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A particle swarm optimization-based ensemble metaheuristic for long-term transmission network expansion planning

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ABSTRACT

To significantly enhance the exploration and exploitation capabilities of particle swarm optimization, it is beneficial to hybridize complementary algorithms, integrate effective methods at specific stages, and trigger a 'bail-in' mechanism when encountering a local optimum. In this study, a Particle Swarm Optimization-Based Hybrid Framework (PSO-BHF) is proposed. The framework includes a probability calculation method that considers particle improvement after using Covariance Matrix Adaptation Evolution Strategy (CMA-ES) to promote the convergence rate. Additionally, an opposition-based learning method is integrated to extend search regions and maintain particle diversity if the search becomes stagnant, while a Success History Intelligent Optimizer (SHIO) is employed to prevent premature convergence. Finally, the Sequential Quadratic Programming (SQP) method is used to enhance exploitation around the best particle in the later stages of the evolutionary process. The novel framework was evaluated on CEC2017 benchmark functions and compared with 15 state-of-the-art PSO-based variants and 11 non-PSO-based algorithms. Ablation tests illustrate the effectiveness of individual mechanisms and their combinations. The framework is also applied to a real-world long-term Transmission Network Expansion Planning (TNEP) problem, which was released by GECCO 2023 and IEEE CEC 2023 for their joint competition on evolutionary computation in the energy domain. Experimental results demonstrate the impressive performance of the proposed framework compared to PSO-based variants and its effectiveness in the TNEP problem. However, it still needs time to catch up with the top tier of metaheuristics, specifically Differential Evolution (DE)-based algorithms.

1. Introduction

Evolutionary computation falls within the realm of artificial intelligence and finds widespread application in tackling intricate optimization challenges, particularly in scenarios involving continuous optimization. Particle swarm optimization (PSO) is a metaheuristic algorithm that operates on a population-based approach. PSO has found extensive use in various real-world applications, including addressing expensive multimodal problems [1], estimating the health of electric vehicles [2], diagnosing plant diseases [3], solving the DNA fragment assembly problem [4], predicting streamflow [5], optimizing design parameters for manufacturing processes [6], performing image segmentation [7], designing mechatronic quadrotor systems [8], and improving power system stability [9].

Recent research on PSO has focused on combining complementary mechanisms or algorithms [10–14], integrating various operators, and dividing the population into subpopulations using different algorithms. Zhao et al. [13] proposed an Elite-Ordinary synergistic PSO (EOPSO), where the elite particles learn from themselves to sustain population diversity and achieve extensive global exploration, while the ordinary particles converge toward a common target and focus on localized exploitation. In [13], confident and humble learning exemplars were proposed to guide the elite and ordinary particles, respectively. According to the observation, the EOPSO has the following aspects that can be improved:

- *The search efficiency is suboptimal, characterized by a slow convergence rate.*
- *The exploitation capability is insufficient to facilitate rapid particle convergence, particularly in proximity to the optima.*
- *The optimization strategy lacks sophistication, as it fails to leverage valuable historical information.*
- *The exploitation capability in the vicinity of the current global best solution is deficient.*

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In [15], Opposition-Based Learning (OBL) suggests that in the pursuit of discovering an unknown optimal solution, conducting simultaneous searches in both a randomly chosen direction and its opposite direction increases the likelihood of uncovering promising regions. In [16], the Success History Intelligent Optimizer (SHIO) was proposed, and its results are rooted in the utilization of the three best particles discovered within the search space, which collectively shape a new movement vector. Each of these top particles is retained in memory and is subtracted from the mean of the three best particles identified at a given point in the optimization process. In [17], a modified Covariance Matrix Adaptation Evolution Strategy (CMA-ES) was introduced, and it specifically employed a crossover operation to enhance its exploration capability. Sequential Quadratic Programming (SQP) is a local search technique that demonstrates excellent exploitation capability around a particular solution [18] and can speed up the convergence rate. These studies have inspired and motivated our research, which proposes a framework that integrates various mechanisms with PSO variants. To overcome the drawbacks of EOPSO, the OBL, SHIO, CMA-ES, and SQP methods are integrated to form a Particle Swarm Optimization-Based Hybrid Framework (PSO-BHF). This framework triggers the appropriate mechanism at the right time to accurately guide the evolution, enable collaborative work among all mechanisms, and better balance the exploitation and exploration capabilities. When evolution ceases or falls into local optima, effective mechanisms will intervene to help particles jump out. The success of the framework and the ideas therein can inspire further improvements for PSO-based variants.

Existing metaheuristics often struggle to balance exploration and exploitation, leading to premature convergence or inefficiency. The novelty of this study lies in proposing an effective combination of various mechanisms with properly timed interventions. These mechanisms are strategically integrated into a single framework, leveraging their respective strengths. The OBL mechanism helps prevent particles from getting trapped in local optima. An improved probability calculation using CMA-ES is introduced to enhance the exploitation capability of ordinary particles. SHIO primarily operates during the initial phases while maintaining influence over all particles throughout the evolution. The SQP method is applied in the later stages of the evolutionary process to significantly enhance exploitation around the current global optimum. This work demonstrates how different mechanisms can be seamlessly integrated and work collaboratively to enhance evolutionary optimization performance.

The long-term Transmission Network Expansion Planning (TNEP) problem is a classic challenge in power systems. It determines the optimal time, location, and quantity of new transmission lines to meet specific technical, economic, and environmental constraints. TNEP ensures a structured and cost-effective expansion of power infrastructure to accommodate future electricity demand while maintaining an acceptable level of reliability. However, achieving a high-quality and reliable energy supply must be balanced with budget constraints. TNEP encompasses various models tailored to different planning needs and uncertainties [19,20]. The TNEP model used in this work was released by GECCO 2023 and IEEE CEC 2023 for their joint competition on evolutionary computation in the energy domain [21].

The main highlights of this research are as follows:

- *To evaluate the performance, we compare the PSO-BHF against 15 of the most recently released PSO-based variants and 11 non-PSO-based algorithms. Additionally, the proposed framework is applied to a real-world application for the TNEP problem. The ablation tests illustrate the effectiveness of individual mechanisms and the combination of mechanisms.*
- *In the proposed framework, an OBL method is used to maintain population diversity and extend the search regions during the evolutionary process. A SHIO records several particles of the swarm to avoid premature convergence. Additionally, the SQP method is applied in the later generations to enable sufficient local exploitation near the global optimum.*
- *A probability calculation method that can balance the exploration and exploitation capabilities well is proposed. The probability can perturb ordinary particles generated via CMA-ES as well as humble learning exemplars.*

The subsequent sections of this paper are structured as follows: Section 2 reviews related work, and Section 3 introduces the integrated individual mechanisms and presents the proposed PSO-BHF algorithm, including the integration and implementation of the OBL mechanism, probability calculation using CMA-ES, the SHIO mechanism, and the SQP method. In Section 4, the experimental results and parameter configurations for the compared algorithms are presented. This section also includes comparisons between the proposed method and 15 recent state-of-the-art PSO-based algorithms and 11 non-PSO algorithms, along with analysis and discussions. Section 5 introduces the TNEP model, experimental results, and the computational complexity of the compared PSO-based variants. Lastly, Section 6 summarizes the research and discusses prospects for future work.

2. Related work

2.1. Particle swarm optimization

The fundamental characteristics of particles in classical PSO, namely, velocity and position, are subject to continuous updates for each individual particle [22,23]. The velocity $V_i^d(t)$ and position $X_i^d(t)$ of the ith particle at generation t are recorded, and their respective formulae are presented as follows:

$$\begin{aligned} V_i^d(t) = V_i^d(t-1) + c_1 * r1_i^d * (Pbest_i^d(t-1) - X_i^d(t-1)) \\ + c_2 * r2_i^d * (Gbest^d(t-1) - X_i^d(t-1)) \end{aligned} \tag{1}$$

$$X_i^d(t) = X_i^d(t-1) + V_i^d(t) \tag{2}$$

where $r1_i^d$ and $r2_i^d$ are uniformly randomly generated values within the range [0,1], with $d = 1, 2, \dots, D$, where D represents the dimension size. The constriction coefficients are set to $c_1 = c_2 = 1.49445$ to prevent the explosion of particle velocities. $Pbest_i^d(t)$ denotes the previous best solution of the ith particle at time t, where $i = 1, 2, \dots, N$, and N represents the population size. $Gbest^d(t)$ represents the best solution of the entire swarm at time t, as defined below:

$$Pbest_i^d(t) = min(f(X_i^d(1)), f(X_i^d(2)), \dots.., f(X_i^d(t))) \tag{3}$$

$$Gbest^d(t) = min(f(Pbest_1^d(t)), f(Pbest_2^d(t)), \dots.., f(Pbest_N^d(t))) \tag{4}$$

2.2. EOPSO

As a variant of PSO, EOPSO offers the following improvements: (1) particles are classified into elite and ordinary members based on their fitness values; (2) a jump-out strategy based on information interaction is devised to address situations in which particles become stagnant. In EOPSO, Eqs. (5) and (6) are used to update the velocities for the elite particles, where confident $E_i^d(t)$ represents $Pbest_i^d(t)$, which is the ith elite particle.

$$V_i^d(t) = \omega(t) * V_i^d(t-1) + c * r1_i^d * (E_i^d(t-1) - X_i^d(t-1)) \tag{5}$$

$$E_i^d(t) = Pbest_i^d(t) \tag{6}$$

where $\omega(t) = 0.9 - 0.5 \times \frac{t}{T}$, where t is the current generation number and T is the maximum generation number. The constriction coefficient c is

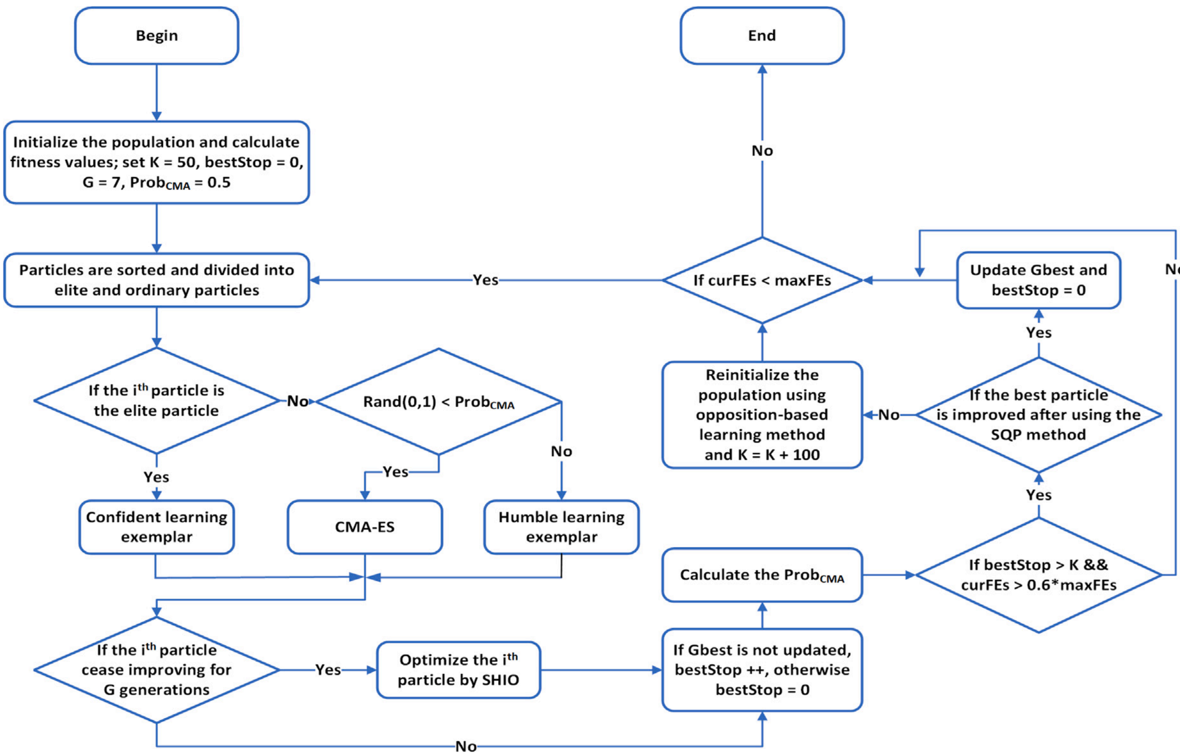

Fig. 1. Main workflow of PSO-BHF.

set to 1.49445, and the humble learning exemplars $E_i^d(t)$ for ordinary particles are evaluated using Eqs. (7) and (8):

$$w_i(t) = \begin{cases} \frac{1}{1+f(X_i^d(t))}, & f(X_i^d(t)) > 0 \\ 1 + |f(X_i^d(t))|, & f(X_i^d(t)) \le 0 \end{cases} \tag{7}$$

where $f(X_i^d(t))$ is the fitness value at tth generation, $1 \le i \le K$, and K is the number of elite particles.

$$Q^d(t) = \sum_{k=1}^{K} \frac{w_k(t) * Pbest_i^d(t)}{\sum_{k=1}^{K} w_k(t)} \tag{8}$$

where $Pbest_i^d(t)$ is the best experienced position of the ith elite particle and $1 \le i \le K$.

$$E_i^d(t) = Q^d(t) \tag{9}$$

In EOPSO, confident learning exemplars offer elite particles multiple promising pathways for extensive exploration, whereas a humble exemplar guides ordinary particles to converge toward a common target, fostering cooperative exploitation. Although EOPSO claims that using ordinary particles to improve local search around promising areas identified by elite particles is hopeful and practical, observations show that ordinary particles contribute insufficiently to the elite particles' local search capability. Additionally, there is a slow convergence rate, a lack of use of historical information, and poor digging ability around the global best particles.

2.3. CMA-ES

In [24], CMA-ES was proposed, and it employs a multivariate normal distribution to represent the search space. It generates new individuals by utilizing a Gaussian distribution that considers the evolutionary path followed by the population across generations. CMA-ES seamlessly and automatically adjusts the mean vector m, covariance matrix C, and step size σ during its optimization process. In [17], a hybridization framework named LSHADE-SPACMA, which combines LSHADE-SPA and a modified version of CMA-ES, is introduced. To balance the usage of LSHADE-SPA and CMA-ES, a probability calculation method was proposed to utilize CMA-ES. A memory of probabilities, M_{Prob}, is created, and its size is set to 5, with the initial probability values set to 0.5. A $Prob_r$ is randomly selected from the memory of probabilities, M_{Prob}, for each particle, and $Prob_r$ is updated using Eqs. (10) to (12).

$$Prob_r(t) = (1-c) * Prob_r(t-1) + c * Prob_{CMA}(t) \tag{10}$$

$$Prob_{CMA}(t) = max(0.2, min(0.8, \frac{IMP_{CMA}(t)}{IMP_{CMA}(t) + IMP_{LSHADE}(t)})) \tag{11}$$

$$IMP_{CMA}(t) = \sum_{i=1}^{n} f(X_i^d(t-1)) - f(X_i^d(t)) \tag{12}$$

where c is the learning rate, and n is the number of particles using CMA-ES. The probability relationship is given by $Prob_{LSHADE}(t) = 1 - Prob_{CMA}(t)$. Here, $Prob_{CMA}(t)$ and $Prob_{LSHADE}(t)$ represent the probabilities for CMA-ES and LSHADE, respectively.

2.4. The timing of SQP method intervention

The superior local search capability of the SQP method has been demonstrated in recent research for both PSO and DE-based variants [18,25,26], owing to its ability to convert nonlinear problems into linear ones [27]. However, the timing of the SQP method's intervention should be evaluated based on the characteristics of each specific algorithm. In [18,25], the intervention condition for the SQP method is set to $\ge 0.75 \times maxFEs$, where $maxFEs$ represents the maximum Function Evaluations (FEs). In [26], the SQP method is intermittently invoked throughout the entire evolutionary process.

3. Particle swarm optimization-based hybrid framework

The workflow of the proposed PSO-BHF is illustrated in Fig. 1. Within the framework, confident learning exemplars are applied to elite particles, whereas both CMA-ES and humble learning exemplars are applied to ordinary particles, based on probability. If the ith particle fails to improve for G generations, the SHIO mechanism using the best three particles will intervene to guide the evolution. If the $Gbest^d(t)$ cannot be further improved, the SQP method will be applied to $Gbest^d(t)$ to seek a better solution. If this approach fails, the OBL mechanism will intervene to extend the search regions. FEs for the OBL mechanism, CMA-ES, SHIO mechanism, and SQP method are included as part of FEs to ensure a fair comparison. The proposed PSO-BHF has the following advantages:

- *The local search capability is greatly enhanced not only during the evolutionary process but also around the global best particle due to the integration of the CMA-ES and SQP methods.*
- *The intervention of the OBL mechanism, which is a 'bail-in' mechanism, will be triggered when a local optimum is encountered or when evolution becomes stagnant. Thus, it can effectively and efficiently prevent particles from falling into local optima.*
- *It leads to effective convergence and a better exploration-exploitation balance because SHIO records successful past solutions and uses this information to guide searches.*

The pseudocode of PSO-BHF is given in Algorithm 1. The source code is written in MATLAB and can be downloaded from the GitHub repository (https://github.com/microhard1999/CODES). It can also be obtained from the first/corresponding author upon request.

3.1. Opposition-based learning method

In [28], two distinct definitions of opposition-based learning (OBL) methods are presented: (1) opposite number; and (2) opposite point in the D space. Since the particles have D dimensions, the opposite point in the D dimensions is selected, as described by Eqs. (13) and (14):

$$\widetilde{X}_i^d(t) = UB + LB - X_i^d(t), i \in \{1, 2, \dots, N\} \tag{13}$$

$$X_i^d(t) = \begin{cases} \widetilde{X}_i^d(t), & f(\widetilde{X}_i^d(t)) < f(X_i^d(t)) \\ X_i^d(t), & f(\widetilde{X}_i^d(t)) \geq f(X_i^d(t)) \end{cases} \tag{14}$$

where N denotes the population size, and UB and LB represent the upper and lower bounds of the fitness function, respectively. Thus, the survival particle has a better fitness value and not only speeds up the convergence rate but also further extends the search regions. In the proposed framework, the OBL mechanism is mainly used when the population is initialized and the optimal solution stagnates in later generations. To implement this mechanism, a variable K is used to store the stagnant times, and the OBL mechanism is triggered after $K + 100$ stagnant periods occur. The essence of the OBL is a 'bail-in' mechanism that helps prevent the algorithm from being trapped in local optima.

3.2. The probability calculation of using CMA-ES

In PSO-BHF, we propose an improved probability calculation method that uses Eqs. (15) and (16), to apply CMA-ES to ordinary particles.

$$IMP_{CMA}(t) = \frac{\sum_{i=1}^{n} max(0, f(X_i^d(t-1)) - f(X_i^d(t)))}{n} \tag{15}$$

where n is the number of particles using CMA-ES. If $f(X_i^d(t-1))$ is worse than $f(X_i^d(t))$, then $max(0, f(X_i^d(t-1)) - f(X_i^d(t)))$ is 0. This implies that the worse improvement will not be counted for $IMP_{CMA}(t)$.

$$Prob_{CMA}(t) = max(0.2, min(0.8, \frac{IMP_{CMA}(t)}{IMP_{CMA}(t) + IMP_{HumbleLearning}(t)})) \tag{16}$$

Algorithm 1 Pseudocode of PSO-BHF

Set $PS = 40$, $G = 7$, $c = 1.49445$, $Prob_{CMA} = 0.5$, $K = 50$, $bestStop = 0$, $t = 0$, and all other parameters required.
for $i : PS$ **do**
  $X_i^d \leftarrow$ uniformly distributed D random numbers.
end for
while termination condition is not satisfied **do**
  $t = t + 1$
  Sort the particles by fitness values and divide them into elite and humble particles.
  for $i : PS$ **do**
    if X_i^d is the elite particle **then**
      Update the velocity by learning exemplar E_i^d using Equations (5) and (6).
    else
      if rand(0,1) $\leq Prob_{CMA}$ **then**
        Update X_i^d using CMA-ES.
      else
        Update the velocity by learning exemplar E_i^d using Equations (5), (7), (8), and (9).
      end if
    end if
    if the X_i^d cease improving for G generations **then**
      Optimize X_i^d using SHIO as described in Equations (17) to (22).
    end if
  end for
  if $Gbest^d(t)$ is not updated **then**
    $bestStop$ ++
  else
    $bestStop = 0$
  end if
  Calculate $Prob_{CMA}$ using Equations (15) and (16).
  if $bestStop \leq$ K and $curFEs > 0.6 \times maxFEs$ **then**
    if the best particle is improved after using the SQP method **then**
      Update $Gbest^d(t)$ and $bestStop = 0$.
    else
      Reinitialize the population using the opposition-based learning method with Equations (13) and (14).
      K = K + 100
    end if
  end if
end while

where $Prob_{CMA}(t)$ and $Prob_{HumbleLearning}(t) = 1 - Prob_{CMA}(t)$ are the probabilities for the CMA-ES and humble learning exemplar, respectively. Based on our observations, it appears that the utilization of humble learning exemplars is less efficient. The motivation behind integrating both CMA-ES and the humble learning exemplars is to further enhance the ordinary particles' exploitation capability.

3.3. Success history intelligent optimizer

In [16], the success history intelligent optimizer (SHIO) mechanism was proposed to guide the ith particle by using information from the best three particles. In our proposed framework, the SHIO mechanism guides the particles if they stop improving for G generations, where G is empirically set to 7. The SHIO mechanism is given in Eqs. (17) to (22).

$$\Delta_1^d(t) = X_{best1}^d(t) + (a(t) * 2 * r1(t) - a(t)) * |r1'(t) * X_{best1}^d(t) - X_i^d(t)| \tag{17}$$

$$\Delta_2^d(t) = X_{best2}^d(t) + (a(t) * 2 * r2(t) - a(t)) * |r2'(t) * X_{best2}^d(t) - X_i^d(t)| \tag{18}$$

Table 1
Parameter configurations for the PSO-based variants under comparison.

Algorithm	Year	Default parameter settings
PSO-BHF	–	Population size is set to 40 for all dimensions, c is 1.49445, $Rmax$ is 0.5, $Rmin$ is 0.4, and G is 7, K is 50.
EAPSO	2023	Population size is set to 100 for all dimensions.
PSO-ELPM	2023	Population size is set to 50 for all dimensions, w_{max} is 0.9, w_{min} is 0.4, λ_d is 0.995, NIt is 30, λ_m is 100.
SpadePSO	2023	Population size is set to 40 for all dimensions, w is 0.99 $\Rightarrow$ 0.2, c_1 is 2.5 $\Rightarrow$ 0.5, c_2 is 0.5 $\Rightarrow$ 2.5, c is 3 $\Rightarrow$ 1.5, k_0 is 2, v_k is 6, n_{exp} is 5.
AMSEPSO	2022	Population size is set to 40 for all dimensions, c_1 is 2.5 $\Rightarrow$ 1.5, c_2 is 1.5 $\Rightarrow$ 2.5, w is 0.9 $\Rightarrow$ 0.4, σ is 0.3 × pop size, $stag\ max$ is 5.
EOPSO	2022	Population size is set to 40 for all dimensions, c is 1.49445, $Rmax$ is 0.5, $Rmin$ is 0.4, G is 7.
PPSO	2022	Population size is set to 64 for all dimensions, ρ is set to 0.008, 0.02, 0.04 and 0.008 for 10, 30, 50, and 100D, respectively.
PSO-sono	2022	Population size is set to 100 for all dimensions, r is 0.5, ϵ is $\frac{D}{PS}\times 0.01$, w is 0.9 $\Rightarrow$ 0.4.
SDPSO	2022	Population size is set to 40 for all dimensions, w is 0.9 $\Rightarrow$ 0.2, c_1 is 2.5 $\Rightarrow$ 0.5, c_2 is 0.5 $\Rightarrow$ 2.5, r is 4, LP is 200.
AHPSO	2021	Population size is set to 60 for all dimensions, w is 0.99 $\Rightarrow$ 0.29, c_1 is 2.5 $\Rightarrow$ 0.5, c_2 is 0.5 $\Rightarrow$ 2.5.
HCLDMS-PSO	2021	Population size is set to 40 for all dimensions, w is 0.99 $\Rightarrow$ 0.29, c_1 is 2.5 $\Rightarrow$ 0.5, c_2 is 0.5 $\Rightarrow$ 2.5, Pm is 0.1.
HIDMS-PSO	2020	Population size is set to 40 for all dimensions, w is 0.99 $\Rightarrow$ 0.20, c_1 is 2.5 $\Rightarrow$ 0.5, c_2 is 0.5 $\Rightarrow$ 2.5.
TAPSO	2020	Population size is set to 40 for all dimensions, w is 0.7298, p_c is 0.5, p_m is 0.02, M is pop size/4.
MPSO	2020	Population size is set to 50 for all dimensions, r is a randomly generated value between 0 and 1, cw is 4 × r × (1 - r), w is 0.9 $\Rightarrow$ 0.4.
FMPSO	2019	Population size is set to 60 for all dimensions, c_h is 1.2, c_m is 0.7, c_l is 0.2.
TCSPSO	2019	Population size is set to 50 for all dimensions, c_1 and c_2 are set to 2, w is 0.9 $\Rightarrow$ 0.4.

$$\Delta_3^d(t) = X_{best3}^d(t) + (a(t) * 2 * r3(t) - a(t)) * |r3'(t) * X_{best3}^d(t) - X_i^d(t)| \quad (19)$$

$$a(t) = a(t-1) - 0.004 \quad (20)$$

where the initial value of a is set to 1.5, $X_{best1}^d(t)$, $X_{best2}^d(t)$, and $X_{best3}^d(t)$ are the best three particles, and $r1(t)$, $r1'(t)$, $r2(t)$, $r2'(t)$, $r3(t)$, and $r3'(t)$ are random numbers in [0, 1).

$$\overline{\Delta}^d(t) = \frac{\Delta_1^d(t) + \Delta_2^d(t) + \Delta_3^d(t)}{3} \quad (21)$$

$$X_i^d(t) = \begin{cases} \overline{\Delta}^d(t), & f(\overline{\Delta}^d(t)) < f(X_i^d(t)) \\ X_i^d(t), & f(\overline{\Delta}^d(t)) \geq f(X_i^d(t)) \end{cases} \quad (22)$$

where d represents the dimension, t is the generation number, and $Gbest^d(t)$ will be replaced by $X_i^d(t)$ if $f(X_i^d(t)) < f(Gbest^d(t))$. Calculating the mean of $\Delta_1^d(t)$, $\Delta_2^d(t)$, and $\Delta_3^d(t)$ generates a new solution at their center point. This new solution may help the particle avoid getting stuck at a local optimum and will also redirect the particles toward the new solution. As the best three particles approach the global optimum, the influence of these optimal particles progressively diminishes for the $Gbest^d(t)$ particle. Consequently, the SHIO mechanism primarily operates during the initial phases, which contributes to a sharp convergence rate in the evolutionary process. Although the SHIO mechanism contributes less to the $Gbest^d(t)$ particle, it still retains influence over all particles during the whole evolution.

3.4. Exploitation capability reinforcement

To reinforce the exploitation capability, the SQP method is introduced in the later generations of the evolution. $maxFEs$ represents the maximum value of the FEs. According to observed results, when the particles are near the global optimum, it is crucial to strongly enhance the exploitation capability. Thus, an experience-based coefficient was proposed and set to 0.6. The activation of the SQP method will occur once the product of $maxFEs$ and 0.6 is reached.

The core principle of the SQP method lies in its capability to convert a nonlinear problem into a linear one [29,30]. At its essence, the SQP method aims to determine a suitable direction and formulate a quadratic optimization problem. Nonlinear optimization problems can be formulated as follows:

$$minimize\ f(x),\ such\ that \begin{cases} h(x) = 0 \\ g(x) \leq 0 \end{cases} \quad (23)$$

The Lagrangian of the aforementioned formulation can be written as follows:

$$L(x, \lambda, \mu) = f(x) + \lambda^T h(x) + \mu^T g(x) \quad (24)$$

where λ and μ represent Lagrangian multipliers. The SQP method is an iterative procedure that constructs the problem for a given iteration x_k by solving a quadratic programming sub-problem. Additionally, x_k is employed to generate a new iteration x_{k+1}. The sub-problem is established by linearizing the constraints at x_k and can be expressed as follows:

$$minimize\ f'(x_k)(x - x_k) + \frac{1}{2}(x - x_k)^T Hf(x_k)(x - x_k),\ such\ that \times \begin{cases} h(x_k) + h'(x_k)(x - x_k) = 0 \\ g(x_k) + g'(x_k)(x - x_k) \leq 0 \end{cases} \quad (25)$$

Here, $Hf(x_k)$ represents the Hessian matrix of the objective function f at $x \in \mathbb{R}^n$. Given the SQP method's significant dependence on the initial estimate [31], This study sets the intervention condition for the SQP method to $> 0.6 \times maxFEs$, where 0.6 is an empirically determined value based on experimental results. In PSO-BHF, the SQP method is applied to the current global best particle, $Gbest^d(t)$, to find a better solution. Since $Gbest^d(t)$ represents elite particles, applying the SQP method to the current global optimum enhances the algorithm's exploitation capability.

4. Performance evaluation

In this study, the performance of PSO-BHF was evaluated against a selection of recent PSO-based variants using tests based on the CEC2017 benchmark functions. The chosen PSO-based variants include Elite Archives-enhanced PSO (EAPSO) [14], PSO with Elite Learning, enhanced Parameter updating, and exponential Mutation operator (PSO-ELPM) [32], Surprisingly Popular Algorithm-Based Adaptive Euclidean Distance Topology Learning PSO (SpadePSO) [33], Adaptive Multi-Strategy Ensemble PSO (AMSEPSO) [34], Elite-Ordinary synergistic PSO (EOPSO) [13], Pyramid PSO (PPSO) [35], PSO for Single-Objective Numerical Optimization (PSO-sono) [12], Strategy Dynamics PSO (SDPSO) [36], Altruistic Heterogeneous PSO (AHPSO) [37], Heterogeneous Comprehensive Learning and Dynamic Multi-Swarm PSO (HCLDMS-PSO) [11], Heterogeneous Improved Dynamic Multi-Swarm PSO (HIDMS-PSO) [38], Triple Archives PSO (TAPSO) [39], Modified PSO (MPSO) [40], Fitness-based Multi-role PSO (FMPSO) [41], and Terminal Crossover and Steering-based PSO (TCSPSO) [10]. To ensure fairness, the maximum value of FEs for each variant with the same dimension size was standardized.

4.1. Parameter settings

The maximum value of FEs is generally represented as $D \times 10^4$, where D is the problem dimension. Therefore, for all tested algorithms,

Table 2
Ranking statistics for all PSO-based variants compared on CEC2017 benchmark functions across different dimensions.

D	Criteria	PSO-BHF	EAPSO	PSO-ELPM	SpadePSO	AMSEPSO	EOPSO	PPSO	PSO-sono	SDPSO	AHPSO	HCLDMS-PSO	HIDMS-PSO	TAPSO	MPSO	FMPSO	TCSPSO
10D	Mean [1st]	**13**	2	0	5	3	3	3	3	9	3	5	2	1	3	2	1
	Min [1st]	9	**12**	3	8	10	5	6	9	7	8	8	6	7	6	9	7
	Median [1st]	**13**	5	0	10	6	4	6	6	8	6	7	5	3	4	4	4
	Total [1st]	**35**	19	3	23	19	12	15	18	24	17	20	13	11	13	15	12
	> | = | <	/	19|5|6	24|2|4	12|8|10	16|6|8	20|6|4	19|7|4	17|5|8	12|8|10	15|7|8	10|9|11	18|5|7	19|4|7	21|3|6	20|7|3	19|5|6
30D	Mean [1st]	**15**	2	2	3	0	5	0	0	1	0	4	1	1	0	2	0
	Min [1st]	**12**	9	6	7	10	8	2	4	9	5	6	6	5	3	6	2
	Median [1st]	**17**	4	4	5	2	6	1	1	7	2	2	3	3	1	2	1
	Total [1st]	**44**	15	12	15	12	19	3	5	17	7	12	10	9	4	10	3
	> | = | <	/	21|3|6	22|6|2	18|5|7	23|4|3	20|3|7	24|4|2	26|3|1	15|8|7	21|4|5	20|3|7	18|7|5	21|4|5	23|3|4	23|3|4	29|0|1
50D	Mean [1st]	**15**	1	0	2	0	4	0	0	1	2	2	0	0	0	3	0
	Min [1st]	**13**	5	2	3	5	9	1	2	4	2	2	3	2	0	7	2
	Median [1st]	**14**	4	1	3	0	5	1	0	2	1	2	1	0	0	4	0
	Total [1st]	**42**	10	3	8	5	18	2	2	7	5	6	4	2	0	14	2
	> | = | <	/	22|3|5	24|3|3	18|5|7	25|2|3	19|2|9	27|2|1	28|1|1	21|5|4	20|4|6	21|2|7	21|2|7	22|5|3	28|2|0	22|3|5	29|1|0
100D	Mean [1st]	**10**	1	1	3	2	5	0	0	0	2	3	0	1	0	2	0
	Min [1st]	**12**	3	3	3	4	5	1	0	2	3	2	1	3	0	1	2
	Median [1st]	**11**	0	1	3	2	7	0	0	0	1	1	0	1	0	4	0
	Total [1st]	**33**	4	5	9	8	17	1	0	2	6	6	1	5	0	7	2
	> | = | <	/	23|2|5	26|1|3	23|1|6	19|1|10	19|2|9	25|2|3	25|2|3	22|3|5	16|5|9	20|3|7	20|6|4	24|2|4	27|2|1	24|3|3	29|1|0

Table 3
The rankings from the Friedman test on the mean values for all PSO-based variants on CEC2017 benchmark functions.

D	Criteria	PSO-BHF	EAPSO	PSO-ELPM	SpadePSO	AMSEPSO	EOPSO	PPSO	PSO-sono	SDPSO	AHPSO	HCLDMS-PSO	HIDMS-PSO	TAPSO	MPSO	FMPSO	TCSPSO
10D	Averaged Friedman ranks	5.5500	10.9500	11.6667	4.1000	8.2667	9.8333	11.2333	8.1000	**3.6000**	7.4500	4.6000	7.7167	11.7000	9.9167	9.5833	11.7333
	Rank	4	12	14	2	8	10	13	7	**1**	5	3	6	15	11	9	16
30D	Averaged Friedman ranks	**4.0333**	10.3667	9.8333	5.7833	9.5333	6.9667	9.4333	10.0333	4.8000	7.0333	5.3500	7.4333	10.1333	11.1333	9.2333	14.9000
	Rank	**1**	14	11	4	10	5	9	12	2	6	3	7	13	15	8	16
50D	Averaged Friedman ranks	**3.7333**	9.6333	9.8000	6.1000	9.4000	5.8333	9.9000	10.2667	6.0333	6.4333	5.4667	6.8333	9.9667	13.0667	8.8000	14.7333
	Rank	**1**	10	11	5	9	3	12	14	4	6	2	7	13	15	8	16
100D	Averaged Friedman ranks	**3.7000**	8.6000	9.6000	6.6000	8.9667	6.1667	9.2000	10.7000	6.1333	6.7000	5.9667	6.5667	9.3333	13.7667	9.0333	14.9667
	Rank	**1**	8	13	6	9	4	11	14	3	7	2	5	12	15	10	16

the maximum value of FEs was configured to be 10×10^4, 30×10^4, 50×10^4, and 100×10^4 for dimensions of 10, 30, 50, and 100, respectively. The parameter settings for the proposed PSO-BHF algorithm and all compared PSO variants are presented in Table 1. We employed the suggested default configurations, which had been fine-tuned by the original researchers and demonstrated the most optimal performance among the state-of-the-art PSO variants.

4.2. Experimental results

To assess the performance and reliability of the proposed algorithm, we present ranking statistics for PSO-BHF, EAPSO, PSO-ELPM, SpadePSO, AMSEPSO, EOPSO, PPSO, PSO-sono, SDPSO, AHPSO, HCLDMS-PSO, HIDMS-PSO, TAPSO, MPSO, FMPSO, and TCSPSO. These algorithms are tested on CEC2017 benchmark functions, and the experimental data are tabulated in Table 2. The rankings are based on the top 1 minimum, median, and mean values for the 10-, 30-, 50-, and 100-dimensional tests across all compared variants on CEC2017. In the table, the highest number of top 1 rankings is highlighted in boldface to identify the most successful algorithms. Friedman tests were conducted, and the ranks are given in Table 3. Additionally, Figs. 6 and 7 display the convergence characteristics of PSO-BHF and the compared PSO variants. It is important to mention that, owing to space constraints, this paper provides the averaged evolutionary processes of f_n over 51 runs exclusively for the 50-dimensional test.

To further assess the individual effectiveness of the OBL mechanism, CMA-ES, SHIO mechanism, and SQP method when used in conjunction with PSO-BHF, separate validations were conducted of PSO-BHF without integrating the OBL mechanism, CMA-ES, SHIO mechanism, and SQP method. Ablation experiments were also conducted without each of the two single mechanisms as well as with all combinations. The ablation experiments, using five fixed pairs of probabilities, were conducted to evaluate the effectiveness of the dynamic probability calculation mechanism for CMA-ES usage. The experimental results are summarized in Tables 4 and 5, respectively. These validations were performed on the CEC2017 benchmark functions, and each was tested in 50 dimensions.

4.3. Comparison and analysis

Within the context of the CEC2017 benchmark functions, there are distinct categories of functions. Specifically, $f_1 - f_3$ belong to the category of unimodal functions, $f_4 - f_{10}$ fall under simple multimodal functions, $f_{11} - f_{20}$ are classified as hybrid functions, and $f_{21} - f_{30}$ are categorized as composition functions. For a more comprehensive understanding of these functions, please refer to the detailed descriptions provided in [42].

