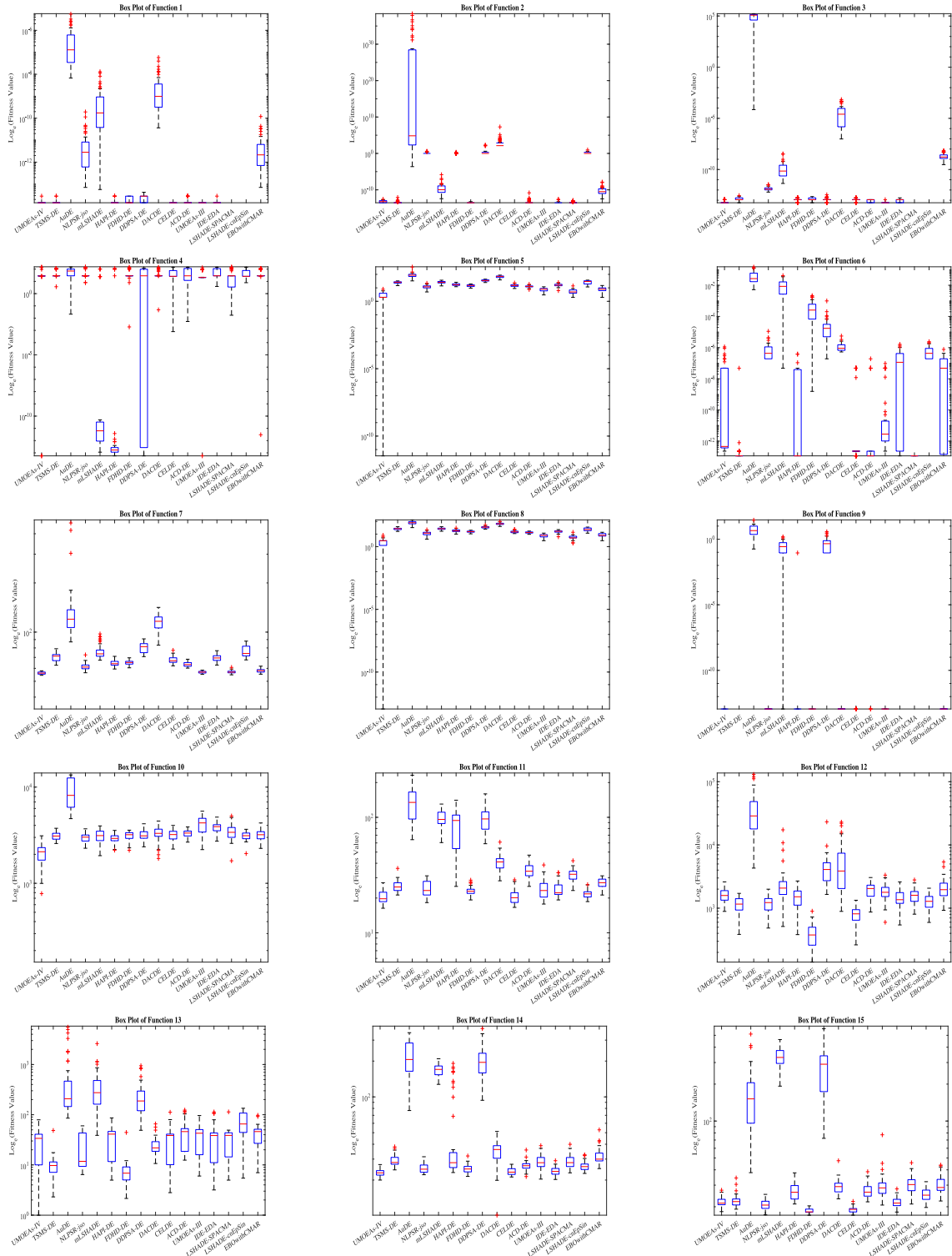


Fig. 9. Box plots of all compared DE-based variants in 50-dimensional experiments on the f_1 – f_{15} of the CEC2017 benchmark.