In Table 2, PSO-BHF achieves the highest number of first-place rankings for both mean (13, 15, 15, and 10) and median (13, 17, 14, and 11) values across experiments of dimensions 10, 30, 50, and 100, respectively. PSO-BHF achieved the highest number of first-place rankings (12, 13, and 12) for the minimum values in the 30-, 50-, and 100-dimensional experiments. In the 10-dimensional experiments, EAPSO and AMSEPSO secured the first and second rankings (12 and 10) for the minimum values, respectively. In this category, both PSO-BHF and PSO-sono obtained the third rank, with a total of 9 occurrences. The total number of first rankings are 35, 44, 42, and 33 in experiments of dimensions 10, 30, 50, and 100, respectively. These results are the best and significantly outperform those of all the compared algorithms. The radar maps depicted in Fig. 2 illustrate a clearer assessment of the rankings. The PSO-BHF rankings for both mean and minimum values are highlighted with red lines; proximity to the center of the circle corresponds to a higher ranking. PSO-BHF outperforms all other algorithms in terms of rankings. In Table 3, the Friedman tests conducted on the mean values across the 30 CEC2017 benchmark functions indicate that PSO-BHF achieves the highest ranking across dimensions 30, 50, and 100, while it ranks fourth in dimension 10. This suggests that PSO-BHF still has room for improvement in low-dimensional tests. To better review the rankings, radar maps are given in Fig. 2. The red points and lines represent the rankings of PSO-BHF. The numbers from 1 to 30 around the outermost circle indicate the function numbers. The closer a point is to the center of the circle, the better the performance for the corresponding function.

The experimental results from tests conducted in dimensions of 10, 30, 50, and 100 on the CEC2017 benchmark functions are presented in

Table 4
The mean best values and standard deviations, averaged over 51 runs, for PSO-BHF employing various mechanisms in the 50-dimensional test on CEC2017 benchmark functions.

f_n	Criteria	PSO-BHF	PSO-BHF without OBL	PSO-BHF without CMA-ES	PSO-BHF without SHIO	PSO-BHF without SQP	PSO-BHF without OBL and CMA-ES	PSO-BHF without OBL and SHIO	PSO-BHF without OBL and SQP	PSO-BHF without CMA-ES and SHIO	PSO-BHF without CMA-ES and SQP	PSO-BHF without SHIO and SQP
f_1	Mean	**1.211863E−02**	1.259882E−02	1.507863E−02	1.505314E−02	5.114933E+03	1.442882E−02	1.395843E−02	5.250817E+03	1.452961E−02	1.366844E+05	2.347079E+03
	(Std Dev)	(5.511680E−03)	(5.517330E−03)	(3.579636E−03)	(3.176194E−03)	(5.781822E+03)	(3.936950E−03)	(4.649211E−03)	(5.524350E+03)	(3.605767E−03)	(2.137322E+04)	(2.769898E+03)
	Friedman rank	**1**	2	7	6	9	4	3	10	5	11	8
f_2	Mean	**6.860784E−04**	1.017451E−03	9.156863E−04	1.180000E−03	2.124695E+12	1.396078E−03	1.170196E−03	2.255112E+07	1.980784E−03	5.467331E+18	6.330212E+02
	(Std Dev)	(1.585369E−03)	(3.270032E−03)	(2.427964E−03)	(2.978406E−03)	(1.517276E+13)	(3.637633E−03)	(3.938219E−03)	(1.396037E+08)	(5.226946E−03)	(3.904163E+19)	(1.865160E+03)
	Friedman rank	**1**	3	2	5	10	6	4	9	7	11	8
f_3	Mean	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	3.711196E−02	**0.000000E+00**	**0.000000E+00**	5.154235E−02	**0.000000E+00**	2.300742E+04	1.006850E+01
	(Std Dev)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(3.062876E−02)	(0.000000E+00)	(0.000000E+00)	(5.714508E−02)	(0.000000E+00)	(4.130429E+03)	(1.782449E+01)
	Friedman rank	**4**	**4**	**4**	**4**	8	**4**	**4**	9	**4**	11	10
f_4	Mean	3.877246E+01	3.910058E+01	4.654843E+01	4.700394E+01	5.177658E+01	3.814942E+01	**3.578859E+01**	7.123106E+01	4.991452E+01	1.384439E+02	5.572939E+01
	(Std Dev)	(4.559231E+01)	(4.672107E+01)	(3.536211E+01)	(4.416544E+01)	(4.793501E+01)	(3.741418E+01)	(4.220871E+01)	(4.566656E+01)	(4.041080E+01)	(3.823559E+01)	(4.432483E+01)
	Friedman rank	3	4	5	6	8	2	**1**	10	7	11	9
f_5	Mean	**1.946998E+01**	2.015279E+01	2.664188E+01	2.021132E+01	1.971891E+01	2.438150E+01	1.968458E+01	2.019518E+01	2.538106E+01	3.631368E+01	1.949079E+01
	(Std Dev)	(5.117047E+00)	(4.257880E+00)	(5.169235E+00)	(5.281685E+00)	(5.259984E+00)	(4.227383E+00)	(4.789520E+00)	(3.271622E+00)	(5.254623E+00)	(4.048943E+01)	(5.105136E+00)
	Friedman rank	**1**	5	10	7	4	8	3	6	9	11	2
f_6	Mean	4.543104E−01	6.405096E−01	6.740902E−02	4.068994E−01	5.516794E−01	6.948941E−02	6.047755E−01	5.422459E−01	**6.539431E−02**	1.190392E−01	4.249943E−01
	(Std Dev)	(3.041868E−01)	(4.368464E−01)	(1.193645E−02)	(3.462704E−01)	(3.352799E−01)	(2.292109E−02)	(4.064964E−01)	(3.899686E−01)	(1.311692E−02)	(1.619694E−02)	(3.354294E−01)
	Friedman rank	7	11	2	5	9	3	10	8	**1**	4	6
f_7	Mean	7.332846E+01	7.350338E+01	8.766141E+01	7.533172E+01	8.125044E+01	9.330189E+01	**7.305678E+01**	7.579727E+01	9.029929E+01	1.410347E+02	8.048212E+01
	(Std Dev)	(6.736992E+00)	(6.190024E+00)	(8.924414E+00)	(6.370119E+00)	(3.438746E+01)	(1.900812E+01)	(5.551993E+00)	(1.415514E+01)	(9.789710E+00)	(8.013643E+01)	(3.305890E+01)
	Friedman rank	2	3	8	4	7	10	**1**	5	9	11	6
f_8	Mean	1.984065E+01	2.099168E+01	2.735407E+01	1.937244E+01	2.025375E+01	2.607232E+01	1.978212E+01	2.015635E+01	2.543973E+01	4.704874E+01	**1.851542E+01**
	(Std Dev)	(5.307568E+00)	(3.643533E+00)	(5.333086E+00)	(4.094362E+00)	(4.922259E+00)	(5.205617E+00)	(4.398783E+00)	(3.934149E+00)	(4.414992E+00)	(5.447699E+01)	(4.650049E+00)
	Friedman rank	4	7	10	2	6	9	3	5	8	11	**1**
f_9	Mean	1.254961E+01	1.699783E+01	3.573787E+00	1.478657E+01	1.923934E+01	3.031143E+00	1.453172E+01	1.104265E+01	**2.762159E+00**	3.604453E+00	1.657865E+01
	(Std Dev)	(1.119452E+01)	(1.865614E+01)	(4.005733E+00)	(1.204662E+01)	(1.653486E+01)	(2.453142E+00)	(1.238262E+01)	(8.521427E+00)	(2.259070E+00)	(3.550913E+00)	(1.200262E+01)
	Friedman rank	6	10	3	8	11	2	7	5	**1**	4	9
f_{10}	Mean	5.762541E+03	5.440114E+03	4.627377E+03	4.459827E+03	7.520474E+03	4.649726E+03	4.400764E+03	7.216633E+03	**4.278481E+03**	5.486529E+03	4.893180E+03
	(Std Dev)	(1.866902E+03)	(1.380294E+03)	(8.585411E+02)	(1.583282E+03)	(3.264887E+03)	(8.538679E+02)	(1.476043E+03)	(2.720917E+03)	(7.313795E+02)	(1.709359E+03)	(2.203257E+03)
	Friedman rank	9	7	4	3	11	5	2	10	**1**	8	6
f_{11}	Mean	1.528567E+02	1.541131E+02	1.429071E+02	1.585062E+02	1.548295E+02	1.403602E+02	1.615696E+02	1.569837E+02	**1.194058E+02**	2.465035E+02	1.598156E+02
	(Std Dev)	(3.524452E+01)	(3.462145E+01)	(4.035660E+01)	(3.413004E+01)	(2.869586E+01)	(3.830815E+01)	(3.649904E+01)	(2.998326E+01)	(2.783598E+01)	(5.756029E+01)	(3.756822E+01)
	Friedman rank	4	5	3	8	6	2	10	7	**1**	11	9
f_{12}	Mean	2.056617E+03	2.169075E+03	**2.035724E+03**	3.062987E+03	3.023209E+05	2.147570E+03	2.110141E+03	2.434109E+05	2.249914E+03	4.070217E+06	2.365481E+05
	(Std Dev)	(4.890698E+02)	(6.112244E+02)	(5.224390E+02)	(7.045490E+03)	(1.704380E+05)	(4.810973E+02)	(4.326176E+02)	(1.484994E+05)	(4.730958E+02)	(2.140951E+06)	(1.247051E+05)
	Friedman rank	2	5	**1**	7	10	4	3	9	6	11	8
f_{13}	Mean	5.282313E+02	**4.918868E+02**	5.416201E+02	5.203222E+02	6.027987E+04	5.933042E+02	5.304533E+02	6.187678E+04	5.259452E+02	1.760667E+04	5.297612E+04
	(Std Dev)	(1.416402E+02)	(1.695532E+02)	(1.848915E+02)	(2.077388E+02)	(2.427645E+04)	(2.532792E+02)	(1.945960E+02)	(2.533694E+04)	(1.551998E+02)	(4.633773E+03)	(2.525650E+04)
	Friedman rank	4	**1**	6	2	10	7	5	11	3	8	9
f_{14}	Mean	2.737460E+02	2.462002E+02	**2.255885E+02**	2.600550E+02	3.044165E+03	2.309782E+02	2.677754E+02	2.216409E+03	2.337562E+02	7.199393E+04	2.334403E+03
	(Std Dev)	(8.598322E+01)	(6.483432E+01)	(4.660834E+01)	(7.196419E+01)	(2.175314E+03)	(4.670082E+01)	(7.358656E+01)	(1.238018E+03)	(4.328152E+01)	(6.771603E+04)	(1.308416E+03)
	Friedman rank	7	4	**1**	5	10	2	6	8	3	11	9
f_{15}	Mean	3.911625E+02	3.900659E+02	5.079451E+02	3.805225E+02	1.400135E+04	4.845148E+02	**3.654134E+02**	1.243514E+04	4.887391E+02	6.952967E+03	1.389739E+04
	(Std Dev)	(1.047539E+02)	(1.057921E+02)	(1.885494E+02)	(1.120110E+02)	(6.024067E+03)	(1.608063E+02)	(1.051466E+02)	(5.372578E+03)	(1.530588E+02)	(5.286483E+03)	(6.708796E+03)
	Friedman rank	4	3	7	2	11	5	**1**	9	6	8	10
f_{16}	Mean	4.581635E+02	4.580366E+02	5.923259E+02	3.901862E+02	4.699891E+02	5.718957E+02	**3.710631E+02**	4.465029E+02	5.969800E+02	6.743905E+02	4.527667E+02
	(Std Dev)	(1.582372E+02)	(2.218423E+02)	(2.233420E+02)	(1.500640E+02)	(2.034819E+02)	(2.295344E+02)	(1.489085E+02)	(2.139101E+02)	(2.045296E+02)	(2.650360E+02)	(1.584478E+02)
	Friedman rank	6	5	9	2	7	8	**1**	3	10	11	4
f_{17}	Mean	4.726377E+02	4.482808E+02	6.503744E+02	4.420297E+02	5.012446E+02	6.253209E+02	4.781112E+02	4.588376E+02	5.812170E+02	7.207700E+02	**4.289456E+02**
	(Std Dev)	(1.801277E+02)	(1.672530E+02)	(1.890039E+02)	(2.174250E+02)	(1.776635E+02)	(2.175480E+02)	(1.574035E+02)	(1.784181E+02)	(1.889371E+02)	(2.822685E+02)	(1.879947E+02)
	Friedman rank	5	3	10	2	7	9	6	4	8	11	**1**
f_{18}	Mean	2.382751E+02	2.645870E+02	1.809134E+02	2.633291E+02	4.073908E+04	**1.802438E+02**	2.440350E+02	3.759708E+04	1.866370E+02	8.893450E+05	4.454043E+04
	(Std Dev)	(8.268008E+01)	(1.041016E+02)	(5.211148E+01)	(1.059916E+02)	(1.890715E+04)	(5.506941E+01)	(8.187007E+01)	(1.531708E+04)	(5.106439E+01)	(4.914163E+05)	(2.124307E+04)
	Friedman rank	4	7	2	6	9	**1**	5	8	3	11	10

(*continued on next page*)

Table 4 (*continued*).

f_{19}	Mean	3.684250E+02	3.875231E+02	**2.053030E+02**	4.088001E+02	1.423894E+04	2.120011E+02	4.325484E+02	1.478534E+04	2.469508E+02	1.379596E+04	1.363696E+04
	(Std Dev)	(1.751089E+02)	(1.719872E+02)	(1.000094E+02)	(2.037026E+02)	(8.116834E+03)	(1.049303E+02)	(2.128829E+02)	(7.952237E+03)	(1.631043E+02)	(6.618427E+03)	(8.647154E+03)
	Friedman rank	4	5	**1**	6	10	2	7	11	3	9	8
f_{20}	Mean	1.750021E+02	2.065192E+02	2.370170E+02	1.703907E+02	1.829843E+02	3.167559E+02	**1.659907E+02**	1.731844E+02	2.965197E+02	3.249502E+02	1.973781E+02
	(Std Dev)	(9.451588E+01)	(1.135884E+02)	(1.366830E+02)	(9.041474E+01)	(9.512022E+01)	(1.481040E+02)	(7.661225E+01)	(8.844477E+01)	(1.549175E+02)	(1.900528E+02)	(9.750756E+01)
	Friedman rank	4	7	8	2	5	10	**1**	3	9	11	6
f_{21}	Mean	2.177670E+02	2.183358E+02	2.290960E+02	2.173060E+02	2.176878E+02	2.278668E+02	2.182516E+02	2.180359E+02	2.291667E+02	2.415363E+02	**2.171093E+02**
	(Std Dev)	(4.208717E+00)	(4.827912E+00)	(6.165137E+00)	(4.017332E+00)	(3.754256E+00)	(5.507486E+00)	(4.407030E+00)	(3.702731E+00)	(6.360305E+00)	(3.908898E+01)	(4.197303E+00)
	Friedman rank	4	7	9	2	3	8	6	5	10	11	**1**
f_{22}	Mean	2.643842E+03	2.797375E+03	3.299911E+03	2.671575E+03	2.494541E+03	1.983861E+03	2.620722E+03	**1.964840E+03**	3.108319E+03	3.458741E+03	2.564511E+03
	(Std Dev)	(2.397566E+03)	(2.843731E+03)	(2.242743E+03)	(2.168515E+03)	(2.954971E+03)	(2.429579E+03)	(2.809856E+03)	(2.889264E+03)	(2.110033E+03)	(3.073061E+03)	(1.988957E+03)
	Friedman rank	6	8	10	7	3	2	5	**1**	9	11	4
f_{23}	Mean	4.605240E+02	4.602496E+02	4.862808E+02	4.637387E+02	4.597621E+02	4.779460E+02	4.604068E+02	**4.582138E+02**	4.823837E+02	4.787479E+02	4.629516E+02
	(Std Dev)	(1.441918E+01)	(1.619456E+01)	(2.941371E+01)	(1.393954E+01)	(1.381702E+01)	(2.986104E+01)	(1.489110E+01)	(1.517122E+01)	(2.763078E+01)	(3.075448E+01)	(1.464601E+01)
	Friedman rank	5	3	11	7	2	8	4	**1**	10	9	6
f_{24}	Mean	5.284961E+02	5.289964E+02	5.498918E+02	**5.268041E+02**	5.282177E+02	5.464593E+02	5.291556E+02	5.285621E+02	5.525548E+02	5.513991E+02	5.293253E+02
	(Std Dev)	(8.286214E+00)	(7.197569E+00)	(1.627614E+01)	(7.986521E+00)	(7.981592E+00)	(1.692101E+01)	(1.029188E+01)	(6.870856E+00)	(2.537310E+01)	(3.322132E+01)	(8.613920E+00)
	Friedman rank	3	5	9	**1**	2	8	6	4	11	10	7
f_{25}	Mean	5.355475E+02	5.249794E+02	5.207287E+02	5.429145E+02	5.269153E+02	**5.184824E+02**	5.357767E+02	5.314155E+02	5.379556E+02	5.356926E+02	5.420020E+02
	(Std Dev)	(3.959992E+01)	(4.546262E+01)	(4.093168E+01)	(3.628676E+01)	(4.030933E+01)	(3.838382E+01)	(3.921700E+01)	(3.884620E+01)	(3.278750E+01)	(3.237325E+01)	(3.806905E+01)
	Friedman rank	6	3	2	11	4	**1**	8	5	9	7	10
f_{26}	Mean	1.303747E+03	**1.119627E+03**	1.304835E+03	1.334718E+03	1.307030E+03	1.292767E+03	1.190124E+03	1.204590E+03	1.293910E+03	1.476118E+03	1.255135E+03
	(Std Dev)	(2.645815E+02)	(4.662170E+02)	(1.338906E+02)	(1.677572E+02)	(6.397841E+02)	(2.690691E+02)	(3.998708E+02)	(4.010803E+02)	(1.089878E+02)	(4.348526E+02)	(2.937639E+02)
	Friedman rank	7	**1**	8	10	9	5	2	3	6	11	4
f_{27}	Mean	7.599767E+02	7.950451E+02	5.991124E+02	8.019484E+02	7.891863E+02	6.105553E+02	7.862810E+02	8.070086E+02	**5.901496E+02**	5.977999E+02	7.799477E+02
	(Std Dev)	(9.391126E+01)	(1.035829E+02)	(4.309799E+01)	(9.508151E+01)	(1.085730E+02)	(5.950240E+01)	(1.026328E+02)	(1.075692E+02)	(5.009570E+01)	(4.594735E+01)	(9.003597E+01)
	Friedman rank	5	9	3	10	8	4	7	11	**1**	2	6
f_{28}	Mean	4.959519E+02	4.996006E+02	4.806189E+02	4.982189E+02	4.967092E+02	4.743736E+02	4.969331E+02	4.956111E+02	**4.727289E+02**	4.806027E+02	4.946377E+02
	(Std Dev)	(2.133913E+01)	(1.415568E+01)	(2.410313E+01)	(1.554802E+01)	(2.210195E+01)	(2.231251E+01)	(2.366284E+01)	(2.393573E+01)	(2.179245E+01)	(2.297984E+01)	(1.806155E+01)
	Friedman rank	7	11	4	10	8	2	9	6	**1**	3	5
f_{29}	Mean	7.729285E+02	8.060021E+02	7.823826E+02	7.852383E+02	7.978555E+02	8.659093E+02	7.771089E+02	8.325199E+02	**7.235759E+02**	8.772474E+02	8.307794E+02
	(Std Dev)	(2.222785E+02)	(2.595849E+02)	(1.973963E+02)	(2.405841E+02)	(1.983621E+02)	(2.173379E+02)	(2.016175E+02)	(2.405805E+02)	(2.022731E+02)	(2.452326E+02)	(3.198888E+02)
	Friedman rank	2	7	4	5	6	10	3	9	**1**	11	8
f_{30}	Mean	6.722871E+05	6.783302E+05	**6.145796E+05**	6.732126E+05	9.616116E+05	6.177760E+05	7.016050E+05	9.771659E+05	6.266045E+05	8.897710E+05	9.715184E+05
	(Std Dev)	(8.075526E+04)	(9.526351E+04)	(3.902257E+04)	(8.408405E+04)	(1.967001E+05)	(5.820201E+04)	(1.191899E+05)	(2.551741E+05)	(4.758319E+04)	(9.360939E+04)	(2.146958E+05)
	Friedman rank	4	6	**1**	5	9	2	7	11	3	8	10
Averaged Friedman ranks		**4.3667**	5.3667	5.4667	5.3333	7.4000	5.1000	4.6667	6.8667	5.5000	9.2667	6.6667

Tables 10 to 13, respectively. These tables provide a summary of the outcomes derived from the final generation of 51 independent runs for each function, including the minimum, median, and mean values, along with their standard deviations. Additionally, the tables feature rankings for these values. The tables highlight the smallest minimum, median, and mean values among all recent PSO-based variants in boldface, as well as the top rankings for each of these values. To enhance reliability and facilitate effective comparisons, the tables display additional decimal places instead of using scientific notation to showcase the robust performances of the variants for specific values.

The two-sided Wilcoxon rank sum test (WRST) [43] was conducted to compare PSO-BHF against EAPSO, PSO-ELPM, SpadePSO, AMSEPSO, EOPSO, PPSO, PSO-sono, SDPSO, AHPSO, HCLDMS-PSO, HIDMS-PSO, TAPSO, MPSO, FMPSO, and TCSPSO. Tables 10 to 13 list the WRST outcomes at the 5% significance level that were obtained by comparing PSO-BHF with EAPSO, PSO-ELPM, SpadePSO, AMSEPSO, EOPSO, PPSO, PSO-sono, SDPSO, AHPSO, HCLDMS-PSO, HIDMS-PSO, TAPSO, MPSO, FMPSO, and TCSPSO. The '<' and '>' symbols indicate that PSO-BHF shows a significant performance decline or improvement compared with the other variants, and the '=' symbol suggests that the performances of the compared variants show no apparent differences. In most cases, PSO-BHF surpasses EAPSO, PSO-ELPM, SpadePSO, AMSEPSO, EOPSO, PPSO, PSO-sono, SDPSO, AHPSO, HCLDMS-PSO, HIDMS-PSO, TAPSO, MPSO, FMPSO, and TCSPSO.

In the 10-dimensional experiments on unimodal functions, PSO-BHF showcased superior performance, attaining the highest mean and median values for both f_1 and f_3, and yielding the lowest minimum values for f_2 through f_3. MPSO excelled in terms of mean values for f_2. On the category of simple multimodal functions, PSO-BHF dominated, achieving the best mean, median, and minimum values for both f_4 and f_9. PPSO outperformed the others in terms of mean values for f_5 and f_7, whereas EOPSO and HCLDMS-PSO exhibited the best mean values for f_8 and f_{10}, respectively. When dealing with hybrid functions, PSO-BHF displayed the best mean values for $f_{12} - f_{15}$ and $f_{18} - f_{19}$ as well as the best median values for $f_{11} - f_{13}$, f_{15}, and f_{19}. Additionally, PSO-BHF provided the best minimum values for $f_{11}-f_{12}$ and f_{15}; AHPSO excelled in terms of the mean value for f_{11}, and SDPSO performed well on $f_{16} - f_{17}$ and f_{20}. For the composition functions, PSO-BHF maintained its dominance with the best mean values for f_{21} and $f_{29} - f_{30}$. PSO-BHF secured the best median values for $f_{21} - f_{22}$ and $f_{29} - f_{30}$ as well as the best minimum values for f_{21} and f_{29}. SDPSO achieved the best mean values for f_{22}, f_{25}, and f_{28}; HCLDMS-PSO, SpadePSO, and PSO-sono demonstrated superior mean values for f_{23}, f_{24}, and f_{27}, respectively. In summary, PSO-BHF consistently performed exceptionally well in the low-dimensional experiments, excelling in both unimodal and hybrid functions and demonstrating remarkable proficiency in several simple multimodal and composition functions.

In the 30-dimensional experiments for unimodal functions, PSO-BHF consistently stood out with the best mean, median, and minimum values for both f_1 and f_3, and it also delivered the best minimum values for f_2. EAPSO excelled with the best mean values for f_2. When it came to simple multimodal functions, PSO-BHF exhibited its dominance by achieving the best mean and median values for f_5, $f_7 - f_8$, and it also secured the best minimum values for $f_4 - f_5$ and $f_8 - f_9$. FMPSO showcased the top mean values for f_4, whereas both SpadePSO and EOPSO displayed exceptional performance with the best mean values for f_6. HCLDMS-PSO and SDPSO outperformed the others with the best mean values for f_9 and f_{10}, respectively. When tackling hybrid functions, PSO-BHF once again demonstrated its superiority, achieving the best mean values for f_{12}, $f_{14} - f_{18}$, and f_{20} as well as the best median values for f_{12}, f_{14}, $f_{16} - f_{18}$, and f_{20}. Additionally, PSO-BHF secured the best minimum values for f_{12} and $f_{17} - f_{18}$. HCLDMS-PSO, SpadePSO, and FMPSO performed exceptionally well with the best mean values for f_{11}, f_{13}, and f_{19}, respectively. For the composition functions, PSO-BHF continued to excel with the best mean values for f_{21} and $f_{23}-f_{24}$, and it also achieved the best median values for $f_{21}-f_{24}$ and $f_{28} - f_{29}$. Additionally, PSO-BHF secured the best minimum values for f_{22} and f_{28}. EOPSO delivered the top mean values for f_{22}, f_{25}, f_{27},

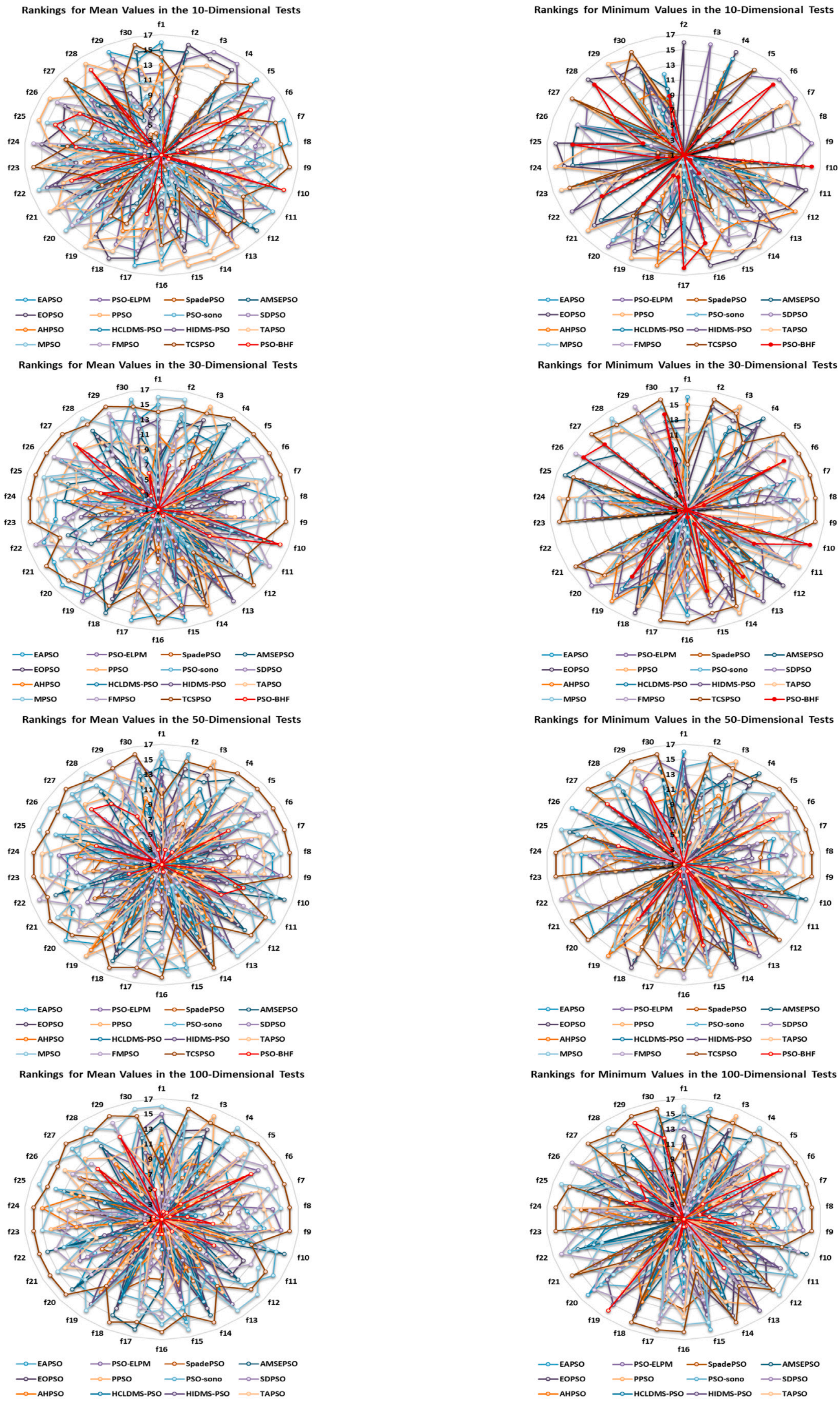

Fig. 2. Radar chart displaying rankings for mean and minimum values across tests conducted in dimensions of 10, 30, 50, and 100. The circumference and radius scales correspond to benchmark functions and rankings, respectively.

Table 5
The mean values and standard deviations, averaged over 51 runs, for PSO-BHF and PSO-BHF with different fixed probabilities of Humble/CMA-ES in the 50-dimensional test on CEC2017 benchmark functions.

f_n	PSO-BHF with Humble/CMA-ES (Dynamic)	PSO-BHF with Humble/CMA-ES (10%/90%)	PSO-BHF with Humble/CMA-ES (30%/70%)	PSO-BHF with Humble/CMA-ES (50%/50%)	PSO-BHF with Humble/CMA-ES (70%/30%)	PSO-BHF with Humble/CMA-ES (90%/10%)
f_1	**1.211863E−02** (5.511680E−03)	1.567216E−02 (3.112674E−03)	1.221647E−02 (6.248567E−03)	1.341059E−02 (5.233620E−03)	1.392000E−02 (4.430632E−03)	1.370039E−02 (4.396686E−03)
f_2	6.860784E−04 (1.585369E−03)	**1.098039E−05** (5.746269E−06)	5.784314E−05 (1.584969E−04)	1.292157E−04 (2.158781E−04)	8.294118E−04 (2.241538E−03)	2.794706E−03 (8.368058E−03)
f_3	**0.000000E+00** (0.000000E+00)	**0.000000E+00** (0.000000E+00)	**0.000000E+00** (0.000000E+00)	**0.000000E+00** (0.000000E+00)	**0.000000E+00** (0.000000E+00)	**0.000000E+00** (0.000000E+00)
f_4	3.877246E+01 (4.559231E+01)	3.667569E+01 (4.226458E+01)	**3.478371E+01** (4.592018E+01)	4.582118E+01 (4.970841E+01)	3.968473E+01 (4.392423E+01)	4.551975E+01 (4.792764E+01)
f_5	1.946998E+01 (5.117047E+00)	**1.841650E+01** (3.798462E+00)	2.060150E+01 (3.885662E+00)	2.126481E+01 (4.691989E+00)	2.278651E+01 (4.608086E+00)	2.482161E+01 (4.752203E+00)
f_6	4.543104E−01 (3.041868E−01)	1.027934E+00 (8.752583E−01)	9.626257E−01 (6.351110E−01)	4.614780E−01 (2.815937E−01)	1.159241E−01 (9.856155E−02)	**4.265451E−02** (2.455473E−02)
f_7	7.332846E+01 (6.736992E+00)	**6.496711E+01** (3.700204E+00)	7.139515E+01 (7.210596E+00)	7.591560E+01 (5.868961E+00)	7.922408E+01 (7.687727E+00)	8.221083E+01 (7.727631E+00)
f_8	1.984065E+01 (5.307568E+00)	1.986016E+01 (4.377641E+00)	**1.968458E+01** (4.108656E+00)	2.060150E+01 (4.760412E+00)	2.331150E+01 (6.531107E+00)	2.423017E+01 (4.997915E+00)
f_9	1.254961E+01 (1.119452E+01)	1.327452E+01 (1.637839E+01)	1.922097E+01 (1.813943E+01)	1.273995E+01 (7.075776E+00)	6.776144E+00 (4.508873E+00)	**1.153055E+00** (1.058458E+00)
f_{10}	5.762541E+03 (1.866902E+03)	7.117825E+03 (2.065150E+03)	6.337731E+03 (1.753807E+03)	6.234438E+03 (1.938027E+03)	5.646287E+03 (1.830542E+03)	**4.763361E+03** (9.931952E+02)
f_{11}	1.528567E+02 (3.524452E+01)	1.618204E+02 (3.727422E+01)	1.579633E+02 (3.296361E+01)	1.574057E+02 (3.595613E+01)	1.328503E+02 (2.533846E+01)	**1.267855E+02** (3.269029E+01)
f_{12}	**2.056617E+03** (4.890698E+02)	2.074732E+03 (5.106128E+02)	5.113752E+03 (2.114937E+04)	2.181744E+03 (5.098475E+02)	2.067059E+03 (3.837220E+02)	2.077864E+03 (4.607341E+02)
f_{13}	5.282313E+02 (1.416402E+02)	5.469429E+02 (2.064160E+02)	5.714680E+02 (2.185200E+02)	5.398656E+02 (1.925827E+02)	**4.928468E+02** (1.987149E+02)	5.337950E+02 (1.836740E+02)
f_{14}	2.737460E+02 (8.598322E+01)	2.540138E+02 (6.588213E+01)	2.356985E+02 (6.705293E+01)	2.680156E+02 (6.738948E+01)	2.315982E+02 (7.923239E+01)	**2.187394E+02** (4.692475E+01)
f_{15}	3.911625E+02 (1.047539E+02)	4.022925E+02 (1.292557E+02)	**3.902093E+02** (1.130784E+02)	4.222917E+02 (1.098659E+02)	3.979307E+02 (9.902043E+01)	4.309780E+02 (1.191393E+02)
f_{16}	4.581635E+02 (1.582372E+02)	5.025827E+02 (2.188705E+02)	4.640389E+02 (2.108946E+02)	**4.252478E+02** (1.854932E+02)	4.521161E+02 (1.973542E+02)	5.476549E+02 (2.288972E+02)
f_{17}	4.726377E+02 (1.801277E+02)	4.778988E+02 (1.430483E+02)	4.587605E+02 (1.459934E+02)	**4.566811E+02** (1.681983E+02)	5.508786E+02 (1.582880E+02)	6.288199E+02 (2.110023E+02)
f_{18}	2.382751E+02 (8.268008E+01)	2.979477E+02 (1.172885E+02)	2.269421E+02 (6.751267E+01)	2.249675E+02 (7.976312E+01)	1.882758E+02 (6.552127E+01)	**1.881697E+02** (6.363692E+01)
f_{19}	3.684250E+02 (1.751089E+02)	3.074661E+02 (1.162562E+02)	4.395755E+02 (1.973345E+02)	4.904236E+02 (2.528452E+02)	2.166462E+02 (1.475400E+02)	**1.942234E+02** (1.042308E+02)
f_{20}	1.750021E+02 (9.451588E+01)	**1.206808E+02** (3.020214E+01)	1.697936E+02 (1.020453E+02)	1.866927E+02 (8.386841E+01)	2.576721E+02 (1.465266E+02)	2.894420E+02 (1.590052E+02)
f_{21}	**2.177670E+02** (4.208717E+00)	2.217185E+02 (4.928978E+00)	2.204728E+02 (4.024193E+00)	2.209948E+02 (7.495319E+00)	2.221682E+02 (5.117666E+00)	2.256792E+02 (6.184917E+00)
f_{22}	2.643842E+03 (2.397566E+03)	3.795954E+03 (2.496873E+03)	3.189473E+03 (2.650180E+03)	3.280338E+03 (2.607651E+03)	2.994593E+03 (2.597967E+03)	**2.289738E+03** (2.309520E+03)
f_{23}	4.605240E+02 (1.441918E+01)	4.668576E+02 (1.408319E+01)	4.614686E+02 (1.352828E+01)	**4.595113E+02** (1.420174E+01)	4.635793E+02 (1.613981E+01)	4.735252E+02 (2.163749E+01)
f_{24}	5.284961E+02 (8.286214E+00)	5.271538E+02 (6.831836E+00)	**5.258918E+02** (6.277476E+00)	5.282481E+02 (8.243028E+00)	5.323350E+02 (1.060021E+01)	5.598835E+02 (2.534833E+01)
f_{25}	5.355475E+02 (3.959992E+01)	5.351090E+02 (3.590053E+01)	5.310815E+02 (3.939841E+01)	5.301350E+02 (4.119788E+01)	**5.285504E+02** (3.867838E+01)	5.302900E+02 (3.742854E+01)
f_{26}	1.303747E+03 (2.645815E+02)	9.804922E+02 (7.418583E+02)	**8.271710E+02** (5.927106E+02)	1.206863E+03 (3.429096E+02)	1.336135E+03 (1.801683E+02)	1.291758E+03 (2.289565E+02)
f_{27}	7.599767E+02 (9.391126E+01)	7.926654E+02 (1.022559E+02)	7.975673E+02 (1.022756E+02)	7.857522E+02 (1.092843E+02)	7.702457E+02 (9.383092E+01)	**6.526211E+02** (6.406927E+01)
f_{28}	4.959519E+02 (2.133913E+01)	4.962462E+02 (1.973620E+01)	4.964043E+02 (1.771943E+01)	**4.938892E+02** (1.965468E+01)	4.985470E+02 (1.683575E+01)	4.966746E+02 (1.906535E+01)
f_{29}	7.729285E+02 (2.222785E+02)	1.115724E+03 (4.703638E+02)	1.008100E+03 (3.216209E+02)	8.301484E+02 (2.334574E+02)	7.898316E+02 (1.998893E+02)	**7.242796E+02** (2.035123E+02)
f_{30}	6.722871E+05 (8.075526E+04)	7.998933E+05 (1.465934E+05)	7.213642E+05 (1.203095E+05)	6.398749E+05 (5.485438E+04)	**6.074958E+05** (3.171657E+04)	6.233892E+05 (4.576799E+04)
Averaged Friedman rank	**2.9833**	3.9833	3.4500	3.5500	3.3833	3.6500

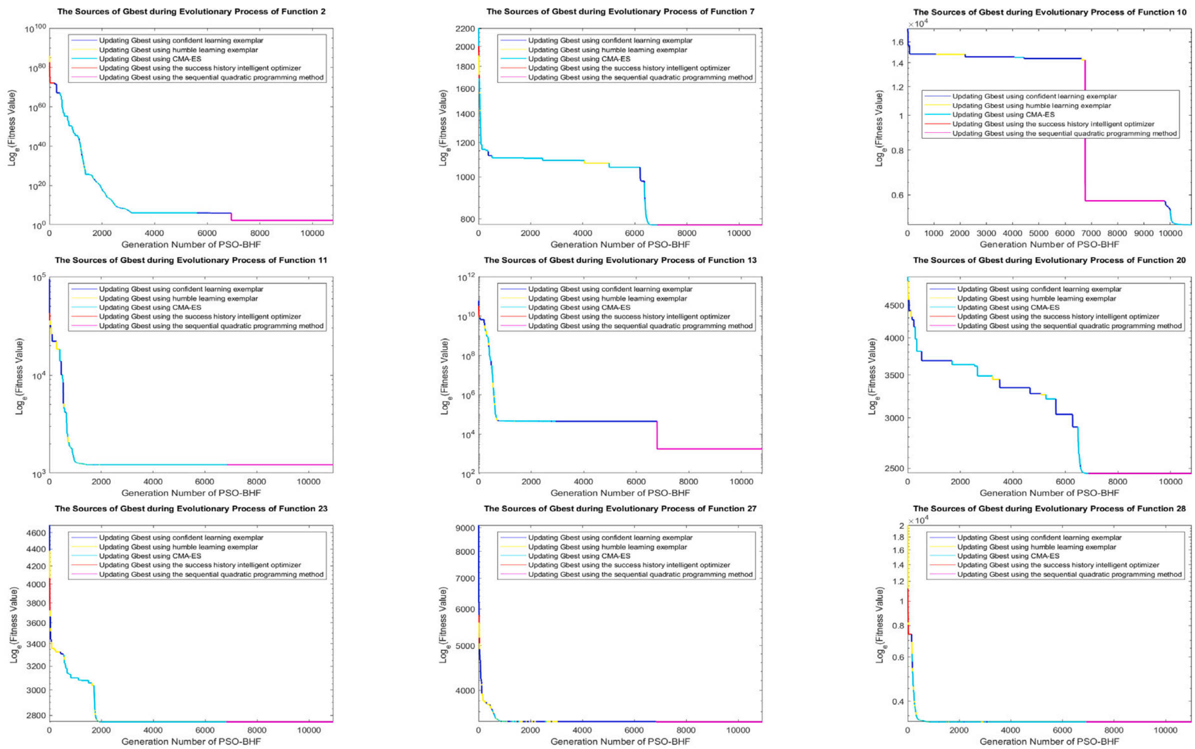

Fig. 3. Sources of the best particle in the 50-dimensional experiments over the evolutionary process of an individual run.

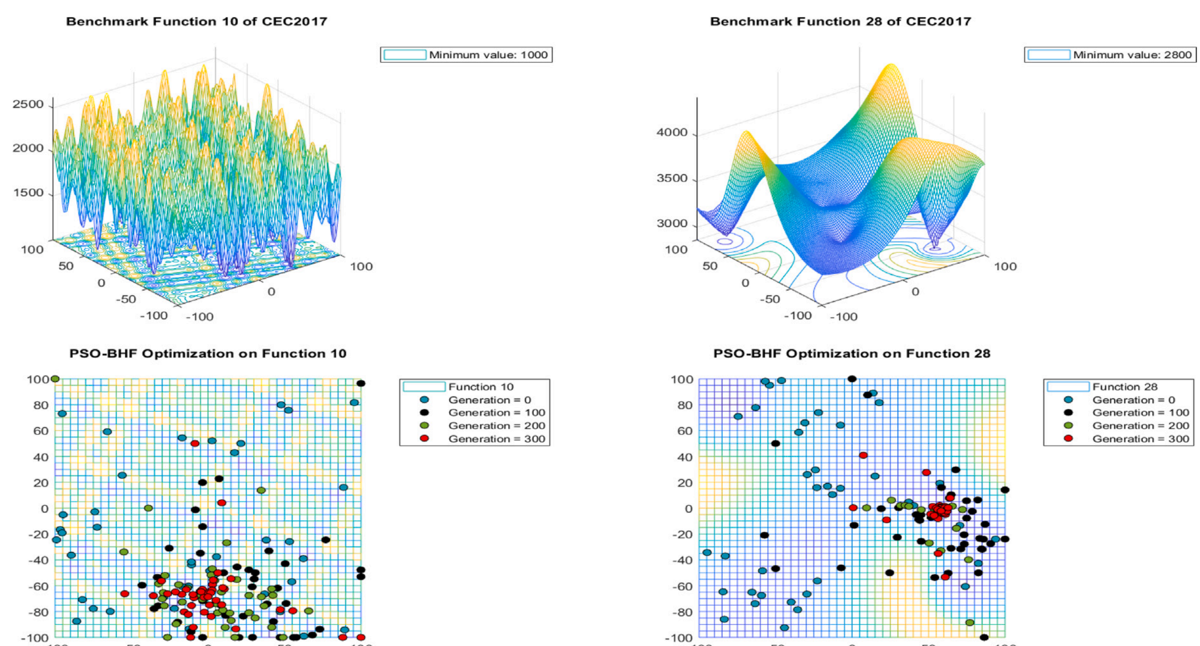

Fig. 4. Plots of 2-dimensional particle evolution over various generations (0, 100, 200, 300) on benchmark functions 10 and 28.

and f_{30}. SpadePSO showcased the best mean values for f_{26}. PSO-ELPM outperformed the others with the best mean values for f_{28}. HCLDMS-PSO exhibited superior mean values for f_{22} and f_{29}. In summary, PSO-BHF consistently outperformed others in the domains of unimodal, simple multimodal, and hybrid functions within the 30-dimensional experiments while also demonstrating competitive performance on the composition functions.

In the 50-dimensional experiments for unimodal functions, PSO-BHF consistently excelled with the best mean, median, and minimum values for both f_1 and f_3, whereas EAPSO achieved the same performance on f_2. In the domain of simple multimodal functions, PSO-BHF demonstrated remarkable performance by achieving the best mean, median, and minimum values for f_5, $f_7 - f_8$ and also securing the best minimum values for f_4 and f_{10}. FMPSO, SpadePSO, HCLDMS-PSO, and SDPSO showcased their strength with the best mean values for f_4, f_6, f_9, and f_{10}, respectively. When dealing with hybrid functions, PSO-BHF displayed the best mean values for $f_{12} - f_{14}$, $f_{16} - f_{18}$, and f_{20} as well as the best median values for f_{12}, f_{14}, $f_{16} - f_{18}$, and f_{20}. Additionally, PSO-BHF secured the best minimum values for f_{12}. EOPSO, SpadePSO,

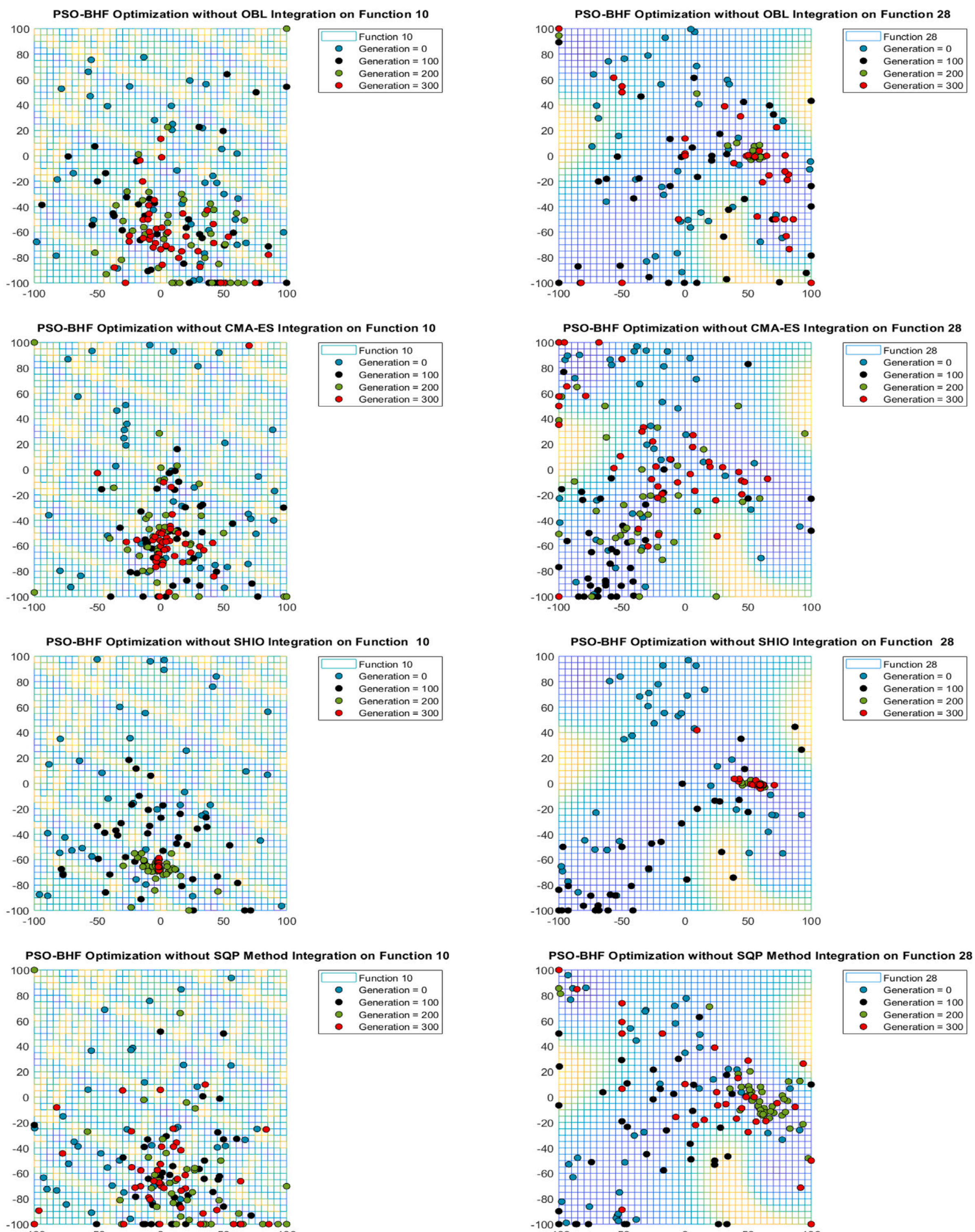

Fig. 5. Plots of 2-dimensional particle evolution obtained without using an individual mechanism over various generations (0, 100, 200, 300) on benchmark functions 10 and 28.

Table 6
Experimental results of PSO-BHF compared with non-PSO algorithms, averaged over 51 runs on 50-dimensional CEC2017 benchmark functions.

f_n	Criteria	PSO-BHF	WSA 2024	SRS 2024	CSA 2024	APO 2024	MSMA 2023	GMO 2023	ELSRao 2023	CCE 2023	EBOwithCMAR 2017	LSHADE-cnEpSin 2017	LSHADE-SPACMA 2017
	Mean	1.211863E−02	7.696477E+02	1.289027E+03	4.869107E+03	3.300562E+10	2.763286E+07	4.756244E+03	3.359788E+03	3.585945E+09	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**
f_1	Rank	4	5| >	6| >	9| >	12| >	10| >	8| >	7| >	11| >	**1**| <	**1**| <	**1**| <
	(Std Dev)	(5.511680E−03)	(9.397932E+02)	(1.829676E+03)	(5.798552E+03)	(5.495426E+09)	(8.840747E+06)	(5.072565E+03)	(4.558688E+03)	(1.576624E+09)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)
	Mean	6.860784E−04	7.225022E+12	1.117951E+07	4.461038E+40	4.005918E+65	4.943615E+28	6.831609E+50	8.301533E+01	4.560295E+41	3.078431E+00	1.000000E+00	**0.000000E+00**
f_2	Rank	2	7| >	6| >	9| >	12| >	8| >	11| >	5| >	10| >	4| >	3| =	**1**| <
	(Std Dev)	(1.585369E−03)	(3.282862E+13)	(7.888614E+07)	(2.971468E+41)	(2.007525E+66)	(2.000373E+29)	(4.864043E+51)	(2.292084E+01)	(3.173511E+42)	(2.084883E+01)	(1.600000E+00)	(0.000000E+00)
	Mean	**0.000000E+00**	2.583414E+04	1.956716E+01	6.739332E+04	1.802561E+05	2.822381E+05	5.344565E+03	2.021402E−01	1.100810E+05	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**
f_3	Rank	**1**	8| >	6| >	9| >	11| >	12| >	7| >	5| >	10| >	**1**| =	**1**| =	**1**| =
	(Std Dev)	(0.000000E+00)	(5.466614E+03)	(3.344376E+01)	(3.577993E+04)	(3.050148E+04)	(3.785985E+04)	(1.719916E+04)	(2.539522E−01)	(2.757273E+04)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)
	Mean	3.877246E+01	1.286918E+02	9.550302E+01	1.106355E+02	5.801488E+02	1.334963E+02	9.052356E+01	1.376705E+02	3.565871E+02	3.897731E+01	4.818944E+01	**3.304318E+01**
f_4	Rank	2	8| >	6| >	7| >	12| >	9| >	5| >	10| >	11| >	3| =	4| =	**1**| =
	(Std Dev)	(4.559231E+01)	(3.926377E+01)	(4.522095E+01)	(5.764770E+01)	(8.903959E+01)	(3.136794E+01)	(5.280754E+01)	(4.341016E+01)	(1.045590E+02)	(3.049788E+01)	(4.506596E+01)	(3.735091E+01)
	Mean	1.946998E+01	6.390370E+01	1.553878E+02	5.175626E+02	2.674779E+02	1.541883E+02	4.318726E+02	1.378750E+02	4.666078E+02	7.842617E+00	2.727418E+01	**6.028281E+00**
f_5	Rank	3	5| >	8| >	12| >	9| >	7| >	10| >	6| >	11| >	2| <	4| >	**1**| <
	(Std Dev)	(5.117047E+00)	(1.064629E+01)	(2.433444E+01)	(6.085974E+01)	(3.597684E+01)	(1.765218E+01)	(1.688491E+02)	(7.680064E+01)	(4.116097E+01)	(2.769112E+00)	(6.193711E+00)	(1.963995E+00)
	Mean	4.543104E−01	2.309804E−04	4.750784E−03	1.329412E−04	4.856770E+01	1.595829E+00	5.919038E+01	5.968992E+00	5.396380E+01	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**
f_6	Rank	7	5| <	6| <	4| <	10| >	8| >	12| >	9| >	11| >	**1**| <	**1**| <	**1**| <
	(Std Dev)	(3.041868E−01)	(1.300406E−03)	(2.324204E−02)	(3.623778E−05)	(5.271533E+00)	(2.634881E−01)	(2.444347E+01)	(3.012852E+00)	(1.178052E+01)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)
	Mean	7.332846E+01	1.099310E+02	2.684578E+02	4.808029E+02	8.140903E+02	2.320499E+02	8.766397E+02	1.829784E+02	8.074277E+02	5.815900E+01	7.565505E+01	**5.668584E+01**
f_7	Rank	3	5| >	8| >	9| >	11| >	7| >	12| >	6| >	10| >	2| <	4| =	**1**| <
	(Std Dev)	(6.736992E+00)	(1.081303E+01)	(4.576699E+01)	(1.129433E+02)	(6.683983E+01)	(1.613441E+01)	(2.356700E+02)	(5.792029E+01)	(1.478609E+02)	(1.489690E+00)	(6.939387E+00)	(1.098208E+00)
	Mean	1.984065E+01	6.136214E+01	1.611633E+02	5.297454E+02	2.810336E+02	1.451970E+02	4.216302E+02	1.330106E+02	4.743721E+02	8.759541E+00	2.723016E+01	**5.560065E+00**
f_8	Rank	3	5| >	8| >	12| >	9| >	7| >	10| >	6| >	11| >	2| <	4| >	**1**| <
	(Std Dev)	(5.307568E+00)	(1.053462E+01)	(2.694719E+01)	(5.921299E+01)	(3.376449E+01)	(1.975459E+01)	(1.656138E+02)	(8.338882E+01)	(4.287822E+01)	(3.329886E+00)	(6.432287E+00)	(1.868577E+00)
	Mean	1.254961E+01	8.144091E+00	1.574330E+03	1.272542E+03	2.080918E+04	2.125628E+03	4.900827E+04	8.297727E+01	2.359814E+04	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**
f_9	Rank	5	4| =	8| >	7| >	10| >	9| >	12| >	6| >	11| >	**1**| <	**1**| <	**1**| <
	(Std Dev)	(1.119452E+01)	(5.455253E+00)	(8.040234E+02)	(7.464093E+02)	(3.924880E+03)	(7.105903E+02)	(1.238122E+04)	(1.212692E+02)	(5.414784E+03)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)
	Mean	5.762541E+03	4.134072E+03	4.161462E+03	1.376966E+04	1.114745E+04	4.386443E+03	1.542581E+04	5.062409E+03	1.373158E+04	**3.080279E+03**	3.148620E+03	3.499482E+03
f_{10}	Rank	8	4| <	5| <	11| >	9| >	6| <	12| >	7| <	10| >	**1**| <	2| <	3| <
	(Std Dev)	(1.866902E+03)	(4.396970E+02)	(5.194384E+02)	(1.396684E+03)	(8.845706E+02)	(3.071073E+02)	(3.986015E+03)	(1.418066E+03)	(3.974055E+02)	(4.387405E+02)	(2.646822E+02)	(7.003823E+02)
	Mean	1.528567E+02	9.174285E+01	1.119206E+02	1.998385E+02	1.121209E+03	5.750398E+03	2.490506E+02	1.556273E+02	5.584210E+03	2.581648E+01	**2.201172E+01**	3.155601E+01
f_{11}	Rank	6	4| <	5| <	8| >	10| >	12| >	9| >	7| =	11| >	2| <	**1**| <	3| <
	(Std Dev)	(3.524452E+01)	(2.893853E+01)	(2.535138E+01)	(9.672813E+01)	(2.983281E+02)	(2.302312E+03)	(6.096277E+01)	(3.559667E+01)	(2.089587E+03)	(3.386325E+00)	(1.749465E+00)	(4.642683E+00)
	Mean	2.056617E+03	1.015044E+06	1.217705E+05	2.624586E+06	5.158559E+09	3.376466E+07	5.946192E+06	9.146521E+06	5.177630E+08	2.063933E+03	**1.398019E+03**	1.601858E+03
f_{12}	Rank	3	6| >	5| >	7| >	12| >	10| >	8| >	9| >	11| >	4| =	**1**| <	2| <
	(Std Dev)	(4.890698E+02)	(5.082084E+05)	(6.455966E+04)	(3.307344E+06)	(1.761538E+09)	(1.245275E+07)	(1.999939E+07)	(4.150951E+06)	(3.431640E+08)	(1.286759E+03)	(3.414158E+02)	(3.747227E+02)
	Mean	5.282313E+02	2.248165E+03	2.546475E+03	1.133959E+04	8.578271E+08	9.211604E+06	2.120991E+05	6.142313E+04	4.813130E+07	4.127150E+01	6.833575E+01	**3.696715E+01**
f_{13}	Rank	4	5| >	6| >	7| >	12| >	10| >	9| >	8| >	11| >	2| <	3| <	**1**| <
	(Std Dev)	(1.416402E+02)	(2.305674E+03)	(4.180869E+03)	(1.927302E+04)	(4.508089E+08)	(4.329372E+06)	(1.295334E+06)	(2.604095E+04)	(5.321863E+07)	(2.186099E+01)	(3.689142E+01)	(1.899799E+01)
	Mean	2.737460E+02	3.156621E+02	1.433947E+03	1.667004E+05	3.033435E+06	1.740941E+06	5.403943E+06	2.023846E+04	7.202666E+05	3.109644E+01	**2.612184E+01**	2.855209E+01
f_{14}	Rank	4	5| >	6| >	8| >	11| >	10| >	12| >	7| >	9| >	3| <	**1**| <	2| <
	(Std Dev)	(8.598322E+01)	(6.644285E+02)	(1.554303E+03)	(1.992531E+05)	(2.516552E+06)	(1.444510E+06)	(1.240920E+07)	(1.122275E+04)	(4.308755E+05)	(3.595953E+00)	(2.553914E+00)	(3.322310E+00)
	Mean	3.911625E+02	1.618162E+03	1.718597E+03	6.144430E+03	6.348181E+07	1.346240E+06	1.624266E+04	2.319108E+04	1.673258E+06	3.055219E+01	**2.603678E+01**	3.056057E+01
f_{15}	Rank	4	5| =	6| >	7| >	12| >	10| >	8| >	9| >	11| >	2| <	**1**| <	3| <
	(Std Dev)	(1.047539E+02)	(2.547279E+03)	(2.645604E+03)	(6.840328E+03)	(4.169925E+07)	(5.589735E+05)	(2.298813E+04)	(1.110109E+04)	(1.567488E+06)	(4.713683E+00)	(2.995517E+00)	(6.148851E+00)
	Mean	4.581635E+02	6.218018E+02	1.060377E+03	3.052380E+03	3.299123E+03	1.398197E+03	2.255021E+03	8.518217E+02	3.050447E+03	3.835203E+02	**2.863799E+02**	3.946762E+02
f_{16}	Rank	4	5| >	7| >	11| >	12| >	8| >	9| >	6| >	10| >	2| <	**1**| <	3| =
	(Std Dev)	(1.582372E+02)	(1.940709E+02)	(2.634897E+02)	(8.541977E+02)	(3.661630E+02)	(1.810630E+02)	(1.666721E+03)	(3.184478E+02)	(4.405777E+02)	(1.503173E+02)	(1.011191E+02)	(2.184436E+02)
	Mean	4.726377E+02	3.936923E+02	7.903810E+02	2.197827E+03	2.248389E+03	9.696669E+02	1.968714E+03	7.979517E+02	1.900357E+03	2.813322E+02	**2.067294E+02**	3.074705E+02
f_{17}	Rank	5	4| <	6| >	11| >	12| >	8| >	10| >	7| >	9| >	2| <	**1**| <	3| <
	(Std Dev)	(1.801277E+02)	(1.118917E+02)	(1.699505E+02)	(5.938942E+02)	(3.628013E+02)	(1.573595E+02)	(1.252202E+03)	(2.074621E+02)	(3.885768E+02)	(9.112134E+01)	(7.331898E+01)	(1.354040E+02)
	Mean	2.382751E+02	3.669344E+04	1.718895E+04	1.373057E+06	1.484690E+07	2.471780E+06	5.469903E+07	9.349668E+04	8.037987E+06	3.213271E+01	**2.441471E+01**	3.346118E+01
f_{18}	Rank	4	6| >	5| >	8| >	11| >	9| >	12| >	7| >	10| >	2| <	**1**| <	3| <
	(Std Dev)	(8.268008E+01)	(1.995693E+04)	(1.107542E+04)	(1.053142E+06)	(9.658522E+06)	(1.285968E+06)	(9.025579E+07)	(2.824434E+04)	(5.433551E+06)	(6.109568E+00)	(1.958079E+00)	(5.903743E+00)

(*continued on next page*)

and FMPSO exhibited exceptional performance with the best mean values for f_{11}, f_{15}, and f_{19}, respectively. For the composition functions, PSO-BHF maintained its dominance with the best mean, median, and minimum values for f_{21} and $f_{23} - f_{24}$, and it also achieved the best minimum values for both f_{22} and f_{26}. AHPSO excelled with the best mean values on both f_{22} and f_{26}. FMPSO and HCLDMS-PSO performed exceptionally well with the best mean values for f_{28} and f_{29}, respectively. EOPSO showcased its strength with the best mean values for f_{25}, f_{27}, and f_{30}. In summary, PSO-BHF consistently outperformed others in the realms of unimodal, simple multimodal, and hybrid functions in the 50-dimensional experiments while also demonstrating competitive performance on composition functions.

In the 100-dimensional experiments for unimodal functions, PSO-BHF showcased its excellence with the best mean values on both f_1 and f_3, the best median values on f_2 and f_3, and the best minimum values across $f_1 - f_3$. EAPSO delivered the top mean values for f_2. In the realm of simple multimodal functions, PSO-BHF excelled with the best mean, median, and minimum values for f_{10} as well as the best median and minimum values for f_4. FMPSO, SpadePSO, AHPSO, and HCLDMS-PSO demonstrated their strength with the best mean values for f_4, f_6, f_7, and f_9, respectively. EOPSO showcased its prowess with the best mean values for both f_5 and f_8 on these functions. When tackling hybrid functions, PSO-BHF continued to shine with the best mean, median, and minimum values for f_{12}, f_{14}, f_{18}, and f_{20} as well as the best mean and median values for f_{15}. TAPSO, EOPSO, and HCLDMS-PSO performed exceptionally well with the best mean values for f_{11}, f_{16}, and f_{17}, respectively. SpadePSO outperformed others with the best mean values for both f_{13} and f_{19}. For the composition functions, PSO-BHF maintained its dominance with the best mean values for f_{21} and f_{28}, the best median values for $f_{21} - f_{22}$, and the best minimum values for $f_{21} - f_{22}$ and f_{28}. AHPSO, FMPSO, PSO-ELPM, and HCLDMS-PSO excelled with the best mean values for f_{22}, f_{25}, f_{26}, and f_{29}, respectively. AMSEPSO demonstrated its strength with the best mean values for $f_{23} - f_{24}$. EOPSO showcased its prowess with the best mean values for both f_{27} and f_{30}. In summary, PSO-BHF consistently performed exceptionally well on unimodal and hybrid functions in the

Table 6 (*continued*).

<table>
<tr><td>f_{19}</td><td>Mean</td><td>3.684250E+02</td><td>7.146260E+03</td><td>1.341720E+02</td><td>1.165479E+04</td><td>1.716275E+07</td><td>6.374296E+05</td><td>2.355517E+04</td><td>4.296784E+05</td><td>1.455817E+06</td><td>2.415382E+01</td><td>1.720179E+01</td><td>2.090607E+01</td></tr>
<tr><td></td><td>Rank</td><td>5</td><td>6| ></td><td>4| <</td><td>7| ></td><td>12| ></td><td>10| ></td><td>8| ></td><td>9| ></td><td>11| ></td><td>3| <</td><td>1| <</td><td>2| <</td></tr>
<tr><td></td><td>(Std Dev)</td><td>(1.751089E+02)</td><td>(4.661347E+03)</td><td>(1.197363E+02)</td><td>(8.626064E+03)</td><td>(1.073264E+07)</td><td>(3.289943E+05)</td><td>(2.341037E+04)</td><td>(2.330771E+05)</td><td>(1.260560E+06)</td><td>(4.904627E+00)</td><td>(3.048008E+00)</td><td>(4.422527E+00)</td></tr>
<tr><td>f_{20}</td><td>Mean</td><td>1.750021E+02</td><td>1.755113E+02</td><td>4.884946E+02</td><td>1.409896E+03</td><td>1.594823E+03</td><td>7.820648E+02</td><td>2.589545E+03</td><td>6.637267E+02</td><td>2.149673E+03</td><td>1.479799E+02</td><td>1.082187E+02</td><td>1.716726E+02</td></tr>
<tr><td></td><td>Rank</td><td>4</td><td>5| =</td><td>6| ></td><td>9| ></td><td>10| ></td><td>8| ></td><td>12| ></td><td>7| ></td><td>11| ></td><td>2| <</td><td>1| <</td><td>3| <</td></tr>
<tr><td></td><td>(Std Dev)</td><td>(9.451588E+01)</td><td>(8.649930E+01)</td><td>(1.962117E+02)</td><td>(6.812516E+02)</td><td>(3.267928E+02)</td><td>(1.421631E+02)</td><td>(1.088061E+03)</td><td>(2.808431E+02)</td><td>(2.761710E+02)</td><td>(7.320722E+01)</td><td>(3.020447E+01)</td><td>(9.969616E+01)</td></tr>
<tr><td>f_{21}</td><td>Mean</td><td>2.177670E+02</td><td>2.503660E+02</td><td>3.403724E+02</td><td>7.341091E+02</td><td>5.172976E+02</td><td>3.674436E+02</td><td>6.306240E+02</td><td>3.174415E+02</td><td>6.752241E+02</td><td>2.120433E+02</td><td>2.260707E+02</td><td>2.140111E+02</td></tr>
<tr><td></td><td>Rank</td><td>3</td><td>5| ></td><td>7| ></td><td>12| ></td><td>9| ></td><td>8| ></td><td>10| ></td><td>6| ></td><td>11| ></td><td>1| <</td><td>4| ></td><td>2| <</td></tr>
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<tr><td>f_{22}</td><td>Mean</td><td>2.643842E+03</td><td>4.352310E+03</td><td>4.701320E+03</td><td>1.385943E+04</td><td>1.161830E+04</td><td>4.545063E+03</td><td>1.368531E+04</td><td>4.346467E+03</td><td>1.379798E+04</td><td>5.107614E+02</td><td>1.713471E+03</td><td>1.604819E+03</td></tr>
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<tr><td>f_{23}</td><td>Mean</td><td>4.605240E+02</td><td>4.825259E+02</td><td>6.091978E+02</td><td>9.376286E+02</td><td>9.996180E+02</td><td>6.117316E+02</td><td>1.016312E+03</td><td>5.522383E+02</td><td>9.587302E+02</td><td>4.356375E+02</td><td>4.401019E+02</td><td>4.400870E+02</td></tr>
<tr><td></td><td>Rank</td><td>4</td><td>5| ></td><td>7| ></td><td>9| ></td><td>11| ></td><td>8| ></td><td>12| ></td><td>6| ></td><td>10| ></td><td>1| <</td><td>3| <</td><td>2| <</td></tr>
<tr><td></td><td>(Std Dev)</td><td>(1.441918E+01)</td><td>(1.143280E+01)</td><td>(3.329427E+01)</td><td>(1.046386E+02)</td><td>(8.856875E+01)</td><td>(1.703454E+01)</td><td>(1.986835E+02)</td><td>(7.617055E+01)</td><td>(4.617797E+01)</td><td>(6.460554E+00)</td><td>(7.092517E+00)</td><td>(6.402721E+00)</td></tr>
<tr><td>f_{24}</td><td>Mean</td><td>5.284961E+02</td><td>5.481476E+02</td><td>7.456651E+02</td><td>1.040457E+03</td><td>1.104877E+03</td><td>8.571712E+02</td><td>1.256468E+03</td><td>6.156131E+02</td><td>1.017520E+03</td><td>5.075703E+02</td><td>5.129716E+02</td><td>5.139322E+02</td></tr>
<tr><td></td><td>Rank</td><td>4</td><td>5| ></td><td>7| ></td><td>10| ></td><td>11| ></td><td>8| ></td><td>12| ></td><td>6| ></td><td>9| ></td><td>1| <</td><td>2| <</td><td>3| <</td></tr>
<tr><td></td><td>(Std Dev)</td><td>(8.286214E+00)</td><td>(1.131007E+01)</td><td>(4.774997E+01)</td><td>(1.697834E+02)</td><td>(1.102766E+02)</td><td>(3.942594E+01)</td><td>(2.755321E+02)</td><td>(6.966489E+01)</td><td>(4.770700E+01)</td><td>(3.252748E+00)</td><td>(6.124631E+00)</td><td>(7.666168E+00)</td></tr>
<tr><td>f_{25}</td><td>Mean</td><td>5.355475E+02</td><td>5.721244E+02</td><td>5.640805E+02</td><td>5.391415E+02</td><td>1.118530E+03</td><td>5.473464E+02</td><td>5.628338E+02</td><td>5.178676E+02</td><td>7.563922E+02</td><td>4.854357E+02</td><td>4.808604E+02</td><td>4.809288E+02</td></tr>
<tr><td></td><td>Rank</td><td>5</td><td>10| ></td><td>9| ></td><td>6| =</td><td>12| ></td><td>7| =</td><td>8| ></td><td>4| <</td><td>11| ></td><td>3| <</td><td>1| <</td><td>2| <</td></tr>
<tr><td></td><td>(Std Dev)</td><td>(3.959992E+01)</td><td>(2.878644E+01)</td><td>(3.183391E+01)</td><td>(3.602767E+01)</td><td>(1.402028E+02)</td><td>(1.888922E+01)</td><td>(4.542440E+01)</td><td>(2.681594E+01)</td><td>(7.783696E+01)</td><td>(1.425723E+01)</td><td>(2.611737E+00)</td><td>(2.755285E+00)</td></tr>
<tr><td>f_{26}</td><td>Mean</td><td>1.303747E+03</td><td>1.809855E+03</td><td>3.724498E+03</td><td>5.655426E+03</td><td>7.561132E+03</td><td>2.484491E+03</td><td>4.382053E+03</td><td>3.284557E+02</td><td>2.545040E+03</td><td>6.732468E+02</td><td>1.207110E+03</td><td>1.148291E+03</td></tr>
<tr><td></td><td>Rank</td><td>5</td><td>6| ></td><td>9| ></td><td>11| ></td><td>12| ></td><td>7| ></td><td>10| =</td><td>1| <</td><td>8| ></td><td>2| <</td><td>4| <</td><td>3| <</td></tr>
<tr><td></td><td>(Std Dev)</td><td>(2.645815E+02)</td><td>(1.756043E+02)</td><td>(4.690820E+02)</td><td>(1.426434E+03)</td><td>(1.105455E+03)</td><td>(1.006789E+03)</td><td>(3.874604E+03)</td><td>(2.032140E+02)</td><td>(2.072836E+03)</td><td>(4.010093E+02)</td><td>(9.353350E+01)</td><td>(5.315677E+01)</td></tr>
<tr><td>f_{27}</td><td>Mean</td><td>7.599767E+02</td><td>5.948035E+02</td><td>7.317939E+02</td><td>6.913875E+02</td><td>1.288331E+03</td><td>6.374461E+02</td><td>1.020045E+03</td><td>6.238766E+02</td><td>5.000123E+02</td><td>5.257122E+02</td><td>5.247738E+02</td><td>5.285145E+02</td></tr>
<tr><td></td><td>Rank</td><td>10</td><td>5| <</td><td>9| =</td><td>8| <</td><td>12| ></td><td>7| <</td><td>11| ></td><td>6| <</td><td>1| <</td><td>3| <</td><td>2| <</td><td>4| <</td></tr>
<tr><td></td><td>(Std Dev)</td><td>(9.391126E+01)</td><td>(3.779944E+01)</td><td>(7.732567E+01)</td><td>(7.687111E+01)</td><td>(1.576215E+02)</td><td>(2.636630E+01)</td><td>(1.966556E+02)</td><td>(5.117015E+01)</td><td>(7.990190E−05)</td><td>(1.326434E+01)</td><td>(7.710838E+00)</td><td>(9.665720E+00)</td></tr>
<tr><td>f_{28}</td><td>Mean</td><td>4.959519E+02</td><td>5.274721E+02</td><td>5.019404E+02</td><td>4.956352E+02</td><td>2.026466E+03</td><td>5.207411E+02</td><td>5.246093E+02</td><td>4.850380E+02</td><td>5.000101E+02</td><td>4.636375E+02</td><td>4.574893E+02</td><td>4.607642E+02</td></tr>
<tr><td></td><td>Rank</td><td>6</td><td>11| ></td><td>8| ></td><td>5| =</td><td>12| ></td><td>9| ></td><td>10| ></td><td>4| =</td><td>7| =</td><td>3| <</td><td>1| <</td><td>2| <</td></tr>
<tr><td></td><td>(Std Dev)</td><td>(2.133913E+01)</td><td>(2.502407E+01)</td><td>(1.135893E+01)</td><td>(1.897294E+01)</td><td>(3.666598E+02)</td><td>(1.150557E+01)</td><td>(9.150455E+01)</td><td>(2.423305E+01)</td><td>(7.180064E−03)</td><td>(1.466975E+01)</td><td>(6.855384E+00)</td><td>(9.575731E+00)</td></tr>
<tr><td>f_{29}</td><td>Mean</td><td>7.729285E+02</td><td>4.190774E+02</td><td>7.866625E+02</td><td>2.002945E+03</td><td>3.603768E+03</td><td>8.858754E+02</td><td>1.927552E+03</td><td>1.137650E+03</td><td>2.172036E+03</td><td>3.540991E+02</td><td>3.532406E+02</td><td>3.777706E+02</td></tr>
<tr><td></td><td>Rank</td><td>5</td><td>4| <</td><td>6| =</td><td>10| ></td><td>12| ></td><td>7| ></td><td>9| ></td><td>8| ></td><td>11| ></td><td>2| <</td><td>1| <</td><td>3| <</td></tr>
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<tr><td>f_{30}</td><td>Mean</td><td>6.722871E+05</td><td>8.015379E+05</td><td>8.483703E+05</td><td>9.971353E+05</td><td>2.270577E+08</td><td>1.843768E+06</td><td>1.446745E+07</td><td>1.459089E+07</td><td>2.613123E+07</td><td>6.276885E+05</td><td>6.392311E+05</td><td>6.364674E+05</td></tr>
<tr><td></td><td>Rank</td><td>4</td><td>5| ></td><td>6| ></td><td>7| ></td><td>12| ></td><td>8| ></td><td>9| ></td><td>10| ></td><td>11| ></td><td>1| <</td><td>3| <</td><td>2| <</td></tr>
<tr><td></td><td>(Std Dev)</td><td>(8.075526E+04)</td><td>(6.793633E+04)</td><td>(1.051731E+05)</td><td>(2.955350E+05)</td><td>(6.520213E+07)</td><td>(3.693491E+05)</td><td>(4.516278E+07)</td><td>(3.505045E+06)</td><td>(1.273343E+07)</td><td>(4.507717E+04)</td><td>(6.546409E+04)</td><td>(5.717783E+04)</td></tr>
<tr><td colspan="2">Averaged Friedman ranks</td><td>4.4166</td><td>5.6333</td><td>6.6333</td><td>8.7333</td><td>11.0333</td><td>8.4666</td><td>9.9000</td><td>6.6333</td><td>10.0000</td><td>2.1500</td><td>2.1833</td><td>2.2167</td></tr>
<tr><td colspan="2">> | = | <</td><td>/</td><td>21|3|6</td><td>24|2|4</td><td>26|2|2</td><td>30|0|0</td><td>27|1|2</td><td>29|1|0</td><td>24|2|4</td><td>28|1|1</td><td>1|3|26</td><td>3|4|23</td><td>0|3|27</td></tr>
</table>

100-dimensional experiments while also remaining competitive on the composition functions.