To ensure a fair comparison, Table 13 reports the computational complexity of UMOEAs-IV and the compared DE-based variants, all evaluated in 30 dimensions. The same system configuration and procedure were applied consistently for each algorithm. As shown in Table 13, UMOEAs-IV ranks 10th, outperforming UMOEAs-III, TSMS-DE, ACD-DE, mLSHADE, AuDE, and CELDE. Considering its notable performance improvements and adaptive evolutionary behavior, the computational cost is regarded as acceptable.

7. Conclusions

This study introduces the fourth generation of the UMOEAs algorithm, named UMOEAs-IV. UMOEAs-IV employs a novel calculation method for the scaling factor F , which provides a flexible adaptive mechanism to control the mutation extent, uses a mutation strategy that better balances exploration and exploitation, integrates an Estimation-of-Distribution Algorithm (EDA) to learn the probabilistic

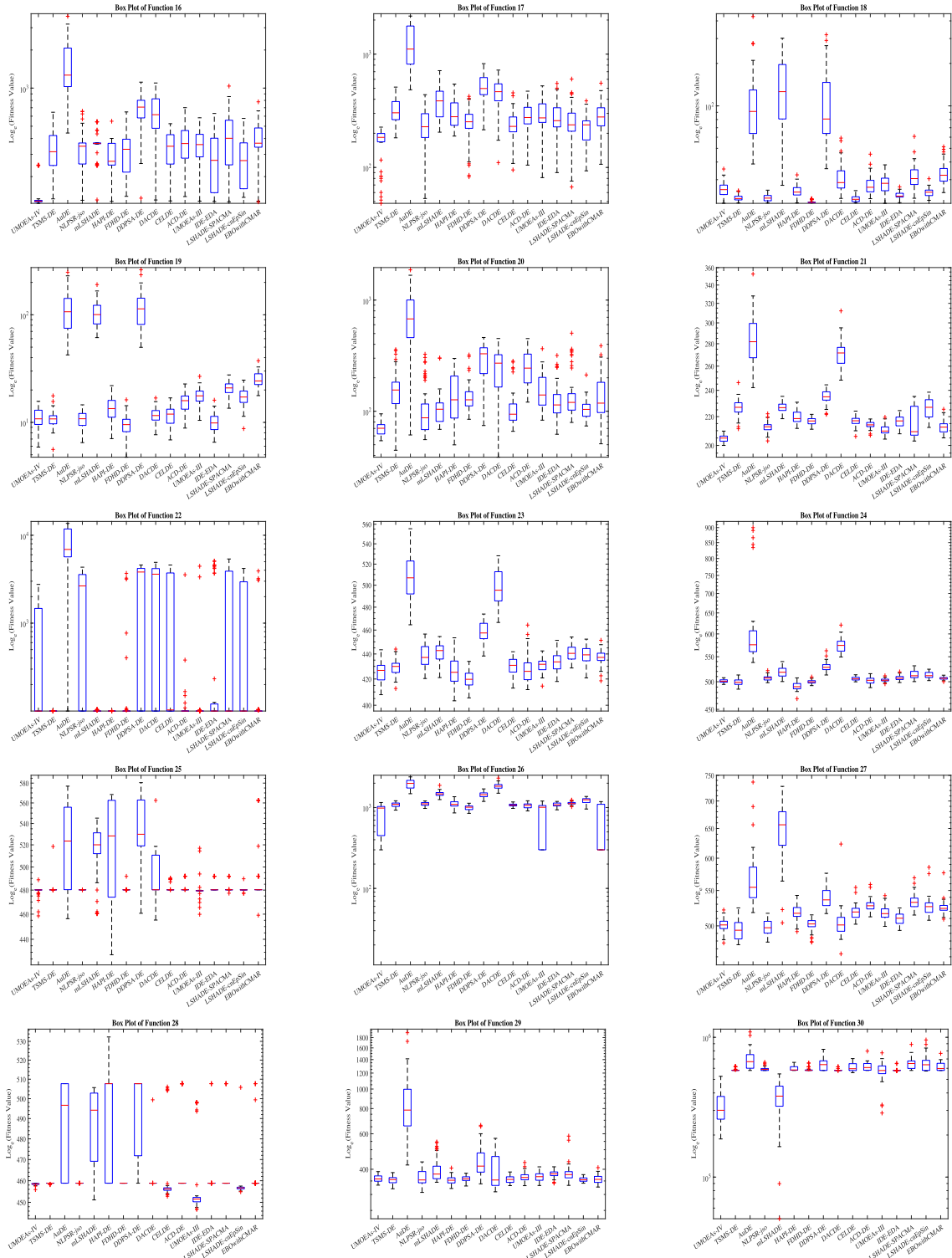


Fig. 10. Box plots of all compared DE-based variants in 50-dimensional experiments on the f_{16} – f_{30} of the CEC2017 benchmark.