Table 4 lists the results of 50-dimensional tests that were conducted to illustrate the effectiveness of different combinations of mechanisms. The degree of contribution of these mechanisms in the framework can also be reflected in Table 4. PSO-BHF achieved the best averaged Friedman rank of 4.3667, which was the lowest among all the compared PSO-based variants. PSO-BHF without using any single mechanism performs significantly worse than does PSO-BHF, and PSO-BHF without using the SQP method had the worst averaged Friedman rank of 7.4000. PSO-BHF without using OBL and SHIO attained the second-best averaged Friedman rank of 4.6667. This implies that the SQP method and CMA-ES make significant contributions to PSO-BHF. While OBL and SHIO slightly improve PSO-BHF, their contributions are also crucial in helping PSO-BHF achieve the best averaged Friedman rank. The mechanisms employed by PSO-BHF demonstrate that both the combinations and timing of the interventions are appropriate on average, illustrating outstanding collaboration.

Table 5 lists the results of ablation experiments that were conducted to better illustrate the effectiveness of calculating probabilities for using CMA-ES in the 50-dimensional tests. Five fixed pairs of probabilities were evaluated for the humble learning exemplars and CMA-ES: 10%–90%, 30%–70%, 50%–50%, 70%–30%, and 90%–10%. PSO-BHF has an averaged Friedman rank of 2.9833, which is the best when compared with these fixed probabilities. The calculation of probabilities for using CMA-ES demonstrates a significant improvement in terms of efficiency.

In general, PSO-BHF performs outstandingly on unimodal functions ($f_1 - f_3$) and simple multimodal functions (except for f_6 and f_{10}), and it also exhibits good performance on most hybrid ($f_{11} - f_{20}$) and composition functions ($f_{21} - f_{30}$). The intervention of the SQP method greatly improves performance, particularly when optimizing unimodal functions, and, on average across all benchmark functions, it has a more significant impact than do OBL, CMA-ES, and SHIO. However, the SQP method only has a negative impact on $f_{21} - f_{25}$, which are composition functions. We believe that the timing of the intervention of the SQP method for these functions can be slightly improved. The intervention of CMA-ES on f_6, $f_9 - f_{10}$ (simple multimodal functions), $f_{11} - f_{12}$, f_{14}, $f_{18} - f_{19}$ (hybrid functions), and f_{25}, $f_{27} - f_{28}$, f_{30} (composition functions) leads to worse performance, possibly because CMA-ES causes the evolution to fall into local optima during the process. PSO-BHF without using OBL has better performance on f_{10} (simple multimodal functions), $f_{13} - f_{17}$ (hybrid functions), f_{23}, and $f_{25} - f_{26}$ (composition functions). Thus, OBL has a negative impact on these functions. SHIO has a negative impact on f_6, f_8, f_{10} (simple multimodal functions), $f_{13} - f_{17}$, f_{20} (hybrid functions), and f_{21}, f_{24} (composition functions), as not using the SHIO mechanism can lead to better performance on these functions. Based on our observations, the collaboration between CMA-ES and the SQP method, with proper triggering timing, plays a major role in improving the proposed algorithm. The other mechanisms and their triggering times serve as minor adjustments to the optimization process.

4.4. Search behavior of PSO-BHF

Fig. 3 depicts the evolution of the best fitness values, classified by their sources, across various functions during individual 50-dimensional runs. Distinct mechanisms will be chosen when dealing with various problems. The search patterns can be described in the following manner:

- *In the early and middle generations, the confident learning exemplar, humble learning exemplar, and CMA-ES alternate their contributions. However, in the later generations, the SQP method takes a dominant role in the effective evolution. Finally, the confident and humble learning exemplars alternate their roles.*
- *In the early and middle generations, the confident learning exemplar, humble learning exemplar, SHIO mechanism, and CMA-ES take turns*

Table 7
Experimental results and statistics using PSO-based variants, run 20 times for the long-term TNEP problem.

No.	PSO-BHF	EAPSO	PSO-ELPM	SpadePSO	AMSEPSO	EOPSO	PPSO	PSO-sono	SDPSO	AHPSO	HCLDMS-PSO	HIDMS-PSO	TAPSO	MPSO	FMPSO	TCSPSO
1	7.347637E+06	7.681562E+06	1.391792E+07	1.434286E+07	1.363462E+07	8.454867E+06	9.827847E+06	1.198271E+07	1.403636E+07	9.648994E+06	1.383920E+07	9.210337E+06	5.463190E+06	9.181059E+06	1.525175E+07	2.937392E+07
2	7.288848E+06	7.325955E+06	1.699717E+07	1.379429E+07	1.259616E+07	7.830475E+06	1.045007E+07	1.290727E+07	1.518017E+07	9.767649E+06	1.360328E+07	8.931632E+06	6.095723E+06	8.960822E+06	1.774490E+07	3.190925E+07
3	6.474236E+06	8.093209E+06	1.329504E+07	1.602022E+07	1.278983E+07	6.928651E+06	1.003468E+07	1.313391E+07	1.582617E+07	1.258353E+07	1.933552E+07	8.776219E+06	6.538759E+06	1.096110E+07	1.447637E+07	2.980978E+07
4	6.437755E+06	7.747467E+06	1.497482E+07	1.554686E+07	1.350849E+07	7.003308E+06	1.049247E+07	1.448237E+07	1.592902E+07	8.789741E+06	1.429593E+07	8.941746E+06	6.402989E+06	9.054602E+06	1.798243E+07	2.307408E+07
5	7.497777E+06	7.761114E+06	1.281720E+07	1.536689E+07	1.327803E+07	8.170947E+06	1.205695E+07	1.554670E+07	1.630647E+07	1.113572E+07	1.482839E+07	8.700186E+06	5.747349E+06	9.193333E+06	1.525895E+07	3.100492E+07
6	6.865612E+06	8.187402E+06	1.462623E+07	1.417936E+07	1.352366E+07	7.743105E+06	9.918603E+06	1.511124E+07	1.572175E+07	1.158220E+07	1.574650E+07	8.118811E+06	6.432108E+06	9.500464E+06	1.728742E+07	3.105913E+07
7	6.401671E+06	7.912664E+06	1.422982E+07	1.405718E+07	1.334427E+07	8.257470E+06	9.882131E+06	1.290830E+07	1.877707E+07	1.060161E+07	1.188984E+07	7.739393E+06	6.919817E+06	1.022353E+07	1.542352E+07	3.114486E+07
8	5.813824E+06	8.305864E+06	1.175440E+07	1.432731E+07	1.454761E+07	7.719167E+06	1.137739E+07	1.221725E+07	1.409953E+07	1.203206E+07	1.380637E+07	8.688111E+06	5.827396E+06	1.078342E+07	1.640002E+07	3.041987E+07
9	6.602379E+06	8.507697E+06	1.101369E+07	1.436492E+07	1.335046E+07	8.436395E+06	9.205701E+06	1.257637E+07	1.604263E+07	1.115815E+07	1.327851E+07	9.967638E+06	5.433534E+06	9.642971E+06	1.548917E+07	2.978122E+07
10	6.947186E+06	7.947653E+06	1.177231E+07	1.344666E+07	1.362519E+07	8.225967E+06	7.868565E+06	1.243463E+07	1.574327E+07	1.317721E+07	2.106677E+07	8.305565E+06	5.900360E+06	8.537493E+06	1.770692E+07	2.904586E+07
11	6.676921E+06	8.395231E+06	1.291518E+07	1.407805E+07	1.372006E+07	8.455241E+06	1.003599E+07	1.461326E+07	1.332294E+07	1.157978E+07	1.269992E+07	1.012365E+07	5.642081E+06	9.561522E+06	1.853593E+07	2.910843E+07
12	5.886693E+06	7.319768E+06	1.189484E+07	1.469413E+07	1.390940E+07	7.602696E+06	8.564568E+06	1.457589E+07	1.599748E+07	1.207153E+07	1.492327E+07	7.934403E+06	5.749646E+06	9.087549E+06	1.702330E+07	3.310695E+07
13	7.639754E+06	7.702918E+06	1.259845E+07	1.415790E+07	1.323278E+07	7.890969E+06	9.643601E+06	1.312468E+07	1.546077E+07	1.191199E+07	1.616473E+07	7.837278E+06	7.548975E+06	8.513156E+06	1.829712E+07	3.080299E+07
14	6.903990E+06	7.518833E+06	1.387008E+07	1.537744E+07	1.263858E+07	7.409026E+06	9.552381E+06	1.311415E+07	1.497282E+07	1.100776E+07	1.340366E+07	9.755249E+06	6.182139E+06	9.349202E+06	1.664786E+07	3.193076E+07
15	6.283235E+06	8.217549E+06	1.775537E+07	1.560391E+07	1.380092E+07	7.891170E+06	8.806002E+06	1.311848E+07	1.755638E+07	9.608314E+06	1.364916E+07	8.597794E+06	7.250929E+06	8.604803E+06	1.776190E+07	2.989245E+07
16	6.328185E+06	7.634567E+06	1.922616E+07	1.484337E+07	1.436868E+07	9.043602E+06	7.961892E+06	1.218018E+07	1.519858E+07	1.145737E+07	1.583425E+07	8.915136E+06	6.394985E+06	9.370728E+06	1.838051E+07	2.821633E+07
17	6.848500E+06	8.151674E+06	1.376109E+07	1.393696E+07	1.350735E+07	7.801055E+06	9.510262E+06	1.413118E+07	1.487306E+07	1.285034E+07	1.428829E+07	9.209538E+06	5.408209E+06	8.900151E+06	1.659592E+07	2.637653E+07
18	6.834094E+06	7.765376E+06	1.203225E+07	1.287346E+07	1.357584E+07	8.277574E+06	8.623897E+06	1.453393E+07	1.518891E+07	9.174781E+06	1.533455E+07	9.549093E+06	5.617754E+06	8.291656E+06	1.917198E+07	2.597638E+07
19	6.994512E+06	7.814315E+06	1.820281E+07	1.370095E+07	1.391189E+07	7.684096E+06	9.687586E+06	1.304466E+07	1.420208E+07	1.033089E+07	1.355700E+07	7.380654E+06	6.504722E+06	9.002177E+06	1.583130E+07	3.321370E+07
20	7.662133E+06	8.437164E+06	1.272078E+07	1.595025E+07	1.309619E+07	8.692252E+06	9.298951E+06	1.234282E+07	1.649813E+07	9.339058E+06	1.513216E+07	9.406068E+06	5.932611E+06	1.065687E+07	1.594560E+07	3.261020E+07
Mean Rank (Std Dev)	6.786747E+06 2 (5.279250E+05)	7.921399E+06 3| > (3.540530E+05)	1.401878E+07 11| > (2.333191E+06)	1.453315E+07 12| > (8.663071E+05)	1.349800E+07 10| > (5.006394E+05)	7.975902E+06 4| > (5.316264E+05)	9.639976E+06 7| > (1.023346E+06)	1.340400E+07 9| > (1.074217E+06)	1.554668E+07 14| > (1.235481E+06)	1.099043E+07 8| > (1.287696E+06)	1.483386E+07 13| > (2.151286E+06)	8.804425E+06 5| > (7.584859E+05)	**6.149664E+06** **1**| < (6.008307E+05)	9.368830E+06 6| > (7.604625E+05)	1.686064E+07 15| > (1.324049E+06)	2.989288E+07 16| > (2.522143E+06)

Table 8
Computational complexity for PSO-BHF.

Dim	$T_0(s)$	$T_1(s)$	$\hat{T}_2(s)$	$(\hat{T}_2 - T_1)/T_0$
$10D$	0.0159	0.5644	6.3670	364.9434
$30D$	0.0159	0.7473	8.4350	483.5031
$50D$	0.0159	1.0733	10.1369	570.0377
$100D$	0.0159	2.5326	23.0961	1293.3019

Table 9
Computational complexities of all PSO-based variants in 30 dimensions.

Algorithm	$T_0(s)$	$T_1(s)$	$\hat{T}_2(s)$	$(\hat{T}_2 - T_1)/T_0$	Rank
PSO-BHF	0.0159	0.7473	8.4350	483.5031	13
EAPSO	0.0159	0.7473	12.2164	721.3270	16
PSO-ELPM	0.0159	0.7473	11.2398	659.9057	15
SpadePSO	0.0159	0.7473	5.9241	325.5849	9
AMSEPSO	0.0159	0.7473	4.5189	237.2075	7
EOPSO	0.0159	0.7473	6.2395	345.4214	11
PPSO	0.0159	0.7473	1.8413	68.8050	4
PSO-sono	0.0159	0.7473	1.8219	67.5849	3
SDPSO	0.0159	0.7473	6.0357	332.6038	10
AHPSO	0.0159	0.7473	3.8531	195.3333	5
HCLDMS-PSO	0.0159	0.7473	4.5130	236.8365	6
HIDMS-PSO	0.0159	0.7473	8.9836	518.0063	14
TAPSO	0.0159	0.7473	6.3465	352.1509	12
MPSO	0.0159	0.7473	1.6414	56.2327	2
FMPSO	0.0159	0.7473	5.7509	314.6918	8
TCSPSO	0.0159	0.7473	1.5148	48.2704	1

working. Subsequently, the SQP method continues to be involved until the end.

- *In the early and middle generations, the confident learning exemplar takes a dominant role in effective evolution. The humble learning exemplar, CMA-ES, and SHIO mechanism also alternate their contributions. Subsequently, the SQP method continues to be involved until the end.*
- *In the early and middle generations, CMA-ES plays a dominant role in the effective evolution, and the confident learning exemplars, humble learning exemplars, and SHIO mechanism take turns contributing to the process. Subsequently, the SQP method continues to be involved until the end.*
- *In general, although the framework does not have a preset timing for enabling the SHIO mechanism, the SHIO mechanism becomes effective in the earliest generations. Conversely, the SQP method becomes effective in the later generations, as it is triggered after reaching the product of 0.6, which is an empirical value, and the maximum function evaluation.*

Figs. 4 and 5 display the two-dimensional particle distributions at various generations (0, 100, 200, and 300) achieved by applying PSO-BHF and PSO-BHF without certain mechanisms to f_{10} and f_{28}, respectively. The findings suggest that particles tend to aggregate more effectively throughout the evolutionary process when PSO-BHF utilizes OBL, indicating that the OBL mechanism enhances the efficiency of evolution. PSO-BHF utilizes CMA-ES, which leads to particles gathering more due to its exploitation capabilities. PSO-BHF incorporates the SHIO mechanism, resulting in particles becoming more dispersed throughout the evolutionary process. This highlights the effectiveness of the SHIO mechanism in promoting particle diversity. PSO-BHF without the SQP method results in the red particles being more scattered, which means that the intervention of the SQP method greatly enhances the exploitation capabilities in the later generations. In general, the absence of the integrated mechanism causes different search behavior, which provides strong evidence of the effectiveness of the integrated mechanisms.

Figs. 6 and 7 depict the evolutionary progress of the PSO-based variants in the 50-dimensional experiments. PSO-BHF demonstrates notably swift and abrupt convergence in most instances, attributable to the SQP method's intervention after completing approximately 61.8% of the overall evolutionary process. As a result, the evolutionary process resembles a steep descent, akin to a cliff fall, which is a common occurrence. Although PSO-BHF has already demonstrated an outstanding convergence rate in most cases, exceptions still exist. PSO-BHF shows inferior performance compared with several other algorithms on a few functions, suggesting potential for improvement. For example, PSO-BHF does not show outstanding performance on f_6, $f_{10} - f_{11}$, f_{25}, and $f_{27} - f_{28}$.

We believe this is due to insufficient exploration, which causes the local search methods to still get stuck in local optima through excessive exploitation.

4.5. PSO-BHF vs. non-PSO algorithms

To better observe the performance of PSO-BHF against different non-PSO algorithms, 11 state-of-the-art, recently proposed metaheuristic and classic algorithms are selected: Wave Search Algorithm (WSA) [44], Strategic Random Search (SRS) [45], Colony-based Search Algorithm (CSA) [46], Artificial Protozoa Optimizer (APO) [47], Multi-strategy improved Slime Mould Algorithm(MSMA) [48], Geometric Mean Optimizer (GMO) [49], Elite Local Search Rao (ELSRao) [50], City Councils Evolution (CCE) [51], EBOwithCMAR [52], LSHADE-cnEpSin [53], and LSHADE-SPACMA [17]. The experimental results are given in Table 6. Both the rank and the averaged Friedman rank are provided in this table. The averaged Friedman rank of PSO-BHF is 4.4166, which is worse than the values of 2.1500, 2.1833, and 2.2167 for EBOwithCMAR, LSHADE-cnEpSin, and LSHADE-SPACMA, respectively. This outcome is expected, as in recent CEC competitions [54, 55], DE-based variants are in the first tier among all metaheuristics. However, PSO-BHF has better performance on 21, 24, 26, 30, 27, 29, 24, and 28 functions compared to WSA, SRS, CSA, APO, MSMA, GMO, ELSRao, and CCE, respectively, demonstrating very competitive performance.

Table 10
Comparison of experimental results of PSO variants, averaged over 51 runs, for 10-dimensional CEC2017 benchmark functions.

f_n	Criteria	PSO-BHF	EAPSO	PSO-ELPM	SpadePSO	AMSEPSO	EOPSO	PPSO	PSO-sono	SDPSO	AHPSO	HCLDMS-PSO	HIDMS-PSO	TAPSO	MPSO	FMPSO	TCSPSO
	Mean	**3.228020E−02**	3.158705E+03	1.740632E+03	3.803984E+01	2.250087E+03	7.645647E+02	9.455910E+02	6.039504E+02	1.258410E+02	1.976141E+03	4.740903E+02	9.735776E+02	1.494868E+03	1.020168E+03	3.731912E+02	2.046047E+03
	Rank|WRST	**1**	16 | >	12 | >	2 | >	15 | >	7 | >	8 | >	6 | >	3 | >	13 | >	5 | >	9 | >	11 | >	10 | >	4 | =	14 | >
	Min	2.900000E−04	7.062210E+00	2.936260E+00	4.027200E−01	3.446400E−01	5.574000E−02	6.440000E−03	4.316900E−01	1.738000E−02	4.822000E−02	4.950000E−03	6.263000E−02	8.838790E+00	8.468400E−01	**2.300000E−04**	9.731000E−02
f_1	Rank	2	15	14	11	10	7	4	12	5	6	3	8	16	13	**1**	9
	Median	**3.700000E−04**	2.074707E+03	9.463764E+02	1.795236E+01	1.658642E+03	1.981179E+02	4.068195E+02	2.651033E+02	6.332345E+01	7.528363E+02	2.339562E+02	5.012730E+02	6.021586E+02	2.434921E+02	**3.700000E−04**	1.396869E+03
	Rank	**1**	16	13	3	15	5	9	8	4	12	6	10	11	7	**1**	14
	(Std Dev)	(2.279129E−01)	(3.102246E+03)	(1.968351E+03)	(5.556195E+01)	(1.982014E+03)	(1.346197E+03)	(1.507724E+03)	(8.700179E+02)	(1.612915E+02)	(3.103261E+03)	(7.566976E+02)	(1.070105E+03)	(2.223930E+03)	(1.516362E+03)	(8.175162E+02)	(2.296248E+03)
	Mean	3.823529E−05	1.019608E−05	3.857059E−03	4.078431E−05	6.170448E+00	1.163648E+01	3.584314E−04	1.568627E−05	2.176471E−05	9.411765E−06	1.039216E−05	2.196078E−05	6.450980E−05	**5.098039E−06**	2.941176E−05	4.274510E−05
	Rank|WRST	9	3 | <	14 | >	10 | =	15 | >	16 | >	13 | >	5 | <	6 | <	2 | <	4 | <	7 | <	12 | =	**1** | <	8 | =	11 | =
	Min	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	1.000000E−05	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**
f_2	Rank	**1**	**1**	**1**	**1**	**1**	16	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
	Median	2.000000E−05	1.000000E−05	1.890000E−03	2.000000E−05	1.000000E−05	1.400000E−04	2.400000E−04	1.000000E−05	2.000000E−05	**0.000000E+00**	1.000000E−05	2.000000E−05	3.000000E−05	**0.000000E+00**	2.000000E−05	3.000000E−05
	Rank	7	3	16	7	3	14	15	3	7	**1**	3	7	12	**1**	7	12
	(Std Dev)	(4.107096E−05)	(1.191473E−05)	(4.737387E−03)	(5.149148E−05)	(3.664987E+01)	(4.719118E+01)	(3.617644E−04)	(1.711782E−05)	(2.405875E−05)	(1.629868E−05)	(8.708797E−06)	(1.949560E−05)	(1.093492E−04)	(9.873712E−06)	(2.715966E−05)	(4.010379E−05)
	Mean	**0.000000E+00**	**0.000000E+00**	6.487647E−03	**0.000000E+00**	**0.000000E+00**	2.313725E−05	2.352941E−06	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**
	Rank|WRST	**1**	**1** | =	16 | >	**1** | =	**1** | =	15 | >	14 | =	**1** | =	**1** | =	**1** | =	**1** | =	**1** | =	**1** | =	**1** | =	**1** | =	**1** | =
	Min	**0.000000E+00**	**0.000000E+00**	1.000000E−05	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**
f_3	Rank	**1**	**1**	16	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
	Median	**0.000000E+00**	**0.000000E+00**	5.450000E−03	**0.000000E+00**	**0.000000E+00**	1.000000E−05	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**
	Rank	**1**	**1**	16	**1**	**1**	15	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
	(Std Dev)	(0.000000E+00)	(0.000000E+00)	(4.641218E−03)	(0.000000E+00)	(0.000000E+00)	(8.952073E−05)	(1.680336E−05)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)
	Mean	**0.000000E+00**	4.352941E−05	5.274065E+00	2.074035E+00	2.346246E+00	4.687036E+00	4.595988E+00	**0.000000E+00**	2.292886E−01	4.308506E−01	2.317751E+00	1.201490E−01	1.309804E−04	1.099688E+00	**0.000000E+00**	2.985986E+00
	Rank|WRST	**1**	4 | >	16 | >	10 | >	12 | >	15 | >	14 | >	**1** | =	7 | >	8 | >	11 | >	6 | >	5 | >	9 | >	**1** | =	13 | >
	Min	**0.000000E+00**	**0.000000E+00**	5.160000E−02	1.033490E+00	2.205500E−01	4.294240E+00	1.403520E+00	**0.000000E+00**	7.985000E−02	2.229300E−01	1.522840E+00	9.070000E−02	2.000000E−05	3.960000E−03	**0.000000E+00**	1.104400E−01
f_4	Rank	**1**	**1**	7	13	11	16	14	**1**	8	12	15	9	5	6	**1**	10
	Median	**0.000000E+00**	1.000000E−05	6.170220E+00	2.083060E+00	2.813810E+00	4.665130E+00	4.703090E+00	**0.000000E+00**	2.088500E−01	3.969000E−01	2.384770E+00	1.182700E−01	3.000000E−05	9.880400E−01	**0.000000E+00**	3.372130E+00
	Rank	**1**	4	16	10	12	14	15	**1**	7	8	11	6	5	9	**1**	13
	(Std Dev)	(0.000000E+00)	(2.166732E−04)	(2.179514E+00)	(3.818654E−01)	(1.895343E+00)	(1.860793E−01)	(6.697028E−01)	(0.000000E+00)	(8.732879E−02)	(2.029064E−01)	(3.595318E−01)	(1.931415E−02)	(5.802525E−04)	(6.564384E−01)	(0.000000E+00)	(1.259747E+00)
	Mean	4.436299E+00	1.346960E+01	1.051915E+01	4.699781E+00	5.705321E+00	3.867686E+00	**2.789789E+00**	7.101275E+00	4.815987E+00	5.090860E+00	3.635898E+00	8.464066E+00	8.837571E+00	1.205749E+01	8.935118E+00	1.271746E+01
	Rank|WRST	4	16 | >	13 | >	5 | =	8 | =	3 | =	**1** | <	9 | >	6 | =	7 | =	2 | <	10 | >	11 | >	14 | >	12 | >	15 | >
	Min	9.949600E−01	3.979840E+00	3.044370E+00	1.989920E+00	**0.000000E+00**	4.700000E−04	**0.000000E+00**	9.949600E−01	2.984880E+00	1.989920E+00	1.989920E+00	2.984880E+00	2.984880E+00	2.984880E+00	1.989920E+00	3.979840E+00
f_5	Rank	4	15	14	6	**1**	3	**1**	4	10	6	6	10	10	10	6	15
	Median	3.979840E+00	1.094454E+01	9.980560E+00	4.830500E+00	3.979840E+00	3.365030E+00	**2.984880E+00**	6.964710E+00	4.974800E+00	3.979840E+00	**2.984880E+00**	7.959670E+00	7.959670E+00	9.949590E+00	7.959670E+00	1.193950E+01
	Rank	4	15	14	7	4	3	**1**	9	8	4	**1**	10	10	13	10	16
	(Std Dev)	(2.000869E+00)	(7.158385E+00)	(5.311722E+00)	(1.370959E+00)	(4.675374E+00)	(2.895596E+00)	(1.541636E+00)	(3.140156E+00)	(1.379442E+00)	(2.573448E+00)	(1.595625E+00)	(3.760524E+00)	(3.535368E+00)	(6.120188E+00)	(3.891749E+00)	(4.720190E+00)
	Mean	1.539804E−03	**0.000000E+00**	8.388431E−02	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	1.061431E−02	**0.000000E+00**	1.470588E−05	**0.000000E+00**	5.882353E−07	7.345098E−04	**0.000000E+00**	4.893922E−03	4.235294E−05
	Rank|WRST	13	**1** | <	16 | >	**1** | <	**1** | <	**1** | <	**1** | <	15 | >	**1** | <	10 | <	**1** | <	9 | <	12 | <	**1** | <	14 | >	11 | <
	Min	5.500000E−04	**0.000000E+00**	9.480000E−03	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**
f_6	Rank	15	**1**	16	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
	Median	1.400000E−03	**0.000000E+00**	7.822000E−02	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	8.000000E−05	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	2.400000E−04	**0.000000E+00**
	Rank	15	**1**	16	**1**	**1**	**1**	**1**	13	**1**	**1**	**1**	**1**	**1**	**1**	14	**1**
	(Std Dev)	(6.827254E−04)	(0.000000E+00)	(4.921295E−02)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(3.395456E−02)	(0.000000E+00)	(3.678875E−05)	(0.000000E+00)	(2.376354E−06)	(5.132758E−03)	(0.000000E+00)	(9.740529E−03)	(2.265797E−04)
	Mean	1.484730E+01	2.210110E+01	2.058538E+01	1.596078E+01	1.825569E+01	1.690126E+01	**1.240072E+01**	1.808682E+01	1.768191E+01	1.799844E+01	1.403841E+01	1.926624E+01	1.903619E+01	1.769866E+01	1.882491E+01	2.170513E+01
	Rank|WRST	3	16 | >	14 | >	4 | >	10 | =	5 | >	**1** | <	9 | >	6 | >	8 | >	2 | =	13 | >	12 | >	7 | >	11 | >	15 | >
	Min	1.098805E+01	5.305550E+00	1.367951E+01	1.277526E+01	1.065845E+01	1.133276E+01	1.072588E+01	1.202749E+01	1.244593E+01	1.142209E+01	**5.237810E+00**	1.262025E+01	1.320496E+01	1.126280E+01	5.484180E+00	1.057856E+01
f_7	Rank	7	2	16	14	5	9	6	11	12	10	**1**	13	15	8	3	4
	Median	1.378287E+01	2.013172E+01	1.879476E+01	1.613914E+01	1.296391E+01	1.681843E+01	**1.237006E+01**	1.768801E+01	1.799909E+01	1.722828E+01	1.380897E+01	1.881549E+01	1.837587E+01	1.705584E+01	1.878971E+01	2.089827E+01
	Rank	3	15	13	5	2	6	**1**	9	10	8	4	14	11	7	12	16
	(Std Dev)	(3.231603E+00)	(7.880745E+00)	(6.062834E+00)	(1.762230E+00)	(8.624754E+00)	(3.972877E+00)	(1.085184E+00)	(4.022275E+00)	(3.047166E+00)	(3.682215E+00)	(1.971712E+00)	(4.387667E+00)	(3.948677E+00)	(4.290744E+00)	(5.467197E+00)	(4.994233E+00)
	Mean	3.819705E+00	1.454267E+01	9.742160E+00	5.036765E+00	6.671880E+00	**2.861028E+00**	2.984879E+00	7.745071E+00	5.189639E+00	5.657610E+00	3.433698E+00	7.940160E+00	9.813022E+00	8.837572E+00	9.754494E+00	1.242723E+01
	Rank|WRST	4	16 | >	12 | >	5 | >	8 | >	**1** | =	2 | =	9 | >	6 | >	7 | >	3 | =	10 | >	14 | >	11 | >	13 | >	15 | >
	Min	9.949600E−01	3.979840E+00	3.001790E+00	1.989920E+00	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	1.989920E+00	1.000000E−05	1.989920E+00	9.949600E−01	2.984880E+00	3.979840E+00	1.989920E+00	2.984880E+00	1.989920E+00
f_8	Rank	5	15	14	7	**1**	**1**	**1**	7	4	7	5	12	15	7	12	7
	Median	2.984880E+00	1.193950E+01	9.022740E+00	4.974800E+00	3.979840E+00	**2.171170E+00**	2.984880E+00	7.959670E+00	4.974800E+00	5.969750E+00	3.979840E+00	7.959670E+00	8.954630E+00	8.954630E+00	8.954630E+00	1.293446E+01
	Rank	2	15	14	6	4	**1**	2	9	6	8	4	9	11	11	11	16
	(Std Dev)	(2.416788E+00)	(6.926143E+00)	(4.473695E+00)	(1.774156E+00)	(5.599443E+00)	(1.961849E+00)	(1.768677E+00)	(3.724458E+00)	(2.185335E+00)	(2.628574E+00)	(1.131362E+00)	(2.866378E+00)	(3.879158E+00)	(3.634781E+00)	(3.576422E+00)	(5.121360E+00)
	Mean	**0.000000E+00**	8.908235E−03	2.699549E−02	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	3.510980E−03	4.616608E−02	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	5.696020E−02	1.068990E−01	3.510980E−03	1.102790E−01
	Rank|WRST	**1**	11 | =	12 | >	**1** | =	**1** | =	**1** | =	9 | =	13 | >	**1** | =	**1** | =	**1** | =	**1** | =	14 | >	15 | >	9 | =	16 | >
	Min	**0.000000E+00**	**0.000000E+00**	1.950000E−03	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**
f_9	Rank	**1**	**1**	16	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
	Median	**0.000000E+00**	**0.000000E+00**	1.074000E−02	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**
	Rank	**1**	**1**	16	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
	(Std Dev)	(0.000000E+00)	(6.361752E−02)	(8.844065E−02)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(1.755139E−02)	(2.001745E−01)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(2.670523E−01)	(3.255782E−01)	(1.755139E−02)	(3.372777E−01)
	Mean	5.786627E+02	4.946436E+02	4.225390E+02	1.555647E+02	3.792467E+02	2.321233E+02	4.575056E+02	3.673565E+02	1.598308E+02	3.304944E+02	**1.039365E+02**	3.327141E+02	4.427364E+02	4.758326E+02	4.747032E+02	4.763625E+02
	Rank|WRST	16	15 | =	9 | <	2 | <	8 | <	4 | <	11 | <	7 | <	3 | <	5 | <	**1** | <	6 | <	10 | <	13 | <	12 | <	14 | <
	Min	2.424432E+02	1.853450E+01	2.858770E+01	3.747000E−01	**1.874000E−01**	2.076000E−01	3.539900E+00	**1.874000E−01**	2.498000E−01	4.372000E−01	**1.874000E−01**	3.123000E−01	2.498000E−01	3.602300E+00	1.859700E+01	1.186257E+02
f_{10}	Rank	16	12	14	8	**1**	4	10	**1**	5	9	**1**	7	5	11	13	15
	Median	5.444329E+02	5.025521E+02	3.629875E+02	1.300701E+02	3.618491E+02	1.280060E+02	3.670525E+02	3.738824E+02	1.451273E+02	3.146413E+02	**1.192030E+02**	3.391179E+02	4.167245E+02	4.587609E+02	3.791712E+02	4.550184E+02
	Rank	16	15	8	3	7	2	9	10	4	5	**1**	6	12	14	11	13
	(Std Dev)	(2.091718E+02)	(2.545554E+02)	(2.224176E+02)	(1.212104E+02)	(2.598052E+02)	(2.446397E+02)	(2.766473E+02)	(2.294647E+02)	(1.253157E+02)	(2.151593E+02)	(1.059598E+02)	(2.326974E+02)	(2.309340E+02)	(2.605149E+02)	(2.863625E+02)	(2.351102E+02)

(*continued on next page*)

Table 10 (*continued*).

f_{11}	Mean	1.014480E+00	4.589325E+00	9.325559E+00	1.833755E+00	1.224361E+00	3.482886E+00	5.415078E+00	1.466918E+01	1.811157E+00	**9.506706E−01**	1.227433E+00	2.692153E+00	8.753351E+00	7.538712E+00	5.577235E+00	6.445147E+00
	Rank|WRST	2	9 | >	15 | >	6 | >	3 | >	8 | >	10 | >	16 | >	5 | >	**1** | <	4 | >	7 | >	14 | >	13 | >	11 | >	12 | >
	Min	**0.000000E+00**	**0.000000E+00**	2.049500E+00	4.830000E−02	1.060000E−02	2.358400E+00	8.303000E−01	9.950000E−01	2.000000E−04	2.500000E−03	9.190000E−02	1.200000E−03	9.950000E−01	3.868000E−01	**0.000000E+00**	6.000000E−03
	Rank	**1**	**1**	15	9	8	16	12	13	4	6	10	5	13	11	**1**	7
	Median	**9.950000E−01**	2.984900E+00	8.714400E+00	2.057100E+00	1.019300E+00	3.401100E+00	3.371500E+00	1.265610E+01	1.990400E+00	1.006600E+00	1.174500E+00	2.028200E+00	8.676200E+00	5.691400E+00	4.974800E+00	5.691600E+00
	Rank	**1**	8	15	7	3	10	9	16	5	2	4	6	14	12	11	13
	(Std Dev)	(9.007562E−01)	(4.197654E+00)	(4.106698E+00)	(9.872077E−01)	(1.172546E+00)	(6.065908E−01)	(6.008953E+00)	(1.125129E+01)	(1.057974E+00)	(1.036624E+00)	(7.963040E−01)	(2.082074E+00)	(4.800341E+00)	(5.439037E+00)	(6.428540E+00)	(4.684830E+00)
f_{12}	Mean	**4.574577E+02**	1.269272E+04	1.374168E+04	8.190242E+03	1.992569E+04	1.172139E+04	7.304708E+03	1.856229E+04	8.004612E+03	1.206322E+04	7.636445E+03	1.495981E+04	8.383990E+03	6.251118E+03	9.015395E+02	1.565400E+04
	Rank|WRST	**1**	11 | >	12 | >	7 | >	16 | >	9 | >	4 | >	15 | >	6 | >	10 | >	5 | >	13 | >	8 | >	3 | >	2 | >	14 | >
	Min	**1.170400E+01**	3.430911E+02	3.840912E+02	6.594346E+02	9.783815E+02	1.273186E+03	7.611419E+02	1.964432E+02	1.226839E+03	1.897977E+03	1.188912E+03	4.137467E+02	2.929091E+02	4.262327E+02	1.269930E+01	4.625892E+02
	Rank	**1**	5	6	10	12	15	11	3	14	16	13	7	4	8	2	9
	Median	**4.484857E+02**	8.883477E+03	9.934373E+03	6.628607E+03	1.944377E+04	8.862179E+03	5.962988E+03	4.448167E+03	5.651499E+03	8.945618E+03	7.265815E+03	1.335826E+04	6.185573E+03	5.272431E+03	5.972128E+02	1.018681E+04
	Rank	**1**	11	13	8	16	10	6	3	5	12	9	15	7	4	2	14
	(Std Dev)	(2.333363E+02)	(1.189175E+04)	(1.110442E+04)	(6.156994E+03)	(1.351724E+04)	(9.719029E+03)	(4.476434E+03)	(6.575299E+04)	(5.789399E+03)	(9.954396E+03)	(4.438365E+03)	(1.083519E+04)	(6.937679E+03)	(3.385366E+03)	(1.818033E+03)	(1.324496E+04)
f_{13}	Mean	**4.119462E+01**	3.717000E+03	6.332973E+03	1.557247E+02	2.276225E+03	3.994476E+03	7.438862E+03	1.338377E+02	1.579501E+02	3.909816E+03	1.352963E+03	3.663269E+03	5.260467E+03	1.546300E+03	7.612641E+01	5.805810E+03
	Rank|WRST	**1**	10 | >	15 | >	4 | >	8 | >	12 | >	16 | >	3 | >	5 | >	11 | >	6 | >	9 | >	13 | >	7 | >	2 | >	14 | >
	Min	2.211100E+00	8.939100E+00	6.990550E+01	9.574700E+00	**7.428000E−01**	1.447889E+02	3.734662E+02	9.131300E+00	6.265700E+00	1.870372E+02	3.365000E+01	4.171013E+02	9.758900E+00	3.125000E+00	4.009000E+00	5.631100E+01
	Rank	2	6	12	8	**1**	13	15	7	5	14	10	16	9	3	4	11
	Median	**2.549100E+01**	2.313965E+03	4.123451E+03	4.941510E+01	1.670126E+03	3.830374E+03	7.327085E+03	8.289200E+01	6.531690E+01	2.703302E+03	3.810910E+02	2.273257E+03	3.455796E+03	6.024730E+01	4.469170E+01	3.747773E+03
	Rank	**1**	10	15	3	8	14	16	6	5	11	7	9	12	4	2	13
	(Std Dev)	(5.424044E+01)	(4.139037E+03)	(6.515041E+03)	(3.531667E+02)	(2.116820E+03)	(3.102944E+03)	(3.985280E+03)	(1.425858E+02)	(2.627881E+02)	(2.960053E+03)	(1.896907E+03)	(3.597548E+03)	(6.752856E+03)	(4.097698E+03)	(8.875615E+01)	(6.153847E+03)
f_{14}	Mean	**2.097356E+01**	2.708157E+02	4.300863E+01	2.163786E+01	2.689711E+01	4.594379E+01	2.461162E+03	4.603178E+01	2.398938E+01	4.353052E+01	2.279355E+01	4.772539E+01	9.149306E+02	5.467970E+01	9.742111E+01	3.330898E+01
	Rank|WRST	**1**	14 | >	7 | >	2 | =	5 | >	9 | >	16 | >	10 | >	4 | =	8 | >	3 | =	11 | >	15 | >	12 | >	13 | >	6 | >
	Min	2.050900E+00	1.011600E+00	1.296290E+01	3.280100E+00	**1.819000E−01**	2.977570E+01	1.313901E+02	1.997860E+01	1.795400E+00	1.917890E+01	3.168400E+00	1.821110E+01	4.780300E+00	6.265000E+00	2.196850E+01	2.796200E+00
	Rank	4	2	10	7	**1**	15	16	13	3	12	6	11	8	9	14	5
	Median	2.248360E+01	1.275271E+02	4.119110E+01	**2.015900E+01**	2.693380E+01	4.072850E+01	1.827971E+03	4.188900E+01	2.152780E+01	4.152750E+01	2.414550E+01	4.566610E+01	8.032802E+02	4.268950E+01	6.086950E+01	3.132360E+01
	Rank	3	14	8	**1**	5	7	16	10	2	9	4	12	15	11	13	6
	(Std Dev)	(6.381853E+00)	(2.819998E+02)	(1.299303E+01)	(1.325518E+01)	(1.162384E+01)	(1.695129E+01)	(2.010222E+03)	(1.913277E+01)	(1.298404E+01)	(1.358220E+01)	(1.000240E+01)	(1.585253E+01)	(9.147109E+02)	(3.830632E+01)	(1.769507E+02)	(2.218594E+01)
f_{15}	Mean	**4.735024E+00**	7.228349E+01	6.990858E+01	1.422510E+01	6.416722E+01	1.248835E+02	2.914737E+03	3.662655E+01	1.343920E+01	4.913700E+01	3.399121E+01	4.947694E+01	1.363207E+03	2.890204E+01	9.534497E+01	8.648999E+01
	Rank|WRST	**1**	11 | >	10 | >	3 | >	9 | >	14 | >	16 | >	6 | >	2 | >	7 | >	5 | >	8 | >	15 | >	4 | >	13 | >	12 | >
	Min	**5.079000E−01**	2.484400E+00	8.124300E+00	2.635000E+00	2.515000E+00	1.579200E+01	1.414130E+01	5.073900E+00	2.453300E+00	5.992800E+00	1.373700E+00	1.350200E+00	5.107000E+00	1.337600E+00	6.899300E+00	4.414300E+00
	Rank	**1**	6	14	8	7	16	15	10	5	12	4	3	11	2	13	9
	Median	**3.734600E+00**	4.420120E+01	5.208710E+01	1.021120E+01	2.219310E+01	8.166660E+01	2.423773E+03	2.960990E+01	1.101810E+01	4.263420E+01	1.835700E+01	4.112110E+01	5.358105E+02	1.327940E+01	7.082530E+01	3.440270E+01
	Rank	**1**	11	12	2	6	14	16	7	3	10	5	9	15	4	13	8
	(Std Dev)	(3.877589E+00)	(7.466278E+01)	(6.147032E+01)	(1.220121E+01)	(2.065565E+02)	(1.838670E+02)	(2.240596E+03)	(3.301230E+01)	(1.032155E+01)	(3.620813E+01)	(4.684046E+01)	(3.800954E+01)	(1.753832E+03)	(3.947470E+01)	(8.378778E+01)	(1.363065E+02)
f_{16}	Mean	4.564808E+00	1.509390E+02	6.064903E+01	7.380667E−01	3.369229E+00	5.548212E+00	2.479896E+01	5.002653E+00	**6.396275E−01**	4.489102E+01	8.499314E−01	1.112423E+01	1.538568E+02	1.170602E+02	3.020497E+01	6.700079E+01
	Rank|WRST	5	15 | >	12 | >	2 | <	4 | <	7 | >	9 | =	6 | =	**1** | <	11 | =	3 | <	8 | >	16 | >	14 | >	10 | >	13 | =
	Min	5.735000E−01	**2.080000E−02**	2.272000E−01	7.130000E−02	1.821000E−01	8.877000E−01	6.334000E−01	5.470000E−02	4.790000E−02	7.220000E−01	2.828000E−01	5.130000E−01	7.810000E−02	2.588000E−01	2.361000E−01	2.002000E−01
	Rank	13	**1**	8	4	6	16	14	3	2	15	11	12	5	10	9	7
	Median	1.635700E+00	1.311411E+02	4.218700E+00	7.429000E−01	1.241400E+00	2.733800E+00	1.879500E+00	2.108200E+00	**6.523000E−01**	1.904900E+00	7.988000E−01	2.768100E+00	1.189696E+02	1.191449E+02	1.912600E+00	9.058000E−01
	Rank	6	16	13	2	5	11	7	10	**1**	8	3	12	14	15	9	4
	(Std Dev)	(4.943871E+00)	(1.339568E+02)	(7.483026E+01)	(2.796228E−01)	(6.761584E+00)	(1.265309E+01)	(5.486460E+01)	(1.665284E+01)	(2.834491E−01)	(7.147255E+01)	(2.889903E−01)	(2.922430E+01)	(1.296770E+02)	(1.397335E+02)	(5.998518E+01)	(1.083758E+02)
f_{17}	Mean	2.798009E+01	4.514072E+01	3.320134E+01	3.675818E+00	2.032598E+01	3.377293E+01	3.236895E+01	2.950655E+01	**2.860363E+00**	3.100695E+01	9.962955E+00	2.582076E+01	2.729704E+01	2.404374E+01	3.280735E+01	2.292280E+01
	Rank|WRST	9	16 | >	14 | >	2 | <	4 | <	15 | >	12 | =	10 | =	**1** | <	11 | =	3 | <	7 | =	8 | <	6 | <	13 | =	5 | <
	Min	1.483580E+01	2.351400E+00	5.462900E+00	**4.160000E−02**	7.519000E−01	1.974500E+00	2.478000E+00	6.548500E+00	1.759000E−01	3.967800E+00	3.319000E−01	4.404000E−01	1.326900E+00	3.122000E−01	1.317100E+00	1.093600E+00
	Rank	16	11	14	**1**	6	10	12	15	2	13	4	5	9	3	8	7
	Median	2.624380E+01	3.907900E+01	3.274130E+01	**1.483100E+00**	2.371800E+01	3.221130E+01	3.440920E+01	2.868270E+01	1.743700E+00	2.870570E+01	4.489300E+00	2.569480E+01	1.879650E+01	2.001550E+01	2.948780E+01	1.922700E+01
	Rank	9	16	14	**1**	7	13	15	10	2	11	3	8	4	6	12	5
	(Std Dev)	(6.128721E+00)	(3.643924E+01)	(1.166804E+01)	(6.389729E+00)	(1.059178E+01)	(9.396509E+00)	(1.612463E+01)	(1.142536E+01)	(3.952455E+00)	(1.096281E+01)	(9.603504E+00)	(9.204157E+00)	(2.344322E+01)	(1.588507E+01)	(1.607364E+01)	(2.043548E+01)
f_{18}	Mean	**8.181575E+01**	3.314386E+03	1.015307E+04	1.063104E+03	2.790358E+03	2.174156E+04	8.698866E+03	2.078675E+02	8.296760E+02	7.640521E+03	1.510689E+03	5.558411E+03	1.120845E+04	3.726716E+02	3.283498E+02	6.817174E+03
	Rank|WRST	**1**	9 | >	14 | >	6 | >	8 | >	16 | >	13 | >	2 | >	5 | >	12 | >	7 | >	10 | >	15 | >	4 | >	3 | >	11 | >
	Min	2.162050E+01	3.205430E+01	3.343469E+02	2.763660E+01	1.672092E+02	2.433462E+02	3.626336E+02	**5.570400E+00**	4.125760E+01	3.752107E+02	3.628790E+01	1.031606E+02	1.590340E+01	1.299420E+01	2.353300E+01	5.189710E+01
	Rank	4	7	14	6	12	13	15	**1**	9	16	8	11	3	2	5	10
	Median	8.120860E+01	2.310272E+03	6.746828E+03	5.476032E+02	2.033403E+03	2.465237E+04	6.758229E+03	**4.747280E+01**	5.976976E+02	6.393486E+03	1.456114E+03	3.397968E+03	4.868086E+03	1.111148E+02	1.122011E+02	2.411882E+03
	Rank	2	9	14	5	8	16	15	**1**	6	13	7	11	12	3	4	10
	(Std Dev)	(4.541098E+01)	(3.153056E+03)	(9.250217E+03)	(1.384571E+03)	(2.883143E+03)	(1.248028E+04)	(6.440079E+03)	(1.049983E+03)	(7.517458E+02)	(6.644086E+03)	(1.271522E+03)	(5.463816E+03)	(1.391647E+04)	(7.910882E+02)	(5.318712E+02)	(1.026365E+04)

(*continued on next page*)

Table 10 (*continued*).

f_{19}	Mean	**6.251527E+00**	6.148574E+02	4.751213E+01	1.498678E+01	1.098759E+02	4.013200E+03	4.770544E+03	1.648036E+01	1.028569E+01	5.993570E+01	2.046064E+01	3.695977E+01	6.054700E+03	2.697226E+01	4.605710E+02	1.105144E+02
	Rank|WRST	**1**	13 | >	8 | >	3 | =	10 | >	14 | >	15 | >	4 | >	2 | >	9 | >	5 | >	7 | >	16 | >	6 | >	12 | >	11 | >
	Min	2.224900E+00	5.205900E+00	5.933200E+00	**7.660000E−02**	4.724200E+00	1.856920E+01	4.562825E+02	3.764200E+00	1.234400E+00	3.063900E+00	2.311100E+00	3.183100E+00	2.400000E+00	2.245000E−01	1.355270E+01	3.419000E+00
	Rank	4	12	13	**1**	11	15	16	10	3	7	5	8	6	2	14	9
	Median	**5.840600E+00**	2.228127E+02	3.071750E+01	6.470500E+00	3.766500E+01	8.094583E+02	3.649135E+03	1.387130E+01	8.200700E+00	2.323100E+01	1.598180E+01	2.329250E+01	4.538507E+03	9.112200E+00	8.529650E+01	4.651990E+01
	Rank	**1**	13	9	2	10	14	15	5	3	7	6	8	16	4	12	11
	(Std Dev)	(2.358782E+00)	(8.970952E+02)	(5.314965E+01)	(2.603298E+01)	(3.022760E+02)	(4.979103E+03)	(3.677732E+03)	(1.184489E+01)	(8.715361E+00)	(1.492337E+02)	(2.368637E+01)	(5.218387E+01)	(6.303289E+03)	(3.813891E+01)	(9.308228E+02)	(1.708414E+02)
f_{20}	Mean	1.143384E+01	1.837891E+01	2.206535E+01	7.863216E−01	1.136135E+01	3.259286E+00	1.969777E+01	2.201994E+01	**3.985667E−01**	1.642120E+01	3.500288E+00	1.929367E+01	1.096170E+01	4.917942E+01	2.580552E+01	1.055523E+01
	Rank|WRST	8	10 | >	14 | >	2 | <	7 | =	3 | =	12 | >	13 | >	**1** | <	9 | >	4 | =	11 | >	6 | =	16 | >	15 | >	5 | =
	Min	2.000000E−04	**0.000000E+00**	1.307100E+00	**0.000000E+00**	3.122000E−01	2.100000E−03	3.122000E−01	1.020900E+00	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	1.000000E−04	**0.000000E+00**	**0.000000E+00**	9.950000E−01	3.122000E−01
	Rank	9	**1**	16	**1**	11	10	11	15	**1**	**1**	**1**	8	**1**	**1**	14	11
	Median	1.035010E+01	2.031220E+01	2.283880E+01	**1.390000E−01**	6.393400E+00	1.307100E+00	2.130710E+01	2.229600E+01	3.122000E−01	2.198970E+01	1.307100E+00	2.161970E+01	5.170400E+00	1.746040E+01	2.130710E+01	5.287000E+00
	Rank	8	10	16	**1**	7	3	11	15	2	14	3	13	5	9	11	6
	(Std Dev)	(1.128308E+01)	(1.548975E+01)	(1.048174E+01)	(2.787961E+00)	(1.158478E+01)	(5.665131E+00)	(1.324018E+01)	(1.044351E+01)	(5.446248E−01)	(1.070699E+01)	(5.788052E+00)	(8.356459E+00)	(1.071846E+01)	(6.089270E+01)	(1.826855E+01)	(1.012247E+01)
f_{21}	Mean	**1.010623E+02**	1.263885E+02	1.709883E+02	1.196539E+02	1.931510E+02	1.934864E+02	2.030420E+02	1.938116E+02	1.169633E+02	1.317257E+02	1.249994E+02	1.382838E+02	2.042394E+02	1.882117E+02	1.945537E+02	1.750241E+02
	Rank|WRST	**1**	5 | =	8 | >	3 | =	11 | >	12 | >	15 | >	13 | >	2 | =	6 | =	4 | =	7 | =	16 | >	10 | >	14 | >	9 | >
	Min	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	1.007007E+02	1.137552E+02	1.295401E+02	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**
	Rank	**1**	**1**	**1**	**1**	14	15	16	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
	Median	**1.000000E+02**	**1.000000E+02**	2.065535E+02	**1.000000E+02**	2.059575E+02	2.030265E+02	2.047099E+02	2.078763E+02	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	2.114406E+02	2.099196E+02	2.132344E+02	2.097931E+02
	Rank	**1**	**1**	11	**1**	10	8	9	12	**1**	**1**	**1**	**1**	15	14	16	13
	(Std Dev)	(1.338491E+00)	(4.928341E+01)	(5.274851E+01)	(3.863323E+01)	(3.812967E+01)	(2.590604E+01)	(1.093102E+01)	(3.771700E+01)	(3.958580E+01)	(4.931435E+01)	(4.527443E+01)	(5.218372E+01)	(3.111719E+01)	(4.633656E+01)	(4.399986E+01)	(5.566288E+01)
f_{22}	Mean	1.000000E+02	9.576979E+01	9.983891E+01	8.538827E+01	8.993846E+01	1.000427E+02	1.000226E+02	9.568739E+01	**7.493854E+01**	9.392352E+01	9.808628E+01	1.016340E+02	9.796116E+01	1.018212E+02	9.590165E+01	9.827356E+01
	Rank|WRST	12	6 | <	11 | <	2 | <	3 | <	14 | >	13 | >	5 | <	**1** | =	4 | <	9 | <	15 | >	8 | <	16 | >	7 | <	10 | <
	Min	1.000000E+02	**0.000000E+00**	1.925960E+01	1.798340E+01	**0.000000E+00**	1.000000E+02	1.000000E+02	1.556050E+01	1.140780E+01	**0.000000E+00**	3.818860E+01	1.005354E+02	1.788660E+01	1.000000E+02	2.607860E+01	**0.000000E+00**
	Rank	12	**1**	9	8	**1**	12	12	6	5	**1**	11	16	7	12	10	**1**
	Median	**1.000000E+02**	1.009808E+02	1.026043E+02	1.006344E+02	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	1.002902E+02	1.005081E+02	1.013775E+02	1.010841E+02	1.013303E+02	1.014972E+02	1.015272E+02
	Rank	**1**	10	16	9	**1**	**1**	**1**	**1**	**1**	7	8	13	11	12	14	15
	(Std Dev)	(0.000000E+00)	(2.181358E+01)	(1.650130E+01)	(2.568268E+01)	(2.680093E+01)	(1.423268E−01)	(7.837650E−02)	(1.842541E+01)	(3.531316E+01)	(2.261969E+01)	(1.222439E+01)	(7.889186E−01)	(1.635262E+01)	(1.409034E+00)	(1.944797E+01)	(1.821987E+01)
f_{23}	Mean	3.061644E+02	3.178093E+02	3.080198E+02	3.071668E+02	3.050072E+02	3.089354E+02	3.079135E+02	3.068427E+02	3.063685E+02	3.087157E+02	**3.047130E+02**	3.085034E+02	3.180427E+02	3.157219E+02	3.109047E+02	3.203850E+02
	Rank|WRST	3	14 | >	8 | >	6 | >	2 | <	11 | >	7 | >	5 | =	4 | =	10 | >	**1** | <	9 | >	15 | >	13 | >	12 | >	16 | >
	Min	3.000000E+02	3.052007E+02	**1.345000E−01**	3.000000E+02	3.000000E+02	3.026732E+02	3.028820E+02	3.025757E+02	3.025757E+02	3.041582E+02	3.000000E+02	3.031197E+02	3.069811E+02	3.040759E+02	3.038227E+02	3.060173E+02
	Rank	2	14	**1**	2	2	8	9	6	6	13	2	10	16	12	11	15
	Median	3.060731E+02	3.164180E+02	3.131135E+02	3.071562E+02	**3.037983E+02**	3.080049E+02	3.075375E+02	3.059072E+02	3.060174E+02	3.084420E+02	3.054982E+02	3.077899E+02	3.158410E+02	3.134318E+02	3.094664E+02	3.206725E+02
	Rank	5	15	12	6	**1**	9	7	3	4	10	2	8	14	13	11	16
	(Std Dev)	(4.151347E+00)	(8.522835E+00)	(4.432750E+01)	(2.554699E+00)	(3.698580E+00)	(4.099879E+00)	(3.177952E+00)	(3.201271E+00)	(2.622622E+00)	(2.811736E+00)	(2.450890E+00)	(2.358466E+00)	(8.061406E+00)	(7.449710E+00)	(4.088035E+00)	(9.346297E+00)
f_{24}	Mean	3.232410E+02	3.211093E+02	3.006669E+02	**1.505275E+02**	3.318393E+02	3.334162E+02	3.328124E+02	2.993648E+02	1.586417E+02	2.491501E+02	3.177378E+02	2.381341E+02	3.137773E+02	3.050531E+02	3.365573E+02	3.274359E+02
	Rank|WRST	11	10 | <	6 | <	**1** | <	13 | >	15 | >	14 | >	5 | <	2 | <	4 | =	9 | =	3 | =	8 | <	7 | <	16 | >	12 | >
	Min	1.000000E+02	**0.000000E+00**	1.000806E+02	5.762680E+01	1.824408E+02	2.576606E+02	2.671716E+02	1.000000E+02	1.000000E+02	1.000000E+02	1.000000E+02	1.000000E+02	**0.000000E+00**	1.000000E+02	1.000000E+02	1.000000E+02
	Rank	4	**1**	13	3	14	15	16	4	4	4	4	4	**1**	4	4	4
	Median	3.314828E+02	3.442346E+02	3.375278E+02	1.193210E+02	3.329883E+02	3.352945E+02	3.349309E+02	3.348850E+02	**1.000000E+02**	3.345428E+02	3.308206E+02	3.345473E+02	3.405555E+02	3.419755E+02	3.410195E+02	3.484694E+02
	Rank	4	15	11	2	5	10	9	8	**1**	6	3	7	12	14	13	16
	(Std Dev)	(4.566756E+01)	(8.180864E+01)	(8.790211E+01)	(6.495547E+01)	(2.193905E+01)	(1.302718E+01)	(1.146939E+01)	(8.692954E+01)	(9.733479E+01)	(1.161126E+02)	(5.500872E+01)	(1.180253E+02)	(8.634089E+01)	(8.964389E+01)	(3.407561E+01)	(7.645208E+01)
f_{25}	Mean	4.361022E+02	4.229405E+02	4.206018E+02	3.898714E+02	4.165352E+02	4.341216E+02	4.407567E+02	4.327675E+02	**3.844919E+02**	4.180121E+02	4.126321E+02	4.025719E+02	4.318430E+02	4.370434E+02	4.163983E+02	4.238695E+02
	Rank|WRST	14	9 | =	8 | =	2 | <	6 | =	13 | <	16 | >	12 | <	**1** | <	7 | <	4 | <	3 | <	11 | <	15 | >	5 | =	10 | =
	Min	3.980090E+02	**1.000302E+02**	3.977429E+02	1.196661E+02	3.980090E+02	3.981162E+02	3.979914E+02	3.977624E+02	1.369428E+02	3.977429E+02	3.977429E+02	3.977429E+02	3.977624E+02	3.979914E+02	3.979289E+02	3.977429E+02
	Rank	14	**1**	4	2	14	16	12	9	3	4	4	4	9	12	11	4
	Median	4.433784E+02	4.435028E+02	3.997546E+02	**3.981194E+02**	3.983225E+02	4.461373E+02	4.461040E+02	4.458453E+02	3.982126E+02	3.996100E+02	3.981342E+02	3.981272E+02	4.440404E+02	4.441961E+02	3.987875E+02	4.437144E+02
	Rank	9	10	8	**1**	5	16	15	14	4	7	3	2	12	13	6	11
	(Std Dev)	(1.781345E+01)	(5.090891E+01)	(2.347213E+01)	(4.484911E+01)	(2.296482E+01)	(2.069498E+01)	(1.518666E+01)	(2.219070E+01)	(5.645842E+01)	(2.260822E+01)	(2.169453E+01)	(1.377196E+01)	(2.183071E+01)	(1.771087E+01)	(2.337383E+01)	(2.390088E+01)
f_{26}	Mean	3.302993E+02	3.049332E+02	3.080725E+02	**2.050198E+02**	3.012061E+02	3.447398E+02	3.728603E+02	3.035862E+02	2.401067E+02	2.764706E+02	3.000000E+02	2.901961E+02	3.411266E+02	3.223201E+02	3.467310E+02	3.128707E+02
	Rank|WRST	12	8 | <	9 | =	**1** | <	6 | <	14 | =	16 | =	7 | <	2 | <	3 | <	5 | <	4 | <	13 | =	11 | <	15 | >	10 | <
	Min	2.000000E+02	2.000000E+02	2.015408E+02	1.005000E−01	3.000000E+02	2.000000E+02	2.000000E+02	2.000000E+02	2.318000E−01	**0.000000E+00**	3.000000E+02	**0.000000E+00**	2.000000E+02	2.000000E+02	3.000000E+02	**0.000000E+00**
	Rank	6	6	13	4	14	6	6	6	5	**1**	14	**1**	6	6	14	**1**
	Median	3.000001E+02	3.000000E+02	3.003548E+02	**2.155234E+02**	3.000000E+02	3.169142E+02	3.554061E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02
	Rank	13	2	14	**1**	2	15	16	2	2	2	2	2	2	2	2	2
	(Std Dev)	(4.210506E+01)	(5.077874E+01)	(1.434909E+02)	(9.644510E+01)	(8.613333E+00)	(1.332412E+02)	(1.329425E+02)	(2.129906E+01)	(9.687483E+01)	(6.507914E+01)	(0.000000E+00)	(4.584715E+01)	(1.423749E+02)	(1.474687E+02)	(1.373543E+02)	(6.482517E+01)
f_{27}	Mean	3.919517E+02	4.015771E+02	3.950413E+02	3.924238E+02	3.937298E+02	3.922673E+02	4.005302E+02	**3.811543E+02**	3.921372E+02	3.919713E+02	3.932860E+02	3.895897E+02	3.930782E+02	4.013219E+02	3.935802E+02	4.020706E+02
	Rank|WRST	3	15 | >	12 | >	7 | =	11 | >	6 | =	13 | >	**1** | <	5 | =	4 | >	9 | >	2 | <	8 | >	14 | >	10 | >	16 | >
	Min	3.890055E+02	3.890055E+02	3.898536E+02	3.873236E+02	3.876723E+02	**3.744167E+02**	3.907519E+02	3.772439E+02	3.873236E+02	3.889780E+02	3.888606E+02	3.889780E+02	3.896415E+02	3.895180E+02	3.886310E+02	3.910442E+02
	Rank	10	10	14	3	5	**1**	15	2	3	8	7	8	13	12	6	16
	Median	3.907849E+02	3.943550E+02	3.948060E+02	3.926833E+02	3.939536E+02	3.907519E+02	3.990668E+02	**3.803153E+02**	3.916195E+02	3.896514E+02	3.938188E+02	3.896496E+02	3.927406E+02	3.986321E+02	3.940181E+02	3.997531E+02
	Rank	5	12	13	7	10	4	15	**1**	6	3	9	2	8	14	11	16
	(Std Dev)	(2.367180E+00)	(2.310090E+01)	(2.340302E+00)	(2.032440E+00)	(2.419504E+00)	(4.481154E+00)	(5.783494E+00)	(3.474171E+00)	(2.374080E+00)	(7.692785E+00)	(1.926786E+00)	(4.048621E−01)	(2.670709E+00)	(8.726520E+00)	(1.632010E+00)	(1.157274E+01)

(*continued on next page*)

Table 10 (*continued*).

f_{28}	Mean	5.227155E+02	3.492099E+02	3.553254E+02	2.862101E+02	3.903608E+02	4.710741E+02	5.745620E+02	4.425007E+02	**2.731003E+02**	4.235555E+02	3.000000E+02	3.384696E+02	5.207400E+02	4.885292E+02	3.671750E+02	3.577416E+02
	Rank|WRST	15	5 | <	6 | <	2 | <	9 | <	12 | <	16 | =	11 | <	**1** | <	10 | <	3 | <	4 | <	14 | <	13 | =	8 | <	7 | <
	Min	3.000001E+02	3.000000E+02	3.000000E+02	5.424230E+01	3.000000E+02	3.000005E+02	3.000000E+02	**0.000000E+00**	**0.000000E+00**	2.465141E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02	3.000000E+02
	Rank	15	5	5	3	5	16	5	**1**	**1**	4	5	5	5	5	5	5
	Median	6.118218E+02	**3.000000E+02**	3.004982E+02	**3.000000E+02**	**3.000000E+02**	4.926373E+02	6.026912E+02	5.000025E+02	**3.000000E+02**	**3.000000E+02**	**3.000000E+02**	**3.000000E+02**	5.837340E+02	5.837452E+02	3.847556E+02	**3.000000E+02**
	Rank	16	**1**	9	**1**	**1**	11	15	12	**1**	**1**	**1**	**1**	13	14	10	**1**
	(Std Dev)	(1.368563E+02)	(9.815957E+01)	(1.077437E+02)	(5.583202E+01)	(1.307612E+02)	(7.315713E+01)	(8.641949E+01)	(1.501130E+02)	(8.273181E+01)	(1.485006E+02)	(0.000000E+00)	(9.498088E+01)	(1.234208E+02)	(1.276089E+02)	(3.802271E+01)	(1.178674E+02)
f_{29}	Mean	**2.425523E+02**	3.445052E+02	2.850738E+02	2.528754E+02	2.558485E+02	2.742843E+02	2.824875E+02	2.633053E+02	2.471804E+02	2.565569E+02	2.445848E+02	2.545899E+02	2.830224E+02	2.786037E+02	2.735905E+02	2.790481E+02
	Rank|WRST	**1**	16 | >	15 | >	4 | >	6 | >	10 | >	13 | >	8 | >	3 | >	7 | >	2 | =	5 | >	14 | >	11 | >	9 | >	12 | >
	Min	**2.285304E+02**	2.408824E+02	2.512761E+02	2.380879E+02	2.350506E+02	2.468451E+02	2.528293E+02	2.307452E+02	2.291071E+02	2.393076E+02	2.323264E+02	2.358280E+02	2.365248E+02	2.438542E+02	2.325666E+02	2.454969E+02
	Rank	**1**	11	15	9	6	14	16	3	2	10	4	7	8	12	5	13
	Median	**2.413416E+02**	3.296742E+02	2.803084E+02	2.525222E+02	2.546448E+02	2.732492E+02	2.751053E+02	2.582122E+02	2.471104E+02	2.538455E+02	2.427814E+02	2.503380E+02	2.682795E+02	2.734516E+02	2.618203E+02	2.740985E+02
	Rank	**1**	16	15	5	7	11	14	8	3	6	2	4	10	12	9	13
	(Std Dev)	(7.280673E+00)	(7.242049E+01)	(2.654345E+01)	(7.938149E+00)	(1.078474E+01)	(1.159919E+01)	(2.199822E+01)	(1.970458E+01)	(8.213283E+00)	(1.241377E+01)	(6.007352E+00)	(1.307482E+01)	(4.345294E+01)	(2.366434E+01)	(3.867273E+01)	(2.221858E+01)
f_{30}	Mean	**7.156225E+02**	2.669590E+05	1.852638E+05	2.956545E+03	4.589651E+05	1.191325E+04	2.096639E+05	1.516069E+05	2.504444E+03	1.235760E+05	2.916310E+03	5.945164E+04	1.595201E+05	1.704282E+05	7.233941E+03	6.100153E+05
	Rank|WRST	**1**	14 | >	12 | >	4 | >	15 | >	6 | >	13 | >	9 | >	2 | >	8 | >	3 | >	7 | >	10 | >	11 | =	5 | >	16 | >
	Min	4.954583E+02	4.842858E+02	8.774893E+02	1.149411E+03	1.295323E+03	1.413661E+03	2.648090E+03	8.672121E+02	6.912596E+02	1.033745E+03	7.858740E+02	1.098191E+03	5.636350E+02	4.110460E+02	**3.514761E+02**	2.832491E+03
	Rank	4	3	9	12	13	14	15	8	6	10	7	11	5	2	**1**	16
	Median	**6.934605E+02**	1.284122E+03	4.469511E+03	2.748495E+03	5.930634E+03	9.405190E+03	1.254885E+04	5.858275E+03	2.116413E+03	3.442386E+03	2.803199E+03	3.049698E+03	1.768218E+03	7.765484E+02	1.055413E+03	8.896057E+04
	Rank	**1**	4	11	7	13	14	15	12	6	10	8	9	5	2	3	16
	(Std Dev)	(1.512251E+02)	(4.257981E+05)	(4.336632E+05)	(1.444776E+03)	(7.308295E+05)	(1.083832E+04)	(4.552121E+05)	(4.020574E+05)	(1.814203E+03)	(3.095751E+05)	(1.796907E+03)	(2.331003E+05)	(3.525702E+05)	(3.796729E+05)	(9.265766E+03)	(8.988542E+05)

Table 11
Comparison of experimental results of PSO variants, averaged over 51 runs, for 30-dimensional CEC2017 benchmark functions.