distribution of promising individuals, and introduces a stagnation strategy to help individuals escape local optima. The search pattern of UMOEAs-IV demonstrates its intelligent adaptability when addressing different problems. Ablation tests confirm that the integrated mechanisms are well-tuned. Comparisons with recently proposed DE-based variants highlight the advantages of UMOEAs-IV: it achieves the best average Friedman ranking in the 30-, 50-, and 100-dimensional tests, and the second-best ranking in the 10-dimensional tests, second only

to UMOEAs-III. When applied to livestock feed ration optimization problems, UMOEAs-IV achieves the best weighted value among all compared DE-based algorithms, demonstrating its competitive performance on real-world problems. Future work will focus on further improving performance on low-dimensional problems for both hybrid and composition functions. This may include integrating more effective local search methods, designing self-adaptive parameters for the current local search strategies, and considering the gradient in the later stages

Table 13

Computational complexities of UMOEAs-IV, TSMS-DE, AuDE, NLPSR-jSO, mLSHADE, HAPI-DE, FDHD-DE, DDPSA-DE, DACDE, CELDE, ACD-DE, UMOEAs-III, IDE-EDA, LSHADE-SPACMA, LSHADE-cnEpSin, and EBOwithCMAR on 30 dimensions.

Algorithm	T_0 (s)	T_1 (s)	$\hat{T}_2$ (s)	$(\hat{T}_2 - T_1)/T_0$	Rank
UMOEAs-IV	0.0601	0.6686	3.2046	42.1968	10
TSMS-DE	0.0601	0.6686	3.8274	52.5598	12
AuDE	0.0601	0.6686	12.1596	191.1987	15
NLPSR-jSO	0.0601	0.6686	1.5921	15.3667	6
mLSHADE	0.0601	0.6686	5.2336	75.9561	14
HAPI-DE	0.0601	0.6686	1.9943	22.0577	8
FDHD-DE	0.0601	0.6686	0.8757	3.4459	3
DDPSA-DE	0.0601	0.6686	0.8668	3.2975	2
DACDE	0.0601	0.6686	0.6769	0.1388	1
CELDE	0.0601	0.6686	28.7374	467.0349	16
ACD-DE	0.0601	0.6686	4.2905	60.2651	13
UMOEAs-III	0.0601	0.6686	3.2581	43.0858	11
IDE-EDA	0.0601	0.6686	1.0957	7.1071	4
LSHADE-SPACMA	0.0601	0.6686	1.6601	16.4974	7
LSHADE-cnEpSin	0.0601	0.6686	1.5826	15.2072	5
EBOwithCMAR	0.0601	0.6686	2.1335	24.3737	9

of evolution to guide the search direction of individuals. Moreover, observations indicate that once a local mechanism produces the best solution, it seldom does so again, particularly when individuals are extremely close to an optimum. Therefore, the use of local mechanisms can be further optimized to save computational resources, which can then be allocated to more intelligent integrated mechanisms to enhance both exploration and exploitation.

CRedit authorship contribution statement

Libin Hong: Writing – review & editing, Writing – original draft, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization. **Dongxu Zhang:** Writing – review & editing, Visualization, Software, Methodology, Conceptualization. **Tianxiang Cui:** Writing – review & editing.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Libin Hong reports financial support was provided by Hangzhou Normal University. Libin Hong reports financial support was provided by University of Nottingham Ningbo China. Libin Hong reports financial support was provided by a project supported by the Scientific Research Fund of the Zhejiang Provincial Education Department (Grant number: Y202456096). Libin Hong reports financial support was provided by the Ningbo Science and Technology Bureau (Ningbo Commonweal Programme, Grant number: 2025S114). If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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