f_n	Criteria	PSO-BHF	EAPSO	PSO-ELPM	SpadePSO	AMSEPSO	EOPSO	PPSO	PSO-sono	SDPSO	AHPSO	HCLDMS-PSO	HIDMS-PSO	TAPSO	MPSO	FMPSO	TCSPSO
f_1	Mean	**8.281373E−03**	3.953348E+03	3.257464E+03	7.245180E+01	2.721290E+03	2.748440E+03	3.150286E+03	1.470640E+03	1.251163E+02	3.180331E+03	5.574399E+02	3.383299E+03	2.851994E+03	5.763369E+06	3.415584E+02	3.481476E+03
	Rank|WRST	**1**	15 | >	12 | >	2 | >	7 | >	8 | >	10 | >	6 | >	3 | >	11 | >	5 | >	13 | >	9 | >	16 | >	4 | >	14 | >
	Min	**4.700000E−04**	2.551686E+01	8.199200E−01	2.790100E−01	3.062720E+00	1.933900E−01	1.236140E+00	1.121960E+00	9.995000E−02	1.419822E+01	4.451600E−01	1.472010E+00	3.749990E+00	1.421430E+00	1.920000E−03	6.799800E−01
	Rank	**1**	16	8	5	13	4	10	9	3	15	6	12	14	11	2	7
	Median	**9.330000E−03**	2.277815E+03	2.443167E+03	2.044338E+01	1.546588E+03	1.786225E+03	1.709966E+03	4.325725E+02	3.716688E+01	1.962000E+03	1.648624E+02	2.015362E+03	1.496334E+03	3.240409E+03	9.970000E−03	1.648093E+03
	Rank	**1**	14	15	3	8	11	10	6	4	12	5	13	7	16	2	9
	(Std Dev)	(2.642091E−03)	(4.248718E+03)	(3.510451E+03)	(1.576771E+02)	(3.119018E+03)	(2.785275E+03)	(3.128829E+03)	(2.062185E+03)	(1.977868E+02)	(3.320895E+03)	(8.559358E+02)	(3.613226E+03)	(3.176635E+03)	(2.734610E+07)	(9.516276E+02)	(4.166250E+03)
f_2	Mean	6.296078E−04	**3.529412E−05**	2.345098E−04	2.284314E−04	5.622436E+15	2.763005E+14	1.752222E+04	1.881118E+16	4.650980E−04	3.006203E+00	3.899216E−03	1.015686E−04	1.196078E−04	5.389766E+18	5.610766E+09	2.342502E+17
	Rank|WRST	7	**1** | <	5 | =	4 | =	13 | >	12 | >	10 | >	14 | >	6 | <	9 | =	8 | >	2 | =	3 | <	16 | >	11 | =	15 | >
	Min	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	1.200000E−04	7.000000E−05	1.000000E−04	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	1.000000E−05	**0.000000E+00**	2.892717E+04
	Rank	**1**	**1**	**1**	**1**	**1**	15	13	14	**1**	**1**	**1**	**1**	**1**	12	**1**	16
	Median	8.000000E−05	**2.000000E−05**	6.000000E−05	8.000000E−05	1.829923E+09	4.234407E+03	5.076337E+01	2.996394E+07	4.000000E−04	1.600000E−04	9.500000E−04	9.000000E−05	3.000000E−05	2.114545E+01	1.900000E−04	2.849137E+11
	Rank	4	**1**	3	4	15	13	12	14	9	7	10	6	2	11	8	16
	(Std Dev)	(1.650062E−03)	(3.986743E−05)	(3.553776E−04)	(3.616594E−04)	(3.132322E+16)	(1.968463E+15)	(8.434220E+04)	(8.734398E+16)	(4.424765E−04)	(1.298664E+01)	(8.955099E−03)	(7.905372E−05)	(2.398329E−04)	(3.849056E+19)	(3.669908E+10)	(1.670521E+18)
f_3	Mean	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	1.207059E−03	2.798987E+02	7.075842E+02	2.534009E+04	7.650451E−02	2.352941E−06	1.167802E−01	2.391713E+01	**0.000000E+00**	**0.000000E+00**	5.003098E−02	5.882353E−07	3.038381E+03
	Rank|WRST	**1**	**1** | =	**1** | =	8 | >	13 | >	14 | >	16 | >	10 | >	7 | =	11 | >	12 | >	**1** | =	**1** | =	9 | >	6 | =	15 | >
	Min	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	1.000000E−05	1.560000E−03	1.184643E+02	8.546804E+03	4.000000E−05	**0.000000E+00**	1.400000E−04	5.840500E−01	**0.000000E+00**	**0.000000E+00**	4.000000E−05	**0.000000E+00**	1.092246E+03
	Rank	**1**	**1**	**1**	8	12	14	16	9	**1**	11	13	**1**	**1**	9	**1**	15
	Median	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	3.700000E−04	7.262867E+01	6.445150E+02	2.306108E+04	4.950000E−03	**0.000000E+00**	2.956000E−02	9.439290E+00	**0.000000E+00**	**0.000000E+00**	5.800000E−04	**0.000000E+00**	2.834042E+03
	Rank	**1**	**1**	**1**	8	13	14	16	10	**1**	11	12	**1**	**1**	9	**1**	15
	(Std Dev)	(0.000000E+00)	(0.000000E+00)	(0.000000E+00)	(2.583261E−03)	(4.818863E+02)	(4.302844E+02)	(1.212012E+04)	(2.469607E−01)	(1.176235E−05)	(2.388362E−01)	(3.552865E+01)	(0.000000E+00)	(0.000000E+00)	(3.467468E−01)	(2.376354E−06)	(1.265838E+03)
f_4	Mean	5.761573E+01	2.631566E+01	4.166882E+01	6.789805E+01	1.203586E+02	5.256508E+01	8.459635E+01	1.140798E+02	2.139150E+01	6.088752E+01	7.971246E+01	5.902762E+01	3.819844E+01	5.567204E+01	**2.345071E−01**	1.390312E+02
	Rank|WRST	8	3 | <	5 | =	11 | >	15 | >	6 | =	13 | >	14 | >	2 | <	10 | >	12 | >	9 | >	4 | <	7 | <	**1** | <	16 | >
	Min	**0.000000E+00**	1.000000E−05	2.900000E−04	3.992330E+00	7.570603E+01	2.221539E+01	4.997000E−01	4.945000E−01	2.840000E−03	1.000000E−03	4.418450E+00	4.830261E+01	1.900000E−04	1.500000E−03	**0.000000E+00**	4.152390E+00
	Rank	**1**	3	5	11	16	14	10	9	8	6	13	15	4	7	**1**	12
	Median	5.856156E+01	3.993390E+00	4.677505E+01	7.145184E+01	1.204452E+02	2.798799E+01	8.633386E+01	1.234387E+02	3.990460E+00	6.711473E+01	8.438808E+01	5.856529E+01	5.856156E+01	6.883100E+01	**0.000000E+00**	1.378544E+02
	Rank	6	3	5	11	14	4	13	15	2	9	12	8	6	10	**1**	16
	(Std Dev)	(1.208270E+01)	(2.947859E+01)	(3.233422E+01)	(1.834068E+01)	(1.457739E+01)	(3.366873E+01)	(2.175097E+01)	(3.658084E+01)	(2.625304E+01)	(2.280312E+01)	(2.024489E+01)	(3.041575E+00)	(2.976669E+01)	(2.801427E+01)	(9.473621E−01)	(4.971207E+01)
f_5	Mean	**1.277510E+01**	6.690506E+01	6.233114E+01	4.155656E+01	3.251135E+01	2.318391E+01	2.224025E+01	3.180220E+01	3.101439E+01	2.461458E+01	2.857485E+01	4.026137E+01	6.097950E+01	6.485030E+01	5.419590E+01	1.041777E+02
	Rank|WRST	**1**	15 | >	13 | >	10 | >	8 | >	3 | >	2 | >	7 | >	6 | >	4 | >	5 | >	9 | >	12 | >	14 | >	11 | >	16 | >
	Min	**3.979840E+00**	3.681342E+01	2.984875E+01	2.487397E+01	7.959670E+00	6.448790E+00	8.954630E+00	1.392942E+01	1.591934E+01	1.193951E+01	1.691430E+01	2.288405E+01	3.681343E+01	2.089413E+01	2.984876E+01	6.367724E+01
	Rank	**1**	14	12	11	3	2	4	6	7	5	8	10	15	9	13	16
	Median	**1.193951E+01**	6.268232E+01	5.969746E+01	3.885037E+01	1.596245E+01	1.607721E+01	2.288405E+01	2.984876E+01	2.984876E+01	2.387901E+01	2.844672E+01	3.979831E+01	6.069241E+01	6.268230E+01	4.875291E+01	1.034755E+02
	Rank	**1**	15	12	9	2	3	4	7	7	5	6	10	13	14	11	16
	(Std Dev)	(4.072676E+00)	(2.289166E+01)	(1.777136E+01)	(9.712889E+00)	(3.199320E+01)	(2.191202E+01)	(5.641766E+00)	(9.409246E+00)	(8.631708E+00)	(7.941303E+00)	(6.234080E+00)	(1.361782E+01)	(1.460770E+01)	(2.299092E+01)	(1.543348E+01)	(3.080254E+01)
f_6	Mean	5.338490E−02	2.941176E−06	1.503500E−01	**0.000000E+00**	1.968235E−03	**0.000000E+00**	3.335294E−02	2.921227E−01	1.960784E−07	7.804314E−03	1.454314E−03	6.672549E−03	1.179941E−02	2.612137E−02	1.544530E+00	4.344842E+00
	Rank|WRST	12	4 | <	13 | =	**1** | <	6 | <	**1** | <	11 | <	14 | =	3 | <	8 | <	5 | <	7 | <	9 | <	10 | <	15 | >	16 | >
	Min	3.560000E−03	**0.000000E+00**	1.000000E−05	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	1.000000E−05	3.000000E−05	**0.000000E+00**	1.300000E−04	5.000000E−05	1.100000E−04	1.570000E−03	**0.000000E+00**	5.844000E−02	9.635000E−02
	Rank	14	**1**	7	**1**	**1**	**1**	7	9	**1**	12	10	11	13	**1**	15	16
	Median	1.744000E−02	**0.000000E+00**	5.274000E−02	**0.000000E+00**	2.800000E−04	**0.000000E+00**	4.250000E−03	3.871000E−02	**0.000000E+00**	2.140000E−03	1.390000E−03	1.290000E−03	6.520000E−03	**0.000000E+00**	9.183100E−01	3.208030E+00
	Rank	12	**1**	14	**1**	6	**1**	10	13	**1**	9	8	7	11	**1**	15	16
	(Std Dev)	(6.508731E−02)	(1.962591E−05)	(2.365699E−01)	(0.000000E+00)	(4.664950E−03)	(0.000000E+00)	(6.907665E−02)	(5.928649E−01)	(1.400280E−06)	(2.233683E−02)	(1.218204E−03)	(8.878903E−03)	(1.447003E−02)	(1.152931E−01)	(1.394965E+00)	(4.255133E+00)
f_7	Mean	**4.602144E+01**	8.734077E+01	6.522802E+01	8.212216E+01	8.428573E+01	8.868521E+01	4.803945E+01	6.708870E+01	8.580279E+01	5.526671E+01	5.842702E+01	8.135221E+01	9.277814E+01	1.026601E+02	1.036427E+02	1.405981E+02
	Rank|WRST	**1**	11 | >	5 | >	8 | >	9 | >	12 | >	2 | >	6 | >	10 | >	3 | >	4 | >	7 | >	13 | >	14 | >	15 | >	16 | >
	Min	3.910173E+01	6.284523E+01	4.786441E+01	5.751976E+01	**3.805196E+01**	4.311246E+01	3.825953E+01	4.362520E+01	5.716051E+01	4.046742E+01	4.373281E+01	6.168886E+01	7.019623E+01	5.854443E+01	6.186328E+01	8.532773E+01
	Rank	3	14	8	10	**1**	5	2	6	9	4	7	12	15	11	13	16
	Median	**4.437461E+01**	8.491543E+01	6.310633E+01	8.220462E+01	4.903203E+01	6.000715E+01	4.782350E+01	6.340080E+01	8.576977E+01	5.391221E+01	5.767108E+01	8.057591E+01	8.732702E+01	9.956503E+01	8.964052E+01	1.323841E+02
	Rank	**1**	11	7	10	3	6	2	8	12	4	5	9	13	15	14	16
	(Std Dev)	(5.064929E+00)	(1.689837E+01)	(1.130974E+01)	(1.026147E+01)	(5.111835E+01)	(4.892717E+01)	(5.377458E+00)	(1.398387E+01)	(1.452710E+01)	(9.643462E+00)	(7.777746E+00)	(1.492448E+01)	(1.675709E+01)	(2.440309E+01)	(3.625976E+01)	(3.299305E+01)
f_8	Mean	**1.043915E+01**	7.182768E+01	5.871009E+01	4.398912E+01	3.645746E+01	2.905603E+01	2.149891E+01	3.488206E+01	3.430754E+01	2.290356E+01	2.709009E+01	4.436136E+01	5.958036E+01	6.547234E+01	5.946329E+01	9.633510E+01
	Rank|WRST	**1**	15 | >	11 | >	9 | >	8 | >	5 | >	2 | >	7 | >	6 | >	3 | >	4 | >	10 | >	13 | >	14 | >	12 | >	16 | >
	Min	**1.989920E+00**	2.586892E+01	3.183866E+01	1.890421E+01	5.969750E+00	5.969810E+00	9.949590E+00	1.293447E+01	1.790926E+01	1.094455E+01	1.591934E+01	1.997253E+01	3.183867E+01	2.885925E+01	2.586890E+01	5.969745E+01
	Rank	**1**	12	14	9	2	3	4	6	8	5	7	10	15	13	11	16
	Median	**9.949590E+00**	6.964684E+01	5.472264E+01	4.377812E+01	1.790926E+01	1.537239E+01	2.089414E+01	3.382856E+01	3.382858E+01	2.188909E+01	2.686388E+01	4.377814E+01	5.969749E+01	6.268219E+01	5.770753E+01	9.452053E+01
	Rank	**1**	15	11	9	3	2	4	7	8	5	6	10	13	14	12	16
	(Std Dev)	(4.278261E+00)	(2.383117E+01)	(1.854029E+01)	(1.139618E+01)	(3.386771E+01)	(3.117346E+01)	(5.057984E+00)	(1.127319E+01)	(1.084816E+01)	(6.294205E+00)	(5.744910E+00)	(1.331870E+01)	(1.593498E+01)	(1.870099E+01)	(2.283161E+01)	(2.004051E+01)
f_9	Mean	6.611957E−01	5.578855E+01	2.739637E+01	3.911800E+00	7.093271E−01	2.132745E−02	9.459712E+00	6.408647E+00	8.822272E+00	4.029441E−01	**5.266471E−03**	3.250478E−01	1.137299E+02	1.549873E+02	5.753744E+01	4.663543E+02
	Rank|WRST	5	12 | >	11 | >	7 | >	6 | =	2 | <	10 | >	8 | >	9 | >	4 | =	**1** | <	3 | <	14 | >	15 | >	13 | >	16 | >
	Min	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	1.790600E−01	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	**0.000000E+00**	4.178420E+00	1.131427E+01	4.903790E+00	1.772587E+01
	Rank	**1**	**1**	**1**	**1**	**1**	**1**	12	**1**	**1**	**1**	**1**	**1**	13	15	14	16
	Median	4.543200E−01	3.180250E+00	1.087700E+00	2.815450E+00	4.543200E−01	**0.000000E+00**	4.903790E+00	6.333800E−01	3.448830E+00	1.790600E−01	**0.000000E+00**	8.953000E−02	7.712613E+01	9.974879E+01	4.517792E+01	4.118202E+02
	Rank	5	10	8	9	5	**1**	12	7	11	4	**1**	3	14	15	13	16
	(Std Dev)	(9.734588E−01)	(2.147384E+02)	(1.641676E+02)	(4.540654E+00)	(9.905665E−01)	(9.007183E−02)	(2.098082E+01)	(1.609794E+01)	(1.012230E+01)	(4.168990E−01)	(2.127550E−02)	(5.521807E−02)	(1.173459E+02)	(1.625729E+02)	(5.541221E+01)	(3.382015E+02)

(*continued on next page*)

Table 11 (*continued*).

f_{10}	Mean	4.055988E+03	3.063951E+03	2.655759E+03	2.137675E+03	3.625168E+03	3.042569E+03	2.350157E+03	3.165080E+03	**2.002393E+03**	2.809525E+03	2.289823E+03	3.259055E+03	2.530451E+03	3.151691E+03	3.335564E+03	3.337437E+03
	Rank|WRST	16	9 | <	6 | <	2 | <	15 | =	8 | <	4 | <	11 | <	**1** | <	7 | <	3 | <	12 | <	5 | <	10 | <	13 | <	14 | <
	Min	2.113543E+03	1.674225E+03	1.518926E+03	1.466478E+03	1.348522E+03	**9.168535E+02**	1.499828E+03	1.988340E+03	9.568164E+02	1.628333E+03	1.387062E+03	1.330413E+03	1.252291E+03	1.769659E+03	1.958859E+03	1.884084E+03
	Rank	16	11	9	7	5	**1**	8	15	2	10	6	4	3	12	14	13
	Median	4.018540E+03	3.034688E+03	2.666863E+03	2.150335E+03	3.709103E+03	2.590735E+03	2.357551E+03	3.246226E+03	**2.062329E+03**	2.645005E+03	2.338876E+03	3.401242E+03	2.469906E+03	3.118329E+03	3.341626E+03	3.431311E+03
	Rank	16	9	8	2	15	6	4	11	**1**	7	3	13	5	10	12	14
	(Std Dev)	(1.058111E+03)	(6.803507E+02)	(5.510090E+02)	(3.152165E+02)	(7.691667E+02)	(1.429599E+03)	(4.885901E+02)	(5.760169E+02)	(3.440389E+02)	(7.421400E+02)	(3.364508E+02)	(7.888041E+02)	(5.021014E+02)	(7.286337E+02)	(6.299104E+02)	(4.832729E+02)
f_{11}	Mean	7.340219E+01	4.795832E+01	9.756679E+01	4.500087E+01	4.104678E+01	5.821927E+01	8.191654E+01	1.358210E+02	7.663090E+01	4.015404E+01	**3.658356E+01**	6.550338E+01	8.286430E+01	8.137216E+01	1.364852E+02	9.024172E+01
	Rank|WRST	8	5 | <	14 | >	4 | <	3 | <	6 | =	11 | =	15 | >	9 | =	2 | <	**1** | <	7 | =	12 | =	10 | =	16 | >	13 | >
	Min	2.288400E+01	6.964800E+00	2.895690E+01	1.149620E+01	**4.334200E+00**	5.221400E+00	2.531790E+01	3.086830E+01	1.592630E+01	1.304940E+01	8.961000E+00	2.414060E+01	3.482490E+01	1.399290E+01	3.183860E+01	1.026870E+01
	Rank	10	3	13	6	**1**	2	12	14	9	7	4	11	16	8	15	5
	Median	7.064130E+01	3.581850E+01	9.059850E+01	3.619960E+01	**1.826270E+01**	7.100920E+01	7.840980E+01	1.267895E+02	7.068900E+01	3.303440E+01	3.415580E+01	5.804170E+01	8.459290E+01	7.784540E+01	1.253629E+02	8.220890E+01
	Rank	7	4	14	5	**1**	9	11	16	8	2	3	6	13	10	15	12
	(Std Dev)	(2.568105E+01)	(3.155685E+01)	(4.106367E+01)	(2.188482E+01)	(3.513017E+01)	(3.149250E+01)	(3.330837E+01)	(5.082577E+01)	(4.395602E+01)	(2.238377E+01)	(1.829220E+01)	(2.594998E+01)	(2.994889E+01)	(3.857209E+01)	(5.425837E+01)	(4.114671E+01)
f_{12}	Mean	**1.533982E+03**	1.856903E+04	2.520721E+04	4.420653E+04	1.477744E+05	4.621533E+05	1.636345E+05	5.540184E+05	1.536814E+04	2.617786E+04	6.671797E+04	4.994651E+04	2.513687E+04	2.099735E+05	5.043622E+03	1.786611E+06
	Rank|WRST	**1**	4 | >	6 | >	8 | >	11 | >	14 | >	12 | >	15 | >	3 | >	7 | >	10 | >	9 | >	5 | >	13 | >	2 | >	16 | >
	Min	**3.561994E+02**	7.318563E+02	1.704527E+03	5.761517E+03	1.297081E+04	5.737104E+04	1.057905E+04	2.567041E+03	3.466993E+03	2.250054E+03	1.252143E+04	1.108346E+04	3.761605E+03	4.752398E+03	7.559371E+02	6.800508E+03
	Rank	**1**	2	4	10	15	16	12	6	7	5	14	13	8	9	3	11
	Median	**1.218273E+03**	1.599075E+04	2.341138E+04	2.473127E+04	6.715628E+04	3.768047E+05	1.271851E+05	2.298742E+04	1.632777E+04	2.202496E+04	5.322633E+04	4.053130E+04	1.750457E+04	3.740584E+04	1.778141E+03	1.491816E+05
	Rank	**1**	3	8	9	13	16	14	7	4	6	12	11	5	10	2	15
	(Std Dev)	(2.286537E+03)	(1.254630E+04)	(1.299573E+04)	(5.420328E+04)	(2.169244E+05)	(4.720190E+05)	(1.120022E+05)	(2.842808E+06)	(7.616679E+03)	(1.341769E+04)	(4.632545E+04)	(3.634410E+04)	(3.290625E+04)	(5.150321E+05)	(2.063579E+04)	(6.872533E+06)
f_{13}	Mean	4.246226E+02	2.088346E+04	1.677249E+04	**3.971455E+02**	8.400702E+03	1.636349E+04	1.517640E+04	1.495491E+04	5.296625E+02	1.754157E+04	5.101707E+03	2.189326E+04	1.539358E+04	1.330821E+04	5.352412E+02	1.422792E+04
	Rank|WRST	2	15 | >	13 | >	**1** | <	6 | >	12 | >	10 | >	9 | >	3 | =	14 | >	5 | >	16 | >	11 | >	7 | >	4 | >	8 | >
	Min	1.386561E+02	6.352600E+01	1.306037E+02	3.153890E+01	**2.752920E+01**	1.000102E+02	1.533176E+02	7.567980E+01	6.606640E+01	3.865292E+02	6.438440E+01	8.383252E+02	9.752390E+01	1.875059E+02	3.695660E+01	1.219465E+02
	Rank	12	4	11	2	**1**	9	13	7	6	15	5	16	8	14	3	10
	Median	3.854997E+02	1.354680E+04	1.053406E+04	**2.535104E+02**	7.141148E+03	1.025316E+04	1.204956E+04	1.083936E+04	3.641488E+02	1.292127E+04	1.562689E+03	1.550398E+04	1.142406E+04	1.124330E+04	2.720450E+02	8.650711E+03
	Rank	4	15	9	**1**	6	8	13	10	3	14	5	16	12	11	2	7
	(Std Dev)	(1.763978E+02)	(2.046868E+04)	(1.693927E+04)	(3.391793E+02)	(6.776984E+03)	(1.590510E+04)	(1.205652E+04)	(1.249229E+04)	(6.066651E+02)	(1.724536E+04)	(7.392116E+03)	(2.154367E+04)	(1.368073E+04)	(1.002432E+04)	(7.378317E+02)	(1.471810E+04)
f_{14}	Mean	**1.295676E+02**	1.220787E+03	2.933115E+03	4.514038E+03	7.422441E+03	1.108902E+04	2.080356E+04	5.278897E+02	2.367885E+03	2.106964E+03	2.063089E+03	1.933253E+03	3.995103E+03	4.497581E+02	1.641117E+03	1.943166E+04
	Rank|WRST	**1**	4 | >	10 | >	12 | >	13 | >	14 | >	16 | >	3 | >	9 | >	8 | >	7 | >	6 | >	11 | >	2 | >	5 | >	15 | >
	Min	6.580150E+01	**3.836400E+01**	2.679757E+02	1.511916E+02	1.454726E+02	2.857769E+02	1.938793E+03	8.548370E+01	1.471850E+02	9.965630E+01	1.268540E+02	2.242711E+02	5.456750E+01	1.419901E+02	6.993130E+01	3.636343E+02
	Rank	3	**1**	13	11	9	14	16	5	10	6	7	12	2	8	4	15
	Median	**1.224877E+02**	6.092462E+02	1.844175E+03	3.189037E+03	4.346771E+03	6.395189E+03	1.165334E+04	1.847230E+02	1.980626E+03	1.456086E+03	1.646353E+03	1.373844E+03	1.867350E+03	3.821685E+02	1.863691E+02	6.766787E+03
	Rank	**1**	5	9	12	13	14	16	2	11	7	8	6	10	4	3	15
	(Std Dev)	(3.159536E+01)	(1.359013E+03)	(2.848100E+03)	(4.568186E+03)	(8.681731E+03)	(1.261529E+04)	(2.953028E+04)	(2.259017E+03)	(1.897279E+03)	(1.894586E+03)	(2.087550E+03)	(1.551084E+03)	(7.446749E+03)	(2.269836E+02)	(3.551831E+03)	(2.935285E+04)
f_{15}	Mean	**2.033790E+02**	1.444743E+04	8.822050E+03	3.085614E+02	3.466381E+03	2.786378E+03	2.152796E+03	6.098171E+03	3.179820E+02	2.292234E+03	1.590107E+03	4.503634E+03	4.773613E+03	1.136014E+03	1.476407E+03	6.340426E+03
	Rank|WRST	**1**	16 | >	15 | >	2 | =	10 | >	9 | >	7 | >	13 | >	3 | =	8 | >	6 | >	11 | >	12 | >	4 | >	5 | >	14 | >
	Min	7.777590E+01	**1.468940E+01**	1.865761E+02	1.866710E+01	2.088520E+01	2.064200E+01	6.155890E+01	1.634380E+01	4.512920E+01	7.918200E+01	2.483200E+01	8.957810E+01	4.629560E+01	2.293040E+01	2.755190E+01	1.478909E+02
	Rank	12	**1**	16	3	5	4	11	2	9	13	7	14	10	6	8	15
	Median	1.845423E+02	8.708345E+03	4.890346E+03	1.593169E+02	2.156125E+03	1.093696E+03	1.217307E+03	3.116973E+03	**1.568228E+02**	1.380101E+03	5.933935E+02	2.225430E+03	1.849951E+03	7.094808E+02	2.450501E+02	2.929479E+03
	Rank	3	16	15	2	11	7	8	14	**1**	9	5	12	10	6	4	13
	(Std Dev)	(8.333432E+01)	(1.281026E+04)	(1.085890E+04)	(4.652906E+02)	(3.794773E+03)	(4.571244E+03)	(2.654714E+03)	(6.497229E+03)	(5.152438E+02)	(2.808616E+03)	(2.699158E+03)	(5.525027E+03)	(7.116449E+03)	(1.312677E+03)	(3.419237E+03)	(7.530942E+03)
f_{16}	Mean	**2.179527E+02**	8.707697E+02	6.778770E+02	5.166436E+02	3.025543E+02	4.721881E+02	7.168085E+02	6.133010E+02	4.036594E+02	4.468565E+02	3.368642E+02	3.257847E+02	8.142546E+02	8.298421E+02	6.408938E+02	9.517269E+02
	Rank|WRST	**1**	15 | >	11 | >	8 | >	2 | =	7 | >	12 | >	9 | >	5 | >	6 | >	4 | >	3 | >	13 | >	14 | >	10 | >	16 | >
	Min	1.442870E+01	2.650602E+02	2.442056E+02	1.218210E+02	1.463450E+01	2.559300E+01	1.369498E+02	1.329556E+02	1.292502E+02	2.967760E+01	**7.968400E+00**	1.659560E+01	1.760130E+01	1.466940E+01	5.183090E+01	2.794695E+02
	Rank	2	15	14	10	3	7	13	12	11	8	**1**	5	6	4	9	16
	Median	**2.450745E+02**	8.687009E+02	6.694539E+02	5.192354E+02	2.700063E+02	4.455527E+02	6.607483E+02	6.170482E+02	3.692284E+02	4.358290E+02	2.822652E+02	2.817584E+02	8.130112E+02	8.484183E+02	6.265914E+02	9.649237E+02
	Rank	**1**	15	12	8	2	7	11	9	5	6	4	3	13	14	10	16
	(Std Dev)	(1.546946E+02)	(2.705165E+02)	(1.953321E+02)	(1.793937E+02)	(2.471202E+02)	(2.397722E+02)	(3.184976E+02)	(2.415217E+02)	(1.934948E+02)	(2.154072E+02)	(1.704454E+02)	(1.644348E+02)	(3.142719E+02)	(2.639390E+02)	(2.667142E+02)	(2.664117E+02)
f_{17}	Mean	**6.732427E+01**	3.855593E+02	1.866264E+02	8.957653E+01	1.201371E+02	1.093247E+02	2.344106E+02	2.286774E+02	8.840797E+01	1.459667E+02	6.816143E+01	1.057102E+02	3.804907E+02	3.332697E+02	3.698223E+02	3.155450E+02
	Rank|WRST	**1**	16 | >	9 | >	4 | >	7 | >	6 | >	11 | >	10 | >	3 | >	8 | >	2 | =	5 | >	15 | >	13 | >	14 | >	12 | >
	Min	**2.558350E+01**	4.465620E+01	4.148710E+01	3.390550E+01	2.577100E+01	3.005850E+01	3.761600E+01	5.107370E+01	3.539790E+01	5.292350E+01	2.888270E+01	4.321830E+01	5.026470E+01	4.225520E+01	6.348460E+01	7.725310E+01
	Rank	**1**	11	8	5	2	4	7	13	6	14	3	10	12	9	15	16
	Median	**5.084620E+01**	3.511651E+02	1.845338E+02	6.758100E+01	8.960380E+01	7.721330E+01	1.885371E+02	1.837309E+02	7.567650E+01	1.310320E+02	5.694640E+01	9.311720E+01	3.872065E+02	3.106087E+02	3.304502E+02	3.243282E+02
	Rank	**1**	15	10	3	6	5	11	9	4	8	2	7	16	12	14	13
	(Std Dev)	(4.903923E+01)	(2.298741E+02)	(1.055035E+02)	(5.927501E+01)	(8.740982E+01)	(6.417188E+01)	(1.662383E+02)	(1.367526E+02)	(3.948029E+01)	(6.873747E+01)	(3.497628E+01)	(4.852837E+01)	(2.026057E+02)	(1.719490E+02)	(1.870031E+02)	(1.653801E+02)
f_{18}	Mean	**1.574323E+02**	2.119812E+04	8.618655E+04	9.011618E+04	2.385432E+05	1.894025E+05	1.606328E+05	3.510468E+04	8.459836E+04	7.031650E+04	8.273923E+04	6.173735E+04	3.304114E+04	2.993616E+04	5.993843E+02	2.313620E+05
	Rank|WRST	**1**	3 | >	11 | >	12 | >	16 | >	14 | >	13 | >	6 | >	10 | >	8 | >	9 | >	7 | >	5 | >	4 | >	2 | >	15 | >
	Min	**3.938410E+01**	1.778791E+03	1.510802E+04	2.842106E+04	3.007649E+04	3.589297E+04	3.341797E+04	1.089864E+03	1.045303E+04	1.659154E+04	2.074425E+04	1.640011E+04	7.190640E+03	1.723861E+03	1.248008E+02	4.608439E+03
	Rank	**1**	5	9	13	14	16	15	3	8	11	12	10	7	4	2	6
	Median	**1.517710E+02**	1.452820E+04	7.275950E+04	7.875521E+04	1.718420E+05	1.598679E+05	1.338357E+05	1.734874E+04	5.153326E+04	6.712698E+04	6.524607E+04	5.922210E+04	2.824930E+04	1.788069E+04	3.419002E+02	1.351823E+05
	Rank	**1**	3	11	12	16	15	13	4	7	10	9	8	6	5	2	14
	(Std Dev)	(6.801566E+01)	(1.787675E+04)	(5.543500E+04)	(5.750975E+04)	(2.226695E+05)	(1.438354E+05)	(1.002512E+05)	(4.645773E+04)	(7.812564E+04)	(3.235108E+04)	(5.728091E+04)	(2.749493E+04)	(2.051787E+04)	(4.654638E+04)	(6.407720E+02)	(2.297823E+05)

(*continued on next page*)

Table 11 (*continued*).

f_{19}	Mean	1.771464E+02	1.012306E+04	1.153102E+04	2.016198E+02	6.432755E+03	4.291493E+03	3.694440E+03	6.466870E+03	2.840311E+02	5.843745E+03	4.678918E+03	6.098351E+03	5.282482E+03	3.735051E+03	**1.297441E+02**	8.053196E+03
	Rank|WRST	2	15 | >	16 | >	3 | >	12 | >	7 | >	5 | >	13 | >	4 | =	10 | >	8 | >	11 | >	9 | >	6 | >	**1** | <	14 | >
	Min	5.154050E+01	1.032980E+01	7.167090E+01	3.124800E+01	**8.863900E+00**	3.268120E+01	3.166000E+01	2.564540E+01	1.616560E+01	2.399114E+02	7.486890E+01	1.272862E+02	2.302200E+01	2.050280E+01	3.344960E+01	2.254600E+01
	Rank	12	2	13	8	**1**	10	9	7	3	16	14	15	6	4	11	5
	Median	1.458276E+02	2.662774E+03	9.195048E+03	**7.968390E+01**	3.263304E+03	3.095637E+03	2.352880E+03	2.338200E+03	1.435632E+02	4.906465E+03	2.341146E+03	3.317826E+03	3.255593E+03	2.478175E+03	1.169408E+02	3.456625E+03
	Rank	4	9	16	**1**	12	10	7	5	3	15	6	13	11	8	2	14
	(Std Dev)	(1.037823E+02)	(1.398254E+04)	(1.195867E+04)	(3.881185E+02)	(9.631548E+03)	(4.609417E+03)	(3.993519E+03)	(8.911590E+03)	(3.247212E+02)	(4.928378E+03)	(6.678281E+03)	(6.169043E+03)	(6.111860E+03)	(3.750415E+03)	(6.913799E+01)	(1.096030E+04)
f_{20}	Mean	**8.873196E+01**	3.600587E+02	2.735098E+02	1.521473E+02	1.142010E+02	1.629417E+02	3.221044E+02	2.267942E+02	1.426478E+02	2.125017E+02	1.292345E+02	1.884693E+02	2.819113E+02	2.494217E+02	2.389407E+02	3.371309E+02
	Rank|WRST	**1**	16 | >	12 | >	5 | >	2 | =	6 | >	14 | >	9 | >	4 | >	8 | >	3 | >	7 | >	13 | >	11 | >	10 | >	15 | >
	Min	2.808530E+01	2.748500E+01	1.688789E+02	1.426300E+01	**1.135700E+01**	3.273760E+01	9.288840E+01	8.699530E+01	3.385610E+01	6.335830E+01	1.860720E+01	4.857030E+01	3.735790E+01	3.003530E+01	3.471040E+01	8.740470E+01
	Rank	5	4	16	2	**1**	7	15	13	8	12	3	11	10	6	9	14
	Median	**5.283640E+01**	3.808090E+02	2.357657E+02	1.690798E+02	8.976190E+01	1.620425E+02	2.613728E+02	2.401667E+02	1.641340E+02	2.217271E+02	9.667430E+01	1.996350E+02	2.673021E+02	2.000025E+02	2.004213E+02	3.287425E+02
	Rank	**1**	16	11	6	2	4	13	12	5	10	3	7	14	8	9	15
	(Std Dev)	(6.628548E+01)	(1.753709E+02)	(8.218239E+01)	(6.495210E+01)	(8.645501E+01)	(5.977384E+01)	(1.510841E+02)	(9.937344E+01)	(6.858693E+01)	(5.339021E+01)	(6.391349E+01)	(8.651069E+01)	(1.541046E+02)	(1.257979E+02)	(1.234574E+02)	(1.574819E+02)
f_{21}	Mean	**2.116206E+02**	2.680331E+02	2.586015E+02	2.369727E+02	2.462763E+02	2.263504E+02	2.246420E+02	2.317609E+02	2.254424E+02	2.260994E+02	2.280428E+02	2.407811E+02	2.631055E+02	2.650334E+02	2.571380E+02	2.928525E+02
	Rank|WRST	**1**	15 | >	12 | >	8 | >	10 | >	5 | >	2 | >	7 | >	3 | >	4 | >	6 | >	9 | >	13 | >	14 | >	11 | >	16 | >
	Min	2.018585E+02	2.328631E+02	2.339604E+02	1.267935E+02	2.104313E+02	2.059516E+02	2.147713E+02	2.203418E+02	**1.000000E+02**	2.110267E+02	2.144612E+02	2.228681E+02	2.292181E+02	2.247801E+02	2.324791E+02	2.600124E+02
	Rank	3	14	15	2	5	4	8	9	**1**	6	7	10	12	11	13	16
	Median	**2.106687E+02**	2.668085E+02	2.584318E+02	2.422631E+02	2.223357E+02	2.156387E+02	2.244550E+02	2.302859E+02	2.298140E+02	2.233216E+02	2.276694E+02	2.416110E+02	2.603733E+02	2.615104E+02	2.553225E+02	2.894867E+02
	Rank	**1**	15	12	10	3	2	5	8	7	4	6	9	13	14	11	16
	(Std Dev)	(5.139010E+00)	(1.678032E+01)	(1.452754E+01)	(2.953892E+01)	(3.636157E+01)	(2.507085E+01)	(6.023139E+00)	(7.770457E+00)	(2.878908E+01)	(1.155425E+01)	(5.018254E+00)	(1.278476E+01)	(1.988908E+01)	(2.294811E+01)	(1.315427E+01)	(1.984854E+01)
f_{22}	Mean	1.000000E+02	2.077912E+03	4.140563E+02	1.003264E+02	4.741923E+02	**1.000000E+02**	1.004680E+02	1.733333E+02	1.001446E+02	1.001925E+02	**1.000000E+02**	1.000482E+02	7.641875E+02	3.105014E+02	3.138761E+03	5.232619E+02
	Rank|WRST	3	15 | >	11 | >	7 | =	12 | >	**1** | <	8 | =	9 | =	5 | =	6 | =	**1** | <	4 | =	14 | >	10 | >	16 | >	13 | >
	Min	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**
	Rank	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
	Median	**1.000000E+02**	2.830781E+03	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	1.047337E+02	3.426594E+03	**1.000000E+02**
	Rank	**1**	15	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	14	16	**1**
	(Std Dev)	(2.715244E−05)	(1.756185E+03)	(9.744475E+02)	(9.144300E−01)	(1.128301E+03)	(0.000000E+00)	(1.046694E+00)	(5.218357E+02)	(5.840291E−01)	(6.665928E−01)	(0.000000E+00)	(3.444409E−01)	(1.226356E+03)	(8.232521E+02)	(1.149005E+03)	(1.202649E+03)
f_{23}	Mean	**3.614743E+02**	4.161146E+02	4.115487E+02	3.940392E+02	3.940320E+02	3.904084E+02	3.873104E+02	3.890222E+02	3.759557E+02	3.883585E+02	3.779886E+02	3.995357E+02	4.273581E+02	4.314160E+02	4.032204E+02	4.917614E+02
	Rank|WRST	**1**	13 | >	12 | >	9 | >	8 | >	7 | >	4 | >	6 | >	2 | >	5 | >	3 | >	10 | >	14 | >	15 | >	11 | >	16 | >
	Min	3.453737E+02	3.724789E+02	3.755187E+02	3.616670E+02	3.554889E+02	3.543575E+02	3.623304E+02	3.578923E+02	**1.000000E+02**	3.591485E+02	3.529677E+02	3.737564E+02	3.826433E+02	3.818133E+02	3.798004E+02	4.249740E+02
	Rank	2	10	12	8	5	4	9	6	**1**	7	3	11	15	14	13	16
	Median	**3.611581E+02**	4.167755E+02	4.094557E+02	3.938658E+02	3.805033E+02	3.786032E+02	3.869476E+02	3.885430E+02	3.808705E+02	3.883993E+02	3.757628E+02	3.994143E+02	4.266487E+02	4.330660E+02	4.013159E+02	4.804502E+02
	Rank	**1**	13	12	9	4	3	6	8	5	7	2	10	14	15	11	16
	(Std Dev)	(8.138678E+00)	(2.295343E+01)	(2.287540E+01)	(1.544656E+01)	(3.389012E+01)	(3.002702E+01)	(1.419898E+01)	(1.483769E+01)	(4.088462E+01)	(1.531535E+01)	(1.074678E+01)	(1.427321E+01)	(2.136984E+01)	(2.697198E+01)	(1.360198E+01)	(4.650092E+01)
f_{24}	Mean	**4.338772E+02**	4.823648E+02	4.715331E+02	4.514949E+02	4.801428E+02	4.468679E+02	4.536449E+02	4.530788E+02	4.478101E+02	4.800710E+02	4.501654E+02	4.702261E+02	5.145091E+02	5.006966E+02	4.785508E+02	5.776389E+02
	Rank|WRST	**1**	13 | >	9 | >	5 | >	12 | >	2 | >	7 | >	6 | >	3 | >	11 | >	4 | >	8 | >	15 | >	14 | >	10 | >	16 | >
	Min	4.232284E+02	4.513394E+02	4.185673E+02	**2.165069E+02**	4.325833E+02	4.272821E+02	4.323518E+02	4.366480E+02	4.323404E+02	4.401534E+02	4.319515E+02	4.447948E+02	4.648988E+02	4.580040E+02	4.542597E+02	4.589338E+02
	Rank	3	12	2	**1**	8	4	7	9	6	10	5	11	16	14	13	15
	Median	**4.323613E+02**	4.801268E+02	4.711539E+02	4.561169E+02	4.876390E+02	4.418755E+02	4.531500E+02	4.529779E+02	4.472127E+02	4.754446E+02	4.493993E+02	4.673003E+02	5.114968E+02	4.960465E+02	4.793754E+02	5.664222E+02
	Rank	**1**	12	9	7	13	2	6	5	3	10	4	8	15	14	11	16
	(Std Dev)	(6.690682E+00)	(1.824182E+01)	(2.149718E+01)	(5.037408E+01)	(4.068255E+01)	(1.775579E+01)	(1.230837E+01)	(8.984416E+00)	(8.381090E+00)	(2.302497E+01)	(8.342827E+00)	(1.247879E+01)	(2.733493E+01)	(2.939084E+01)	(1.544382E+01)	(5.849941E+01)
f_{25}	Mean	3.872729E+02	3.866583E+02	3.896385E+02	3.868106E+02	3.947871E+02	**3.792360E+02**	3.948479E+02	3.960688E+02	3.869245E+02	3.883933E+02	3.864726E+02	3.872183E+02	3.902438E+02	4.136321E+02	3.831093E+02	4.209037E+02
	Rank|WRST	8	4 | =	10 | >	5 | =	12 | >	**1** | <	13 | >	14 | >	6 | =	9 | >	3 | <	7 | <	11 | >	15 | >	2 | <	16 | >
	Min	3.834709E+02	3.834207E+02	3.834612E+02	3.842151E+02	3.886977E+02	3.783211E+02	3.838625E+02	3.851536E+02	3.834077E+02	3.834200E+02	3.836627E+02	3.868311E+02	3.834781E+02	3.844746E+02	**3.752681E+02**	3.870677E+02
	Rank	7	5	6	11	16	2	10	13	3	4	9	14	8	12	**1**	15
	Median	3.871032E+02	3.870955E+02	3.884514E+02	3.871113E+02	3.924139E+02	**3.787855E+02**	3.893619E+02	3.913030E+02	3.872438E+02	3.872824E+02	3.869450E+02	3.872043E+02	3.878083E+02	4.110675E+02	3.802874E+02	4.230408E+02
	Rank	5	4	11	6	14	**1**	12	13	8	9	3	7	10	15	2	16
	(Std Dev)	(1.405255E+00)	(1.306918E+00)	(5.372870E+00)	(9.291699E−01)	(6.361657E+00)	(1.109111E+00)	(1.344956E+01)	(1.216782E+01)	(1.265930E+00)	(7.810751E+00)	(1.230683E+00)	(1.495123E−01)	(9.532551E+00)	(2.108869E+01)	(1.476605E+01)	(2.164211E+01)
f_{26}	Mean	1.037212E+03	1.591173E+03	7.533985E+02	**3.651100E+02**	1.210515E+03	1.118077E+03	1.484340E+03	1.255623E+03	5.064928E+02	7.097522E+02	1.171646E+03	9.312108E+02	1.331123E+03	1.300358E+03	1.617511E+03	1.804299E+03
	Rank|WRST	6	14 | >	4 | <	**1** | <	9 | >	7 | >	13 | >	10 | >	2 | <	3 | =	8 | >	5 | =	12 | >	11 | =	15 | >	16 | >
	Min	8.707514E+02	**2.000000E+02**	**2.000000E+02**	**2.000000E+02**	6.336622E+02	3.000000E+02	**2.000000E+02**	**2.000000E+02**	**2.000000E+02**	**2.000000E+02**	**2.000000E+02**	**2.000000E+02**	**2.000000E+02**	**2.000000E+02**	1.186565E+03	**2.000000E+02**
	Rank	15	**1**	**1**	**1**	14	13	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	16	**1**
	Median	1.023598E+03	1.624500E+03	**3.000000E+02**	**3.000000E+02**	1.185428E+03	1.115059E+03	1.530227E+03	1.393225E+03	**3.000000E+02**	**3.000000E+02**	1.209551E+03	1.274005E+03	1.642038E+03	6.809009E+02	1.633061E+03	2.261542E+03
	Rank	6	13	**1**	**1**	8	7	12	11	**1**	**1**	9	10	15	5	14	16
	(Std Dev)	(1.078029E+02)	(4.436628E+02)	(7.154144E+02)	(2.901132E+02)	(2.255641E+02)	(2.756059E+02)	(3.055194E+02)	(4.348334E+02)	(4.310462E+02)	(5.267275E+02)	(2.238720E+02)	(6.219761E+02)	(8.058292E+02)	(1.232250E+03)	(3.228925E+02)	(1.104066E+03)
f_{27}	Mean	5.346450E+02	5.179106E+02	5.333451E+02	5.151096E+02	5.206053E+02	**4.899236E+02**	5.344194E+02	5.293215E+02	5.166244E+02	5.129671E+02	5.083476E+02	5.048162E+02	5.208768E+02	5.398051E+02	5.264687E+02	6.509456E+02
	Rank|WRST	14	7 | <	12 | =	5 | <	8 | <	**1** | <	13 | =	11 | =	6 | <	4 | <	3 | <	2 | <	9 | <	15 | =	10 | =	16 | >
	Min	5.087058E+02	4.850645E+02	4.976117E+02	4.936867E+02	5.007947E+02	**4.366696E+02**	5.096050E+02	4.919922E+02	4.993827E+02	4.785744E+02	4.912086E+02	4.946460E+02	4.912621E+02	5.072837E+02	4.533584E+02	5.875619E+02
	Rank	14	4	10	8	12	**1**	15	7	11	3	5	9	6	13	2	16
	Median	5.297378E+02	5.158191E+02	5.311504E+02	5.144848E+02	5.206335E+02	**5.000065E+02**	5.330106E+02	5.264398E+02	5.166447E+02	5.124108E+02	5.111340E+02	5.045511E+02	5.210076E+02	5.388860E+02	5.202995E+02	6.460744E+02
	Rank	12	6	13	5	9	**1**	14	11	7	4	3	2	10	15	8	16
	(Std Dev)	(1.905417E+01)	(1.175509E+01)	(1.536746E+01)	(6.210907E+00)	(9.023625E+00)	(1.893077E+01)	(1.449115E+01)	(2.967146E+01)	(6.321709E+00)	(1.594762E+01)	(1.190386E+01)	(5.238451E+00)	(9.697410E+00)	(1.685507E+01)	(3.422753E+01)	(4.208517E+01)

(*continued on next page*)

Table 11 (*continued*).

f_{28}	Mean	3.309141E+02	3.424846E+02	**3.138135E+02**	3.523670E+02	4.327660E+02	3.987592E+02	3.960880E+02	3.931034E+02	3.256978E+02	3.612311E+02	3.753815E+02	3.265552E+02	3.391186E+02	4.969484E+02	3.779474E+02	4.333159E+02
	Rank|WRST	4	6 | =	**1** | =	7 | >	14 | >	13 | >	12 | >	11 | >	2 | =	8 | >	9 | >	3 | =	5 | =	16 | >	10 | >	15 | >
	Min	**3.000000E+02**	**3.000000E+02**	**3.000000E+02**	**3.000000E+02**	**3.000000E+02**	**3.000000E+02**	3.000005E+02	**3.000000E+02**	**3.000000E+02**	**3.000000E+02**	**3.000000E+02**	**3.000000E+02**	**3.000000E+02**	4.218535E+02	**3.000000E+02**	3.000009E+02
	Rank	**1**	**1**	**1**	**1**	**1**	**1**	14	**1**	**1**	**1**	**1**	**1**	**1**	16	**1**	15
	Median	**3.000000E+02**	**3.000000E+02**	**3.000000E+02**	3.596126E+02	4.466438E+02	4.017723E+02	3.998511E+02	4.069450E+02	**3.000000E+02**	3.908853E+02	3.921277E+02	**3.000000E+02**	**3.000000E+02**	4.995946E+02	4.032857E+02	4.126388E+02
	Rank	**1**	**1**	**1**	7	15	11	10	13	**1**	8	9	**1**	**1**	16	12	14
	(Std Dev)	(5.450915E+01)	(5.680593E+01)	(3.899487E+01)	(5.084974E+01)	(4.358682E+01)	(3.066687E+01)	(3.759474E+01)	(6.511026E+01)	(4.687115E+01)	(5.437422E+01)	(4.294522E+01)	(4.668943E+01)	(5.684269E+01)	(3.739587E+01)	(7.378986E+01)	(5.757145E+01)
f_{29}	Mean	5.029800E+02	7.794500E+02	6.354845E+02	4.998253E+02	5.472815E+02	5.082797E+02	7.242737E+02	7.415938E+02	5.297026E+02	5.455455E+02	**4.901671E+02**	5.451576E+02	7.168077E+02	7.420105E+02	8.492377E+02	8.993185E+02
	Rank|WRST	3	14 | >	9 | >	2 | =	8 | >	4 | =	11 | >	12 | >	5 | >	7 | >	**1** | =	6 | >	10 | >	13 | >	15 | >	16 | >
	Min	3.954189E+02	4.319178E+02	4.689278E+02	4.247940E+02	3.746563E+02	**3.719953E+02**	4.776120E+02	5.227154E+02	4.388860E+02	4.401804E+02	4.456183E+02	4.792106E+02	4.851552E+02	4.542426E+02	5.438987E+02	5.263814E+02
	Rank	3	5	10	4	2	**1**	11	14	6	7	8	12	13	9	16	15
	Median	**4.780324E+02**	7.784576E+02	6.167739E+02	4.782433E+02	5.235137E+02	4.786493E+02	7.206124E+02	7.279722E+02	5.100602E+02	5.296505E+02	4.781063E+02	5.368134E+02	6.728859E+02	7.426869E+02	8.250848E+02	9.225410E+02
	Rank	**1**	14	9	3	6	4	11	12	5	7	2	8	10	13	15	16
	(Std Dev)	(8.413597E+01)	(1.970614E+02)	(1.126339E+02)	(6.103970E+01)	(1.104167E+02)	(1.280805E+02)	(1.486435E+02)	(1.433829E+02)	(6.657914E+01)	(8.481520E+01)	(3.498626E+01)	(4.750474E+01)	(1.949100E+02)	(2.028314E+02)	(1.915063E+02)	(2.096004E+02)
f_{30}	Mean	4.224251E+03	6.563051E+03	7.139867E+03	3.912271E+03	5.438968E+03	**1.813105E+03**	4.732747E+03	2.098822E+04	4.004680E+03	4.156783E+03	4.648988E+03	5.601691E+03	5.170407E+03	3.614544E+03	4.434269E+03	2.009337E+04
	Rank|WRST	6	13 | >	14 | >	3 | <	11 | >	**1** | <	9 | =	16 | >	4 | <	5 | <	8 | =	12 | =	10 | =	2 | <	7 | >	15 | >
	Min	3.088844E+03	1.998470E+03	2.374733E+03	2.286983E+03	3.044680E+03	**2.192603E+02**	2.546369E+03	4.418745E+03	2.446088E+03	2.167558E+03	2.519349E+03	2.614292E+03	2.279211E+03	2.151063E+03	2.405276E+03	4.823222E+03
	Rank	14	2	7	6	13	**1**	11	15	9	4	10	12	5	3	8	16
	Median	4.099548E+03	5.612953E+03	5.761279E+03	3.489647E+03	5.023733E+03	**1.093225E+03**	4.179461E+03	1.225378E+04	3.473519E+03	3.406497E+03	4.354575E+03	4.054195E+03	3.729406E+03	3.250896E+03	3.515749E+03	1.744957E+04
	Rank	9	13	14	5	12	**1**	10	15	4	3	11	8	7	2	6	16
	(Std Dev)	(7.135678E+02)	(3.528320E+03)	(3.509846E+03)	(1.458890E+03)	(2.078104E+03)	(1.766839E+03)	(1.759015E+03)	(3.389226E+04)	(1.191705E+03)	(2.250601E+03)	(1.502425E+03)	(3.443573E+03)	(3.314098E+03)	(9.681774E+02)	(2.648544E+03)	(1.268450E+04)

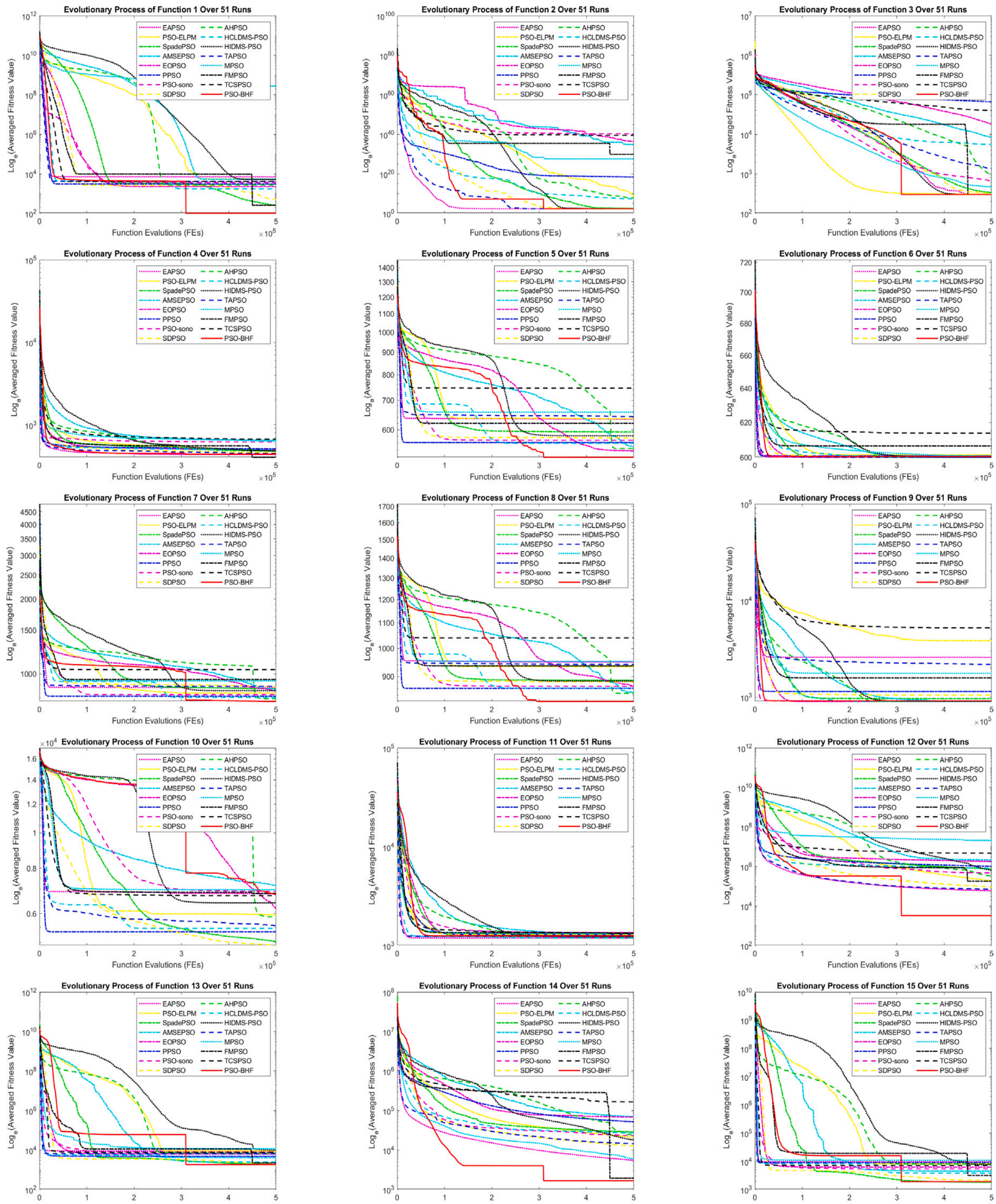

Fig. 6. Evolutionary processes of 50-dimensional experiments, averaged over 51 runs, were generated by the proposed PSO-BHF and all compared PSO-based variants for f_1 to f_{15} on the CEC2017 benchmark functions. The Y-axis represents fitness values, while the X-axis represents Function Evaluations (FEs).

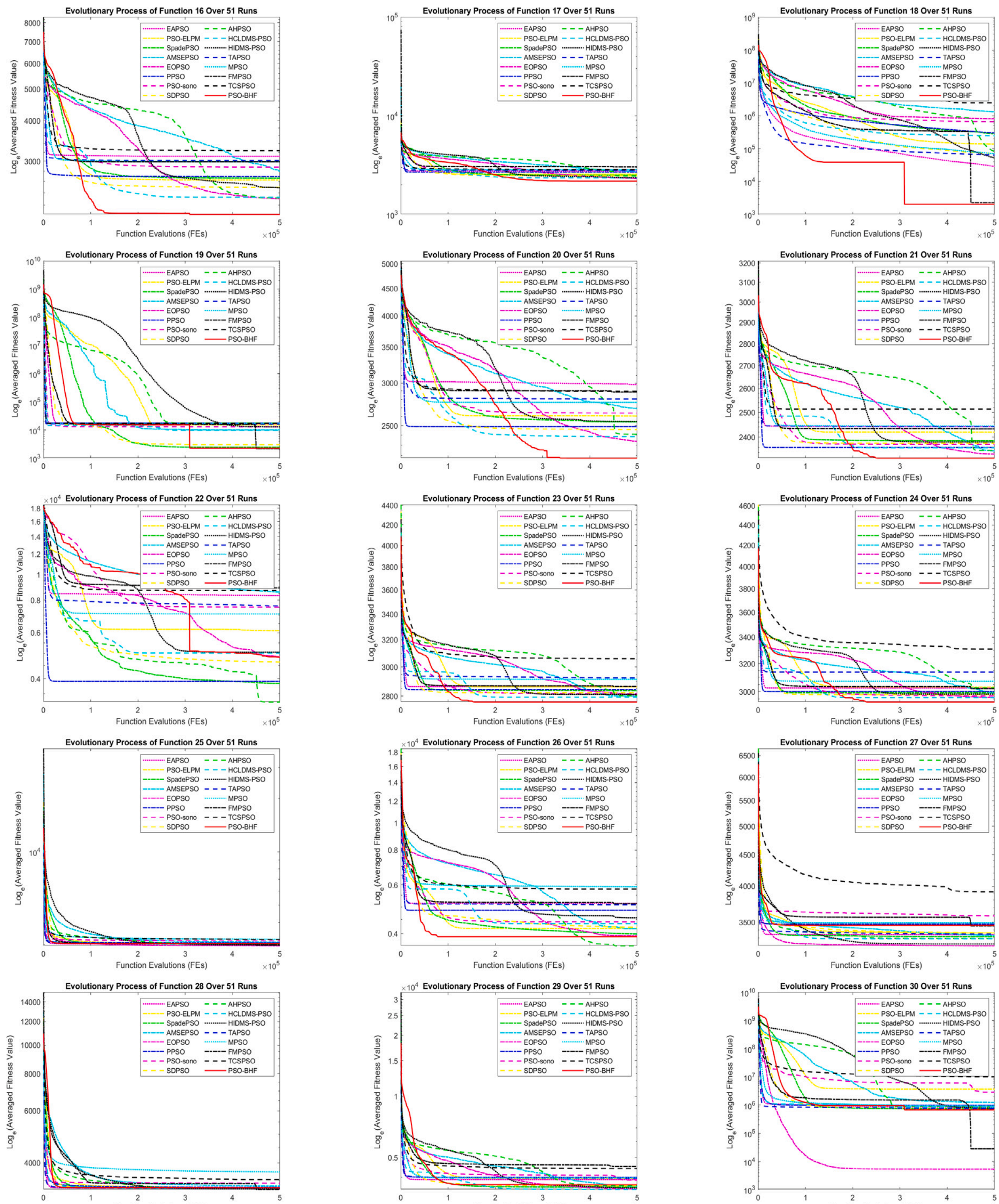

Fig. 7. Evolutionary processes of 50-dimensional experiments, averaged over 51 runs, were generated by the proposed PSO-BHF and all compared PSO-based variants for f_{16} to f_{30} on the CEC2017 benchmark functions. The Y-axis represents fitness values, while the X-axis represents Function Evaluations (FEs).

Table 12
Comparison of experimental results of PSO variants, averaged over 51 runs, for 50-dimensional CEC2017 benchmark functions.

f_n	Criteria	PSO-BHF	EAPSO	PSO-ELPM	SpadePSO	AMSEPSO	EOPSO	PPSO	PSO-sono	SDPSO	AHPSO	HCLDMS-PSO	HIDMS-PSO	TAPSO	MPSO	FMPSO	TCSPSO
f_1	Mean	**1.211863E−02**	6.907258E+03	4.974704E+03	1.544704E+02	5.222602E+03	2.165722E+03	2.952488E+03	3.507356E+03	3.908164E+02	2.619762E+03	1.628472E+03	5.027334E+03	3.999655E+03	2.777660E+08	1.539350E+02	3.945177E+03
	Rank|WRST	**1**	15 | >	12 | >	3 | >	14 | >	6 | >	8 | >	9 | >	4 | >	7 | >	5 | >	13 | >	11 | >	16 | >	2 | >	10 | >
	Min	**1.580000E−03**	4.838411E+01	6.837670E+00	8.679200E−01	1.064000E−01	5.931740E+00	4.782350E+00	7.669060E+00	1.571750E+00	5.284640E+00	3.087960E+00	3.029324E+01	3.355730E+00	1.299180E+00	4.120000E−03	9.668100E−01
	Rank	**1**	16	13	4	3	12	10	14	7	11	8	15	9	6	2	5
	Median	**1.565000E−02**	4.260851E+03	2.663590E+03	7.384339E+01	3.849059E+03	1.343610E+03	1.571922E+03	1.461353E+03	2.091275E+02	9.343652E+02	1.335900E+03	2.292345E+03	2.100825E+03	1.112851E+08	1.626000E−02	1.764468E+03
	Rank	**1**	15	13	3	14	7	9	8	4	5	6	12	11	16	2	10
	(Std Dev)	(5.511680E−03)	(7.224478E+03)	(5.870773E+03)	(2.075984E+02)	(4.907660E+03)	(3.351224E+03)	(3.407468E+03)	(5.235311E+03)	(4.356287E+02)	(3.902301E+03)	(2.008301E+03)	(7.264972E+03)	(5.892203E+03)	(4.528372E+08)	(3.928101E+02)	(5.182760E+03)
f_2	Mean	6.860784E−04	**2.431373E−05**	1.560378E+08	9.204527E+01	5.358826E+34	1.952705E+36	2.423302E+18	1.840141E+40	1.253149E+01	8.535897E+07	1.705277E+07	3.114398E−01	2.537255E−04	3.333523E+27	7.189154E+29	3.296846E+39
	Rank|WRST	3	**1** | <	9 | >	6 | >	13 | >	14 | >	10 | >	16 | >	5 | >	8 | >	7 | >	4 | >	2 | =	11 | >	12 | >	15 | >
	Min	1.000000E−05	**0.000000E+00**	6.395100E+00	2.500000E−04	1.164214E+04	9.391506E+01	3.493115E+06	2.067459E+13	**0.000000E+00**	6.100000E−04	1.100067E+02	3.000000E−05	**0.000000E+00**	2.708905E+03	1.000000E−05	7.832887E+15
	Rank	4	**1**	9	7	13	10	14	15	**1**	8	11	6	**1**	12	4	16
	Median	1.300000E−04	**1.000000E−05**	2.516373E+02	8.091043E+01	1.877251E+27	2.815412E+08	1.291427E+12	6.485768E+23	1.690000E−03	1.858541E+02	1.059285E+04	5.400000E−04	1.100000E−04	1.089685E+09	4.613000E−02	6.309630E+27
	Rank	3	**1**	9	7	15	11	13	14	5	8	10	4	2	12	6	16
	(Std Dev)	(1.585369E−03)	(3.061078E−05)	(1.095690E+09)	(5.124978E+01)	(2.526225E+35)	(1.385264E+37)	(1.671218E+19)	(1.269410E+41)	(2.633526E+01)	(2.908135E+08)	(5.155965E+07)	(2.218128E+00)	(3.217699E−04)	(2.380611E+28)	(5.134083E+30)	(2.354237E+40)
f_3	Mean	**0.000000E+00**	8.486725E−02	5.786153E+00	3.184617E+01	7.947899E+03	1.761420E+04	6.610250E+04	3.226580E+02	2.580242E+01	5.923983E+02	5.194396E+03	3.868627E−04	1.046594E+03	1.690232E+02	2.096078E−04	4.000362E+04
	Rank|WRST	**1**	4 | >	5 | >	7 | >	13 | >	14 | >	16 | >	9 | >	6 | >	10 | >	12 | >	3 | >	11 | >	8 | >	2 | >	15 | >
	Min	**0.000000E+00**	**0.000000E+00**	8.520000E−03	3.814960E+00	9.738930E+02	9.894686E+03	3.802442E+04	5.584170E+00	3.350000E−03	7.878636E+01	2.449486E+03	**0.000000E+00**	1.210390E+00	2.178200E−01	**0.000000E+00**	2.502815E+04
	Rank	**1**	**1**	6	9	12	14	16	10	5	11	13	**1**	8	7	**1**	15
	Median	**0.000000E+00**	**0.000000E+00**	5.262800E−01	1.499130E+01	6.917774E+03	1.667849E+04	6.368189E+04	1.707044E+02	1.836400E−01	5.096468E+02	5.346894E+03	**0.000000E+00**	3.382164E+02	4.846580E+00	6.000000E−05	3.997531E+04
	Rank	**1**	**1**	6	8	13	14	16	9	5	11	12	**1**	10	7	4	15
	(Std Dev)	(0.000000E+00)	(5.713674E−01)	(1.587817E+01)	(3.494908E+01)	(4.855800E+03)	(5.067547E+03)	(1.585390E+04)	(4.769393E+02)	(1.780170E+02)	(4.084084E+02)	(1.769583E+03)	(2.621868E−03)	(2.559900E+03)	(9.031552E+02)	(4.695613E−04)	(7.660565E+03)
f_4	Mean	3.877246E+01	3.495586E+01	8.250315E+01	7.638575E+01	2.467052E+02	9.956136E+01	1.099923E+02	2.164492E+02	5.717500E+01	8.452085E+01	9.359321E+01	7.985508E+01	4.754140E+01	2.359591E+02	**1.641549E+00**	2.681173E+02
	Rank|WRST	3	2 | =	8 | >	6 | >	15 | >	11 | >	12 | >	13 | >	5 | >	9 | >	10 | >	7 | >	4 | =	14 | >	**1** | <	16 | >
	Min	**0.000000E+00**	1.100000E−04	3.341300E−01	2.565587E+01	1.523403E+02	4.483861E+01	8.023170E+00	1.360000E−03	9.290000E−03	8.051540E+00	7.982100E−01	8.684980E+00	3.700000E−04	2.851278E+01	**0.000000E+00**	6.404422E+01
	Rank	**1**	3	7	12	16	14	9	5	6	10	8	11	4	13	**1**	15
	Median	2.851271E+01	2.851271E+01	7.392323E+01	7.668974E+01	2.501659E+02	9.795314E+01	1.010590E+02	2.275905E+02	3.443694E+01	7.716757E+01	9.993269E+01	9.603668E+01	2.851271E+01	1.440249E+02	**0.000000E+00**	2.490403E+02
	Rank	2	2	6	7	16	10	12	14	5	8	11	9	2	13	**1**	15
	(Std Dev)	(4.559231E+01)	(4.104052E+01)	(5.635495E+01)	(4.389365E+01)	(2.315838E+01)	(3.954097E+01)	(5.315863E+01)	(6.964548E+01)	(3.834210E+01)	(4.733779E+01)	(4.361719E+01)	(4.955622E+01)	(4.493374E+01)	(1.879278E+02)	(1.981550E+00)	(1.265969E+02)
f_5	Mean	**1.946998E+01**	1.339761E+02	1.355286E+02	9.314425E+01	4.896962E+01	3.676698E+01	6.143378E+01	6.801224E+01	7.530644E+01	4.422590E+01	5.903571E+01	8.176449E+01	1.426094E+02	1.579920E+02	1.208775E+02	2.462803E+02
	Rank|WRST	**1**	12 | >	13 | >	10 | >	4 | >	2 | >	6 | >	7 | >	8 | >	3 | >	5 | >	9 | >	14 | >	15 | >	11 | >	16 | >
	Min	**1.094455E+01**	7.064197E+01	8.357645E+01	5.571767E+01	1.790926E+01	1.591984E+01	3.880338E+01	3.581852E+01	3.482356E+01	2.586893E+01	4.377818E+01	4.477309E+01	8.357641E+01	1.004906E+02	6.566723E+01	1.572030E+02
	Rank	**1**	12	14	10	3	2	7	6	5	4	8	9	13	15	11	16
	Median	**1.890422E+01**	1.343190E+02	1.363090E+02	9.153607E+01	3.382859E+01	2.885141E+01	6.069246E+01	6.865210E+01	7.163697E+01	4.278322E+01	5.870254E+01	7.760669E+01	1.402888E+02	1.581979E+02	1.233746E+02	2.457537E+02
	Rank	**1**	12	13	10	3	2	6	7	8	4	5	9	14	15	11	16
	(Std Dev)	(5.117047E+00)	(3.106899E+01)	(2.997390E+01)	(1.799749E+01)	(4.770649E+01)	(3.507538E+01)	(1.250300E+01)	(1.249509E+01)	(1.946256E+01)	(9.786952E+00)	(7.474412E+00)	(2.082065E+01)	(2.783898E+01)	(3.481238E+01)	(2.626029E+01)	(3.938548E+01)
f_6	Mean	4.543104E−01	1.044196E−02	1.460278E+00	**0.000000E+00**	2.592706E−02	1.372549E−06	8.081090E−01	6.474202E−01	1.588235E−05	4.758235E−02	9.970588E−03	5.329294E−02	3.377784E−02	6.004904E−01	6.447012E+00	1.377126E+01
	Rank|WRST	10	5 | <	14 | =	**1** | <	6 | <	2 | <	13 | >	12 | >	3 | <	8 | <	4 | <	9 | <	7 | <	11 | >	15 | >	16 | >
	Min	8.589000E−02	**0.000000E+00**	1.460000E−03	**0.000000E+00**	1.640000E−03	**0.000000E+00**	1.037600E−01	9.000000E−05	**0.000000E+00**	2.850000E−03	1.530000E−03	9.900000E−04	5.420000E−03	1.980000E−03	1.922090E+00	3.525840E+00
	Rank	13	**1**	7	**1**	9	**1**	14	5	**1**	11	8	6	12	10	15	16
	Median	3.760000E−01	**0.000000E+00**	6.248500E−01	**0.000000E+00**	1.488000E−02	**0.000000E+00**	7.439700E−01	8.241000E−02	**0.000000E+00**	2.483000E−02	7.970000E−03	4.575000E−02	2.604000E−02	1.137500E−01	4.985820E+00	1.338594E+01
	Rank	12	**1**	13	**1**	6	**1**	14	10	**1**	7	5	9	8	11	15	16
	(Std Dev)	(3.041868E−01)	(3.383232E−02)	(2.883163E+00)	(0.000000E+00)	(2.926920E−02)	(3.475404E−06)	(5.751857E−01)	(1.059595E+00)	(2.089751E−05)	(5.751080E−02)	(5.003815E−03)	(5.343277E−02)	(2.100800E−02)	(1.012344E+00)	(4.458605E+00)	(5.758623E+00)
f_7	Mean	**7.332846E+01**	1.784630E+02	1.364645E+02	1.698247E+02	1.612920E+02	1.504573E+02	1.123223E+02	1.243260E+02	1.871767E+02	9.209707E+01	1.025296E+02	1.542710E+02	1.901545E+02	2.361220E+02	2.478332E+02	3.391336E+02
	Rank|WRST	**1**	11 | >	6 | >	10 | >	9 | >	7 | >	4 | >	5 | >	12 | >	2 | >	3 | >	8 | >	13 | >	14 | >	15 | >	16 | >
	Min	**6.083154E+01**	1.168317E+02	9.553572E+01	1.108569E+02	7.281943E+01	8.612146E+01	9.015687E+01	8.088268E+01	1.078253E+02	7.205328E+01	8.406094E+01	1.026696E+02	1.333738E+02	1.418965E+02	1.397400E+02	2.164054E+02
	Rank	**1**	12	8	11	3	6	7	4	10	2	5	9	13	15	14	16
	Median	**7.275533E+01**	1.774897E+02	1.355831E+02	1.703803E+02	1.064092E+02	1.076511E+02	1.135524E+02	1.178493E+02	1.945010E+02	8.856757E+01	9.891163E+01	1.490886E+02	1.924779E+02	2.363018E+02	2.458324E+02	3.404768E+02
	Rank	**1**	11	8	10	4	5	6	7	13	2	3	9	12	14	15	16
	(Std Dev)	(6.736992E+00)	(2.535986E+01)	(2.116508E+01)	(2.230821E+01)	(1.038152E+02)	(8.774058E+01)	(1.324694E+01)	(3.040601E+01)	(2.797436E+01)	(1.419038E+01)	(1.075955E+01)	(2.810315E+01)	(3.029710E+01)	(4.766138E+01)	(5.529172E+01)	(6.167176E+01)
f_8	Mean	**1.984065E+01**	1.501074E+02	1.335973E+02	8.747289E+01	4.632649E+01	6.663134E+01	6.118485E+01	6.768418E+01	8.252270E+01	4.552331E+01	6.219374E+01	8.479442E+01	1.418159E+02	1.522270E+02	1.366796E+02	2.395498E+02
	Rank|WRST	**1**	14 | >	11 | >	10 | >	3 | >	6 | >	4 | >	7 | >	8 | >	2 | >	5 | >	9 | >	13 | >	15 | >	12 | >	16 | >
	Min	**9.949590E+00**	6.865210E+01	8.059160E+01	5.372775E+01	1.890422E+01	2.069194E+01	2.984876E+01	3.283363E+01	5.074288E+01	2.387901E+01	4.676305E+01	5.273279E+01	9.253101E+01	7.465096E+01	8.656126E+01	1.542183E+02
	Rank	**1**	11	13	10	2	3	5	6	8	4	7	9	15	12	14	16
	Median	**1.989918E+01**	1.422787E+02	1.313343E+02	8.556630E+01	3.183867E+01	3.226575E+01	6.069242E+01	6.566721E+01	8.059155E+01	4.278322E+01	6.168740E+01	8.457137E+01	1.343191E+02	1.512778E+02	1.243695E+02	2.258547E+02
	Rank	**1**	14	12	10	2	3	5	7	8	4	6	9	13	15	11	16
	(Std Dev)	(5.307568E+00)	(4.245824E+01)	(2.781038E+01)	(1.815952E+01)	(4.797230E+01)	(7.096237E+01)	(1.333847E+01)	(1.840515E+01)	(1.838517E+01)	(1.348777E+01)	(1.050602E+01)	(1.972911E+01)	(2.817159E+01)	(3.657339E+01)	(3.173195E+01)	(5.102861E+01)
f_9	Mean	1.254961E+01	1.677723E+03	2.915678E+03	5.950935E+01	9.437429E+00	7.296094E−01	2.431939E+02	9.172388E+00	1.460517E+02	2.172907E+00	**1.689457E−01**	6.350228E+00	1.269031E+03	8.577996E+02	6.764093E+02	4.309806E+03
	Rank|WRST	7	14 | >	15 | >	8 | >	6 | =	2 | <	10 | >	5 | <	9 | >	3 | <	**1** | <	4 | <	13 | >	12 | >	11 | >	16 | >
	Min	2.685800E−01	7.812990E+00	2.685800E−01	7.723470E+00	**0.000000E+00**	**0.000000E+00**	4.162871E+01	6.333800E−01	4.075550E+00	8.953000E−02	**0.000000E+00**	8.953000E−02	2.973143E+02	1.792336E+02	7.823293E+01	1.747884E+03
	Rank	6	11	6	10	**1**	**1**	12	8	9	4	**1**	4	15	14	13	16
	Median	1.051400E+01	2.316689E+02	1.797489E+03	4.168588E+01	6.542010E+00	5.438500E−01	1.583120E+02	5.080370E+00	7.077645E+01	1.721090E+00	**0.000000E+00**	3.724100E+00	1.017097E+03	6.546252E+02	4.372796E+02	3.928866E+03
	Rank	7	11	15	8	6	2	10	5	9	3	**1**	4	14	13	12	16
	(Std Dev)	(1.119452E+01)	(3.441218E+03)	(4.582382E+03)	(5.248124E+01)	(7.806693E+00)	(8.023932E−01)	(1.772527E+02)	(1.423777E+01)	(1.554561E+02)	(1.419039E+00)	(3.118597E−01)	(9.245812E+00)	(8.900145E+02)	(8.369574E+02)	(9.856873E+02)	(2.092247E+03)

(*continued on next page*)

Table 12 (*continued*).

f_{10}	Mean	5.762541E+03	5.808170E+03	4.951142E+03	4.032362E+03	6.167711E+03	5.192070E+03	4.348818E+03	5.912406E+03	**3.905014E+03**	4.870362E+03	4.465653E+03	5.427184E+03	4.559603E+03	5.983978E+03	5.818845E+03	5.722706E+03
	Rank|WRST	11	12 | =	7 | <	2 | <	16 | >	8 | <	3 | <	14 | =	**1** | <	6 | <	4 | <	9 | =	5 | <	15 | =	13 | =	10 | =
	Min	**2.243457E+03**	4.054304E+03	3.277627E+03	3.032401E+03	4.456186E+03	2.484119E+03	2.284630E+03	4.301515E+03	3.213824E+03	2.693417E+03	3.551384E+03	2.856234E+03	2.784691E+03	3.721537E+03	3.267179E+03	4.049441E+03
	Rank	**1**	14	10	7	16	3	2	15	8	4	11	6	5	12	9	13
	Median	5.428501E+03	5.785403E+03	4.997103E+03	4.044207E+03	6.133740E+03	4.553992E+03	4.323718E+03	6.042675E+03	**3.869423E+03**	4.920563E+03	4.349014E+03	5.527385E+03	4.528151E+03	5.979801E+03	5.912912E+03	5.755975E+03
	Rank	9	12	8	2	16	6	3	15	**1**	7	4	10	5	14	13	11
	(Std Dev)	(1.866902E+03)	(8.431391E+02)	(7.197116E+02)	(4.191298E+02)	(6.463472E+02)	(2.208236E+03)	(9.035232E+02)	(8.897179E+02)	(3.956893E+02)	(1.121631E+03)	(5.116969E+02)	(9.752777E+02)	(7.543531E+02)	(8.689766E+02)	(1.029865E+03)	(8.433574E+02)
f_{11}	Mean	1.528567E+02	8.787645E+01	1.753536E+02	1.297489E+02	1.485282E+02	**8.774823E+01**	1.812188E+02	2.527967E+02	1.922001E+02	9.126666E+01	9.634406E+01	1.247738E+02	1.400601E+02	2.130928E+02	2.233559E+02	2.467970E+02
	Rank|WRST	9	2 | <	10 | >	6 | <	8 | =	**1** | <	11 | >	16 | >	12 | >	3 | <	4 | <	5 | <	7 | =	13 | >	14 | >	15 | >
	Min	9.587140E+01	4.378420E+01	9.189150E+01	5.507830E+01	8.228580E+01	**3.004530E+01**	1.072750E+02	1.176454E+02	9.979430E+01	5.806310E+01	5.502130E+01	7.793400E+01	6.900770E+01	9.542590E+01	1.084498E+02	7.123230E+01
	Rank	12	2	10	4	9	**1**	14	16	13	5	3	8	6	11	15	7
	Median	1.515883E+02	**7.497730E+01**	1.762546E+02	1.267148E+02	1.412868E+02	8.681560E+01	1.684464E+02	2.509591E+02	1.844220E+02	8.990170E+01	9.637650E+01	1.257201E+02	1.346989E+02	1.920976E+02	2.208783E+02	2.011975E+02
	Rank	9	**1**	11	6	8	2	10	16	12	3	4	5	7	13	15	14
	(Std Dev)	(3.524452E+01)	(3.411454E+01)	(4.394534E+01)	(4.070432E+01)	(3.646061E+01)	(4.235841E+01)	(5.352022E+01)	(8.103663E+01)	(5.387791E+01)	(1.539942E+01)	(2.172029E+01)	(2.057416E+01)	(4.496771E+01)	(9.006835E+01)	(5.630799E+01)	(1.257290E+02)
f_{12}	Mean	**2.056617E+03**	5.707407E+04	1.895392E+05	7.415231E+05	2.078920E+06	1.791132E+06	1.014245E+06	4.604689E+05	9.004456E+04	3.204411E+05	8.051730E+05	6.318319E+05	6.817407E+04	2.094171E+07	1.762734E+05	4.747903E+06
	Rank|WRST	**1**	2 | >	6 | >	10 | >	14 | >	13 | >	12 | >	8 | >	4 | >	7 | >	11 | >	9 | >	3 | >	16 | >	5 | >	15 | >
	Min	**7.365631E+02**	1.048309E+04	2.059829E+04	9.027420E+04	1.764383E+05	2.147187E+05	3.607271E+04	2.686734E+04	2.895204E+04	6.591619E+04	2.012388E+05	1.403879E+05	9.377485E+03	2.480572E+05	1.266910E+03	5.027012E+05
	Rank	**1**	4	5	10	12	14	8	6	7	9	13	11	3	15	2	16
	Median	**2.103858E+03**	5.579301E+04	1.891149E+05	7.032877E+05	1.428397E+06	1.411219E+06	7.753459E+05	2.412940E+05	7.667345E+04	2.771675E+05	7.149273E+05	5.053012E+05	4.609814E+04	5.716949E+06	2.381650E+03	2.053892E+06
	Rank	**1**	4	6	10	14	13	12	7	5	8	11	9	3	16	2	15
	(Std Dev)	(4.890698E+02)	(2.765969E+04)	(8.630352E+04)	(4.995318E+05)	(1.793260E+06)	(1.083940E+06)	(7.595226E+05)	(5.480331E+05)	(4.888638E+04)	(1.693620E+05)	(4.861690E+05)	(3.973171E+05)	(5.826378E+04)	(4.305929E+07)	(6.092664E+05)	(1.272634E+07)
f_{13}	Mean	**5.282313E+02**	7.654534E+03	7.547213E+03	5.335142E+02	2.714761E+03	5.024979E+03	3.480063E+03	6.577875E+03	6.343826E+02	5.519535E+03	1.134840E+03	8.545912E+03	5.136173E+03	1.033409E+04	9.182589E+02	6.266735E+03
	Rank|WRST	**1**	14 | >	13 | >	2 | =	6 | >	8 | >	7 | >	12 | >	3 | =	10 | >	5 | >	15 | >	9 | >	16 | >	4 | =	11 | >
	Min	2.896249E+02	8.122260E+01	1.752605E+02	6.791700E+01	1.627801E+02	1.801146E+02	3.552323E+02	2.247615E+02	5.348330E+01	6.996300E+01	1.988095E+02	1.203517E+03	1.071135E+02	2.068915E+02	**4.761310E+01**	1.712643E+02
	Rank	14	5	9	3	7	10	15	13	2	4	11	16	6	12	**1**	8
	Median	5.253018E+02	4.880373E+03	4.339108E+03	4.898759E+02	1.714177E+03	3.820988E+03	2.525874E+03	3.050027E+03	4.985929E+02	3.327943E+03	8.042112E+02	5.530980E+03	2.107777E+03	4.213726E+03	**4.448241E+02**	4.335710E+03
	Rank	4	15	14	2	6	11	8	9	3	10	5	16	7	12	**1**	13
	(Std Dev)	(1.416402E+02)	(7.614147E+03)	(8.689813E+03)	(3.278304E+02)	(3.264644E+03)	(4.020663E+03)	(3.106971E+03)	(7.527727E+03)	(5.067661E+02)	(7.153567E+03)	(9.660109E+02)	(7.635258E+03)	(7.325765E+03)	(2.419778E+04)	(1.116216E+03)	(6.622999E+03)
f_{14}	Mean	**2.737460E+02**	4.111766E+03	1.949765E+04	2.694958E+04	6.866027E+04	6.701746E+04	5.052144E+04	2.174938E+04	1.120360E+04	2.189901E+04	2.587102E+04	1.650672E+04	1.305593E+04	4.588573E+03	5.238075E+02	1.640792E+05
	Rank|WRST	**1**	3 | >	8 | >	12 | >	15 | >	14 | >	13 | >	9 | >	5 | >	10 | >	11 | >	7 | >	6 | >	4 | >	2 | >	16 | >
	Min	1.503495E+02	2.332040E+02	1.315623E+03	1.048314E+03	8.084406E+02	8.053560E+03	2.357537E+03	2.664729E+02	6.119869E+02	1.374784E+03	3.049395E+03	1.895907E+03	8.123747E+02	5.725233E+02	**1.163833E+02**	2.797584E+03
	Rank	2	3	10	9	7	16	13	4	6	11	15	12	8	5	**1**	14
	Median	**2.538806E+02**	3.480461E+03	1.462052E+04	2.210333E+04	4.135819E+04	5.562119E+04	4.885778E+04	2.492878E+03	8.782515E+03	1.849012E+04	2.647762E+04	1.398094E+04	5.683711E+03	3.469662E+03	5.201366E+02	9.581663E+04
	Rank	**1**	5	9	11	13	15	14	3	7	10	12	8	6	4	2	16
	(Std Dev)	(8.598322E+01)	(2.700158E+03)	(1.708786E+04)	(2.033042E+04)	(5.393560E+04)	(5.921797E+04)	(3.022262E+04)	(6.212062E+04)	(8.886025E+03)	(1.602578E+04)	(1.272223E+04)	(1.096233E+04)	(1.916667E+04)	(3.776169E+03)	(3.947410E+02)	(2.478344E+05)
f_{15}	Mean	3.911625E+02	8.167565E+03	6.647665E+03	**3.125650E+02**	2.991869E+03	4.652461E+03	7.474708E+03	3.866819E+03	6.283433E+02	6.416789E+03	2.274783E+03	6.143634E+03	7.352317E+03	9.311853E+03	1.643305E+03	5.579759E+03
	Rank|WRST	2	15 | >	12 | >	**1** | <	6 | >	8 | >	14 | >	7 | >	3 | =	11 | >	5 | >	10 | >	13 | >	16 | >	4 | >	9 | >
	Min	2.095867E+02	**1.758780E+01**	4.237437E+02	5.579140E+01	5.015660E+01	1.030644E+02	4.908750E+02	5.733040E+01	3.061720E+01	1.341495E+02	4.417220E+01	3.659486E+02	1.401875E+02	4.446943E+02	1.021750E+02	1.528388E+02
	Rank	12	**1**	14	5	4	8	16	6	2	9	3	13	10	15	7	11
	Median	3.771976E+02	6.998448E+03	3.242535E+03	**2.040800E+02**	1.863833E+03	3.215854E+03	7.208185E+03	2.101424E+03	3.135748E+02	4.262054E+03	9.677715E+02	5.414045E+03	6.003390E+03	9.870573E+03	4.704140E+02	3.345949E+03
	Rank	3	14	9	**1**	6	8	15	7	2	11	5	12	13	16	4	10
	(Std Dev)	(1.047539E+02)	(7.080700E+03)	(7.929228E+03)	(2.408318E+02)	(3.120957E+03)	(5.305780E+03)	(4.992127E+03)	(4.164476E+03)	(9.869124E+02)	(6.378644E+03)	(3.278562E+03)	(5.381181E+03)	(5.923301E+03)	(5.568486E+03)	(2.690786E+03)	(5.588057E+03)
f_{16}	Mean	**4.581635E+02**	1.508133E+03	1.026234E+03	1.059525E+03	1.195755E+03	7.006328E+02	1.091257E+03	1.276876E+03	8.943738E+02	7.121896E+02	7.237839E+02	8.859968E+02	1.414892E+03	1.389890E+03	1.387270E+03	1.633205E+03
	Rank|WRST	**1**	15 | >	7 | >	8 | >	10 | >	2 | >	9 | >	11 | >	6 | >	3 | >	4 | >	5 | >	14 | >	13 | >	12 | >	16 | >
	Min	1.967209E+02	6.368773E+02	3.713135E+02	3.740174E+02	**1.297706E+02**	1.334583E+02	2.454673E+02	3.567848E+02	3.081358E+02	2.636371E+02	2.475284E+02	1.482878E+02	6.090702E+02	5.873126E+02	7.751747E+02	6.902525E+02
	Rank	4	14	10	11	**1**	2	5	9	8	7	6	3	13	12	16	15
	Median	**4.379209E+02**	1.419685E+03	1.009893E+03	1.093649E+03	1.126332E+03	6.601395E+02	1.116316E+03	1.243531E+03	8.852604E+02	6.176386E+02	7.250438E+02	8.294048E+02	1.321120E+03	1.491759E+03	1.405657E+03	1.704583E+03
	Rank	**1**	14	7	8	10	3	9	11	6	2	4	5	12	15	13	16
	(Std Dev)	(1.582372E+02)	(4.457160E+02)	(3.485215E+02)	(2.696898E+02)	(5.139494E+02)	(3.036316E+02)	(3.653241E+02)	(3.546065E+02)	(2.682346E+02)	(2.780479E+02)	(1.971852E+02)	(3.912615E+02)	(4.400894E+02)	(3.580428E+02)	(3.493073E+02)	(4.657307E+02)
f_{17}	Mean	**4.726377E+02**	1.054167E+03	8.572470E+02	7.910805E+02	9.867298E+02	6.136393E+02	9.820872E+02	1.101039E+03	7.647154E+02	7.555468E+02	6.483866E+02	6.579953E+02	1.088995E+03	1.134927E+03	1.335076E+03	1.143997E+03
	Rank|WRST	**1**	11 | >	8 | >	7 | >	10 | >	2 | >	9 | >	13 | >	6 | >	5 | >	3 | >	4 | >	12 | >	14 | >	16 | >	15 | >
	Min	1.690147E+02	3.363047E+02	4.520553E+02	2.599313E+02	**1.499658E+02**	1.826435E+02	2.289057E+02	4.669868E+02	1.608576E+02	4.048082E+02	2.558618E+02	2.668170E+02	5.446143E+02	4.288838E+02	4.348763E+02	4.886007E+02
	Rank	3	9	13	7	**1**	4	5	14	2	10	6	8	16	11	12	15
	Median	**4.558485E+02**	1.093251E+03	8.436306E+02	7.983814E+02	1.057375E+03	6.271686E+02	9.944157E+02	1.093579E+03	8.199681E+02	7.216069E+02	6.702810E+02	6.551636E+02	1.136862E+03	1.176169E+03	1.401545E+03	1.123462E+03
	Rank	**1**	11	8	6	10	2	9	12	7	5	4	3	14	15	16	13
	(Std Dev)	(1.801277E+02)	(3.319499E+02)	(2.308603E+02)	(1.836002E+02)	(3.594072E+02)	(2.268145E+02)	(3.322954E+02)	(2.845092E+02)	(1.978265E+02)	(2.043382E+02)	(2.080052E+02)	(2.427862E+02)	(2.800557E+02)	(3.152117E+02)	(4.181378E+02)	(3.222637E+02)
f_{18}	Mean	**2.382751E+02**	2.707580E+04	1.370186E+05	2.783057E+05	1.316124E+06	8.052250E+05	2.969294E+05	6.556192E+05	6.921462E+04	7.962985E+04	2.458095E+05	5.055158E+04	6.212070E+04	6.831439E+04	4.687488E+02	2.461347E+06
	Rank|WRST	**1**	3 | >	9 | >	11 | >	15 | >	14 | >	12 | >	13 | >	7 | >	8 | >	10 | >	4 | >	5 | >	6 | >	2 | >	16 | >
	Min	1.032141E+02	5.136122E+03	2.432397E+04	3.687350E+04	1.152659E+05	1.530804E+05	7.731597E+04	5.945220E+03	7.382826E+03	1.996903E+04	3.236521E+04	1.636399E+04	1.100057E+04	7.761134E+03	**9.137760E+01**	5.395757E+04
	Rank	2	3	10	12	15	16	14	4	5	9	11	8	7	6	**1**	13
	Median	**2.283632E+02**	2.409345E+04	9.591259E+04	1.518436E+05	8.636799E+05	6.222777E+05	2.683426E+05	2.541490E+05	3.898329E+04	7.456988E+04	2.168842E+05	4.817199E+04	4.246107E+04	5.908776E+04	4.066039E+02	1.279170E+06
	Rank	**1**	3	9	10	15	14	13	12	4	8	11	6	5	7	2	16
	(Std Dev)	(8.268008E+01)	(1.472178E+04)	(1.456076E+05)	(3.032070E+05)	(1.274653E+06)	(5.380996E+05)	(1.314891E+05)	(1.137324E+06)	(1.653416E+05)	(4.005162E+04)	(1.668082E+05)	(2.577660E+04)	(1.012638E+05)	(4.760454E+04)	(2.821429E+02)	(2.902518E+06)

(*continued on next page*)

Table 12 (*continued*).

	Mean	3.684250E+02	1.562898E+04	1.345927E+04	5.512349E+02	8.249007E+03	1.470079E+04	1.585345E+04	1.103454E+04	1.113345E+03	1.581507E+04	7.752571E+03	1.099990E+04	1.422945E+04	1.534337E+04	**2.892583E+02**	1.538720E+04
	Rank|WRST	2	14 | >	9 | >	3 | =	6 | >	11 | >	16 | >	8 | >	4 | >	15 | >	5 | >	7 | >	10 | >	12 | >	**1** | <	13 | >
	Min	1.362888E+02	5.656700E+01	1.649093E+02	8.622830E+01	**2.015590E+01**	1.334691E+03	1.758083E+03	4.823260E+01	5.242760E+01	3.304251E+03	1.304593E+02	1.355035E+02	2.792020E+02	2.234014E+03	1.115234E+02	4.660750E+01
f_{19}	Rank	10	5	11	6	**1**	13	14	3	4	16	8	9	12	15	7	2
	Median	3.260635E+02	1.130961E+04	1.054529E+04	2.266216E+02	8.051645E+03	1.551837E+04	1.362864E+04	9.944553E+03	5.197609E+02	1.360873E+04	5.127449E+03	1.019686E+04	1.230109E+04	1.468309E+04	**2.098247E+02**	1.388851E+04
	Rank	3	10	9	2	6	16	13	7	4	12	5	8	11	15	**1**	14
	(Std Dev)	(1.751089E+02)	(1.278604E+04)	(1.088277E+04)	(8.353987E+02)	(6.917040E+03)	(6.609805E+03)	(7.344924E+03)	(8.715137E+03)	(1.496074E+03)	(8.280504E+03)	(6.519410E+03)	(9.594111E+03)	(8.807605E+03)	(7.878672E+03)	(2.480404E+02)	(1.158718E+04)
	Mean	**1.750021E+02**	9.828794E+02	6.043582E+02	5.427270E+02	6.909364E+02	3.375933E+02	4.899674E+02	6.383831E+02	4.587485E+02	4.105430E+02	3.850680E+02	5.452355E+02	8.029303E+02	7.624893E+02	8.872023E+02	8.972701E+02
	Rank|WRST	**1**	16 | >	9 | >	7 | >	11 | >	2 | >	6 | >	10 | >	5 | >	4 | >	3 | >	8 | >	13 | >	12 | >	14 | >	15 | >
	Min	8.466300E+01	3.432191E+02	1.718332E+02	1.047029E+02	1.448287E+02	**7.372040E+01**	1.106913E+02	2.291055E+02	1.550302E+02	1.376770E+02	8.786620E+01	1.333170E+02	8.058630E+01	1.503407E+02	1.191268E+02	3.740795E+02
f_{20}	Rank	3	15	13	5	10	**1**	6	14	12	9	4	8	2	11	7	16
	Median	**1.489812E+02**	9.717174E+02	5.958940E+02	5.779062E+02	6.694227E+02	3.062348E+02	4.670318E+02	6.537836E+02	4.383844E+02	3.578047E+02	3.847845E+02	5.375229E+02	8.577939E+02	7.406045E+02	8.108269E+02	9.309628E+02
	Rank	**1**	16	9	8	11	2	6	10	5	3	4	7	14	12	13	15
	(Std Dev)	(9.451588E+01)	(2.966397E+02)	(2.428301E+02)	(1.961749E+02)	(2.516191E+02)	(1.940335E+02)	(2.405003E+02)	(2.369351E+02)	(1.776528E+02)	(2.033310E+02)	(1.574439E+02)	(1.965895E+02)	(3.100276E+02)	(2.560790E+02)	(4.247312E+02)	(2.630639E+02)
	Mean	**2.177670E+02**	3.419129E+02	3.196780E+02	2.858407E+02	2.458975E+02	2.338347E+02	2.591023E+02	2.715597E+02	2.758124E+02	2.481934E+02	2.579643E+02	2.806531E+02	3.421105E+02	3.446760E+02	3.324939E+02	4.137902E+02
	Rank|WRST	**1**	13 | >	11 | >	10 | >	3 | >	2 | >	6 | >	7 | >	8 | >	4 | >	5 | >	9 | >	14 | >	15 | >	12 | >	16 | >
	Min	**2.094628E+02**	2.882041E+02	2.707382E+02	2.518689E+02	2.189657E+02	2.174364E+02	2.338679E+02	2.416729E+02	2.437023E+02	2.313170E+02	2.402223E+02	2.515161E+02	2.986621E+02	2.824589E+02	2.731089E+02	3.343162E+02
f_{21}	Rank	**1**	14	11	10	3	2	5	7	8	4	6	9	15	13	12	16
	Median	**2.177088E+02**	3.435447E+02	3.195743E+02	2.841102E+02	2.361111E+02	2.307413E+02	2.581115E+02	2.704873E+02	2.704757E+02	2.435145E+02	2.540277E+02	2.804244E+02	3.426567E+02	3.412521E+02	3.357666E+02	4.082433E+02
	Rank	**1**	15	11	10	3	2	6	8	7	4	5	9	14	13	12	16
	(Std Dev)	(4.208717E+00)	(3.174985E+01)	(2.728021E+01)	(1.550713E+01)	(3.991233E+01)	(2.387790E+01)	(1.110816E+01)	(1.687158E+01)	(1.960566E+01)	(1.662266E+01)	(1.106435E+01)	(2.052457E+01)	(2.505981E+01)	(3.141937E+01)	(3.118413E+01)	(4.928424E+01)
	Mean	2.643842E+03	6.130147E+03	3.907923E+03	1.636101E+03	6.394236E+03	2.660656E+03	1.710657E+03	5.317439E+03	2.441249E+03	**1.057137E+03**	2.819488E+03	2.860529E+03	5.402233E+03	4.884546E+03	6.709245E+03	6.510825E+03
	Rank|WRST	5	13 | >	9 | >	2 | =	14 | >	6 | =	3 | =	11 | >	4 | =	**1** | <	7 | =	8 | =	12 | >	10 | >	16 | >	15 | >
	Min	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	**1.000000E+02**	3.894896E+03	1.026429E+02	4.285483E+03	**1.000000E+02**
f_{22}	Rank	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	15	14	16	**1**
	Median	3.172168E+03	6.109104E+03	4.951461E+03	**1.000000E+02**	6.693020E+03	**1.000000E+02**	**1.000000E+02**	5.767970E+03	3.723423E+03	**1.000000E+02**	4.125839E+03	3.999266E+03	5.628396E+03	6.271520E+03	6.619630E+03	6.776844E+03
	Rank	5	12	9	**1**	15	**1**	**1**	11	6	**1**	8	7	10	13	14	16
	(Std Dev)	(2.397566E+03)	(1.253149E+03)	(2.590403E+03)	(2.220471E+03)	(1.636555E+03)	(2.848047E+03)	(2.572581E+03)	(2.096555E+03)	(2.341945E+03)	(2.104879E+03)	(2.533854E+03)	(2.791229E+03)	(7.473250E+02)	(3.160488E+03)	(1.099800E+03)	(1.767912E+03)
	Mean	**4.605240E+02**	5.628223E+02	5.616752E+02	5.368365E+02	4.956525E+02	5.056185E+02	5.428377E+02	5.160183E+02	5.226668E+02	5.032834E+02	4.928853E+02	5.115234E+02	6.235702E+02	6.141888E+02	5.652850E+02	7.601764E+02
	Rank|WRST	**1**	12 | >	11 | >	9 | >	3 | >	5 | >	10 | >	7 | >	8 | >	4 | >	2 | >	6 | >	15 | >	14 | >	13 | >	16 | >
	Min	**4.348198E+02**	4.940509E+02	4.846143E+02	4.938735E+02	4.451734E+02	4.564802E+02	4.830655E+02	4.625332E+02	4.656743E+02	4.619246E+02	4.705375E+02	4.716642E+02	5.417501E+02	4.982673E+02	4.923069E+02	6.501198E+02
f_{23}	Rank	**1**	13	10	12	2	3	9	5	6	4	7	8	15	14	11	16
	Median	**4.591543E+02**	5.545446E+02	5.670870E+02	5.371433E+02	4.750296E+02	4.909106E+02	5.396960E+02	5.166180E+02	5.216075E+02	5.039572E+02	4.906803E+02	5.112264E+02	6.170164E+02	6.176097E+02	5.638786E+02	7.384729E+02
	Rank	**1**	11	13	9	2	4	10	7	8	5	3	6	14	15	12	16
	(Std Dev)	(1.441918E+01)	(3.261278E+01)	(3.707463E+01)	(2.211125E+01)	(6.666676E+01)	(5.384826E+01)	(3.054042E+01)	(2.773730E+01)	(2.472202E+01)	(2.167084E+01)	(1.418726E+01)	(2.244852E+01)	(3.992425E+01)	(6.275053E+01)	(2.974375E+01)	(8.291898E+01)
	Mean	**5.284961E+02**	6.251684E+02	6.313280E+02	5.914337E+02	5.871521E+02	5.593973E+02	5.994867E+02	5.713138E+02	5.789002E+02	6.248497E+02	5.586945E+02	5.832111E+02	7.370704E+02	6.717752E+02	6.198605E+02	9.082403E+02
	Rank|WRST	**1**	12 | >	13 | >	8 | >	7 | >	3 | >	9 | >	4 | >	5 | >	11 | >	2 | >	6 | >	15 | >	14 | >	10 | >	16 | >
	Min	**5.149999E+02**	5.687859E+02	5.629614E+02	5.555907E+02	5.241273E+02	5.234994E+02	5.592352E+02	5.364632E+02	5.367958E+02	5.568973E+02	5.362694E+02	5.352273E+02	6.380433E+02	5.734387E+02	5.584615E+02	6.806493E+02
f_{24}	Rank	**1**	13	12	8	3	2	11	6	7	9	5	4	15	14	10	16
	Median	**5.277552E+02**	6.182771E+02	6.289503E+02	5.848933E+02	5.475075E+02	5.486172E+02	5.984157E+02	5.668198E+02	5.759549E+02	6.079545E+02	5.607546E+02	5.819233E+02	7.467851E+02	6.564188E+02	6.098635E+02	8.759490E+02
	Rank	**1**	12	13	8	2	3	9	5	6	10	4	7	15	14	11	16
	(Std Dev)	(8.286214E+00)	(3.492515E+01)	(3.747758E+01)	(2.648551E+01)	(7.649307E+01)	(3.628892E+01)	(2.613277E+01)	(2.283193E+01)	(1.718090E+01)	(6.183823E+01)	(1.423736E+01)	(2.579888E+01)	(5.477075E+01)	(7.353092E+01)	(3.819891E+01)	(1.400143E+02)
	Mean	5.355475E+02	5.207729E+02	5.349321E+02	5.085341E+02	5.886819E+02	**4.475792E+02**	5.619719E+02	6.430059E+02	5.107923E+02	5.433778E+02	5.085556E+02	4.875155E+02	5.437966E+02	7.106127E+02	4.759193E+02	7.020122E+02
	Rank|WRST	9	7 | <	8 | =	4 | <	13 | >	**1** | <	12 | >	14 | >	6 | <	10 | =	5 | <	3 | <	11 | =	16 | >	2 | <	15 | >
	Min	4.609338E+02	4.588138E+02	4.604332E+02	4.599093E+02	5.525464E+02	**4.311382E+02**	4.649234E+02	5.481412E+02	4.561725E+02	4.706829E+02	4.602582E+02	4.802451E+02	4.609040E+02	5.751060E+02	4.311484E+02	5.472955E+02
f_{25}	Rank	9	4	7	5	15	**1**	10	14	3	11	6	12	8	16	2	13
	Median	5.555886E+02	5.252423E+02	5.305398E+02	5.185919E+02	5.903561E+02	**4.312419E+02**	5.700839E+02	6.330028E+02	5.207774E+02	5.628066E+02	5.160900E+02	4.803412E+02	5.528793E+02	7.010974E+02	4.758620E+02	6.905851E+02
	Rank	10	7	8	5	13	**1**	12	14	6	11	4	3	9	16	2	15
	(Std Dev)	(3.959992E+01)	(4.203314E+01)	(3.595755E+01)	(2.395212E+01)	(1.588668E+01)	(2.671615E+01)	(2.945179E+01)	(3.658626E+01)	(2.920811E+01)	(3.310211E+01)	(3.302690E+01)	(2.013009E+01)	(4.439460E+01)	(6.295816E+01)	(2.669512E+01)	(6.864517E+01)
	Mean	1.303747E+03	2.545642E+03	1.583413E+03	1.389511E+03	1.576250E+03	1.328190E+03	2.257271E+03	1.814109E+03	1.620824E+03	**1.017423E+03**	1.753355E+03	1.963264E+03	2.474122E+03	3.311206E+03	2.529589E+03	3.191896E+03
	Rank|WRST	2	14 | >	6 | =	4 | =	5 | >	3 | =	11 | >	9 | >	7 | >	**1** | =	8 | >	10 | >	12 | >	16 | >	13 | >	15 | >
	Min	**3.000000E+02**	1.952109E+03	**3.000000E+02**	**3.000000E+02**	1.266855E+03	**3.000000E+02**	1.657889E+03	**3.000000E+02**	**3.000000E+02**	**3.000000E+02**	1.364551E+03	**3.000000E+02**	**3.000000E+02**	5.875717E+02	1.938198E+03	**3.000000E+02**
f_{26}	Rank	**1**	16	**1**	**1**	12	**1**	14	**1**	**1**	**1**	13	**1**	**1**	11	15	**1**
	Median	1.362892E+03	2.481646E+03	**3.000000E+02**	1.304747E+03	1.569304E+03	1.325758E+03	2.218095E+03	1.800793E+03	1.966624E+03	1.417908E+03	1.764404E+03	2.058127E+03	2.865057E+03	3.429462E+03	2.498143E+03	4.272438E+03
	Rank	4	12	**1**	2	6	3	11	8	9	5	7	10	14	15	13	16
	(Std Dev)	(2.645815E+02)	(3.937699E+02)	(1.642267E+03)	(8.077078E+02)	(1.348185E+02)	(1.964069E+02)	(3.007413E+02)	(3.012421E+02)	(7.185881E+02)	(6.665220E+02)	(1.503041E+02)	(5.283906E+02)	(1.388994E+03)	(2.246737E+03)	(3.350094E+02)	(2.332128E+03)

(*continued on next page*)

Table 12 (*continued*).

	Mean	7.599767E+02	6.404991E+02	6.632036E+02	6.175021E+02	6.337102E+02	**5.052588E+02**	7.738732E+02	8.849041E+02	6.648433E+02	5.932409E+02	5.911652E+02	5.274433E+02	6.485558E+02	7.916662E+02	7.519313E+02	1.218856E+03
	Rank|WRST	12	7 | <	9 | <	5 | <	6 | <	**1** | <	13 | =	15 | >	10 | <	4 | <	3 | <	2 | <	8 | <	14 | =	11 | =	16 | >
	Min	6.102542E+02	5.568382E+02	5.828731E+02	5.725749E+02	5.637065E+02	4.609147E+02	6.288014E+02	6.650815E+02	5.817832E+02	5.121118E+02	5.322502E+02	5.060030E+02	5.435106E+02	5.804005E+02	**4.311155E+02**	9.387455E+02
f_{27}	Rank	13	7	12	9	8	2	14	15	11	4	5	3	6	10	**1**	16
	Median	7.480311E+02	6.137844E+02	6.574348E+02	6.161204E+02	6.275313E+02	**5.000115E+02**	7.583687E+02	8.777443E+02	6.673215E+02	5.722223E+02	5.913784E+02	5.274771E+02	6.362741E+02	7.686610E+02	7.891212E+02	1.190147E+03
	Rank	11	5	9	6	7	**1**	12	15	10	3	4	2	8	13	14	16
	(Std Dev)	(9.391126E+01)	(6.709556E+01)	(5.409031E+01)	(2.934374E+01)	(4.278479E+01)	(1.817658E+01)	(7.982943E+01)	(9.625619E+01)	(3.837182E+01)	(8.698279E+01)	(3.550891E+01)	(1.258672E+01)	(6.715009E+01)	(1.249914E+02)	(2.101458E+02)	(1.714364E+02)
	Mean	4.959519E+02	4.898001E+02	4.951483E+02	4.783754E+02	5.379280E+02	4.786276E+02	5.076740E+02	6.153694E+02	4.860450E+02	4.865861E+02	4.816044E+02	4.670321E+02	4.925733E+02	9.252256E+02	**4.485066E+02**	7.122213E+02
	Rank|WRST	11	8 | =	10 | <	3 | <	13 | >	4 | <	12 | >	14 | >	6 | =	7 | =	5 | =	2 | <	9 | =	16 | >	**1** | <	15 | >
	Min	4.533516E+02	4.588487E+02	4.588487E+02	4.588487E+02	4.804417E+02	4.436342E+02	4.623376E+02	5.000119E+02	4.579260E+02	4.588487E+02	4.588488E+02	4.588487E+02	4.533516E+02	6.079612E+02	**4.384488E+02**	5.375930E+02
f_{28}	Rank	3	6	6	6	13	2	12	14	5	6	11	6	3	16	**1**	15
	Median	4.994279E+02	4.966693E+02	5.076947E+02	4.630235E+02	5.411182E+02	4.948959E+02	5.081135E+02	6.116229E+02	4.966693E+02	5.002797E+02	4.654729E+02	4.588487E+02	4.966693E+02	9.218224E+02	**4.436320E+02**	6.728767E+02
	Rank	9	6	11	3	13	5	12	14	6	10	4	2	6	16	**1**	15
	(Std Dev)	(2.133913E+01)	(2.644383E+01)	(1.962439E+01)	(2.179723E+01)	(2.519260E+01)	(2.364110E+01)	(1.822166E+01)	(4.585614E+01)	(2.369547E+01)	(2.307505E+01)	(2.368772E+01)	(1.794155E+01)	(3.190169E+01)	(1.914480E+02)	(1.213423E+01)	(1.298999E+02)
	Mean	7.729285E+02	9.930761E+02	1.048024E+03	6.786213E+02	6.998133E+02	5.995765E+02	1.097523E+03	9.693804E+02	7.254423E+02	7.421691E+02	**5.783676E+02**	6.487147E+02	1.050893E+03	1.068745E+03	1.622588E+03	1.511318E+03
	Rank|WRST	8	10 | >	11 | >	4 | =	5 | <	2 | <	14 | >	9 | >	6 | =	7 | =	**1** | <	3 | <	12 | >	13 | >	16 | >	15 | >
	Min	4.734783E+02	3.803063E+02	5.553245E+02	3.516066E+02	3.567285E+02	**3.314960E+02**	6.790696E+02	3.563171E+02	4.210917E+02	4.330757E+02	4.172302E+02	4.229522E+02	3.897914E+02	4.689064E+02	6.223316E+02	8.563492E+02
f_{29}	Rank	12	5	13	2	4	**1**	15	3	8	10	7	9	6	11	14	16
	Median	7.236509E+02	9.525078E+02	1.037909E+03	6.821492E+02	6.394440E+02	5.779834E+02	1.105814E+03	9.034268E+02	7.242863E+02	6.974551E+02	**5.175061E+02**	6.289737E+02	1.033163E+03	1.058335E+03	1.711543E+03	1.457216E+03
	Rank	7	10	12	5	4	2	14	9	8	6	**1**	3	11	13	16	15
	(Std Dev)	(2.222785E+02)	(3.014081E+02)	(2.511346E+02)	(1.608875E+02)	(3.050484E+02)	(2.405263E+02)	(2.338257E+02)	(3.346909E+02)	(1.659568E+02)	(1.805145E+02)	(1.331558E+02)	(1.466388E+02)	(2.905740E+02)	(3.268608E+02)	(4.255998E+02)	(3.414416E+02)
	Mean	6.722871E+05	8.254685E+05	3.691367E+06	7.216475E+05	1.229365E+06	**2.348132E+03**	9.743130E+05	2.861568E+06	7.237882E+05	8.696271E+05	8.641795E+05	7.718090E+05	7.479445E+05	9.264322E+05	2.501847E+04	1.006945E+07
	Rank|WRST	3	8 | >	15 | >	4 | >	13 | >	**1** | <	12 | >	14 | >	5 | >	10 | >	9 | >	7 | >	6 | >	11 | >	2 | <	16 | >
	Min	5.797151E+05	6.016902E+05	1.755387E+06	6.035570E+05	8.394813E+05	**2.927198E+02**	7.496894E+05	4.189394E+05	5.932636E+05	6.043987E+05	6.968855E+05	5.950914E+05	5.876390E+05	7.267423E+05	2.484967E+03	3.931392E+06
f_{30}	Rank	4	8	15	9	14	**1**	13	3	6	10	11	7	5	12	2	16
	Median	6.524580E+05	8.319744E+05	3.245409E+06	7.093557E+05	1.139346E+06	**1.792851E+03**	9.559350E+05	2.437381E+06	7.034558E+05	8.280860E+05	8.677471E+05	7.791189E+05	7.492897E+05	8.880069E+05	1.592094E+04	8.205154E+06
	Rank	3	9	15	5	13	**1**	12	14	4	8	10	7	6	11	2	16
	(Std Dev)	(8.075526E+04)	(1.188262E+05)	(1.478785E+06)	(7.334816E+04)	(3.616630E+05)	(1.797908E+03)	(1.504359E+05)	(2.132050E+06)	(7.552983E+04)	(1.836249E+05)	(7.565014E+04)	(1.128984E+05)	(9.557702E+04)	(1.737977E+05)	(2.287388E+04)	(5.874015E+06)

Table 13
Comparison of experimental results of PSO variants, averaged over 51 runs, for 100-dimensional CEC2017 benchmark functions.

f_n	Criteria	PSO-BHF	EAPSO	PSO-ELPM	SpadePSO	AMSEPSO	EOPSO	PPSO	PSO-sono	SDPSO	AHPSO	HCLDMS-PSO	HIDMS-PSO	TAPSO	MPSO	FMPSO	TCSPSO
f_1	Mean	**1.320604E−01**	8.934093E+03	5.101754E+04	2.682429E+02	3.993502E+04	6.339720E+03	4.283643E+03	4.427587E+03	5.130445E+02	8.791334E+03	7.411075E+03	7.394810E+03	8.964829E+03	7.214332E+09	6.395016E+01	6.378310E+03
	Rank|WRST	**1**	12 | >	15 | >	3 | >	14 | >	7 | >	5 | >	6 | >	4 | >	11 | >	10 | >	9 | >	13 | >	16 | >	2 | =	8 | >
	Min	**5.380000E−03**	4.132390E+00	2.866139E+01	2.327794E+01	1.403428E+02	2.656974E+01	1.587098E+01	4.975399E+01	7.219450E+00	3.155080E+00	3.569100E+00	4.727790E+00	8.616230E+00	8.847891E+08	5.600000E−03	1.546690E+00
	Rank	**1**	6	13	11	15	12	10	14	8	4	5	7	9	16	2	3
	Median	1.446500E−01	3.467675E+03	3.457642E+03	1.247369E+02	5.409451E+03	4.466473E+03	2.893918E+03	2.209570E+03	2.666017E+02	5.708981E+03	2.323558E+03	4.128658E+03	5.123170E+03	6.078572E+09	**1.440700E−01**	3.926069E+03
	Rank	2	9	8	3	14	12	7	5	4	15	6	11	13	16	**1**	10
	(Std Dev)	(3.043126E−02)	(1.120005E+04)	(3.119699E+05)	(3.529662E+02)	(1.073274E+05)	(6.169054E+03)	(4.154250E+03)	(5.558079E+03)	(7.311492E+02)	(1.010606E+04)	(9.500731E+03)	(8.899438E+03)	(9.818684E+03)	(5.560928E+09)	(2.253745E+02)	(6.662585E+03)
f_2	Mean	4.752941E−04	**1.635294E−04**	1.703727E+48	1.173242E+20	1.659229E+91	4.695377E+107	1.342100E+66	1.566466E+109	1.247832E+12	3.681100E+35	3.949843E+28	1.160363E−01	1.896114E+36	4.091797E+109	2.767574E+85	1.362256E+110
	Rank|WRST	2	**1** | <	9 | >	5 | >	12 | >	13 | >	10 | >	14 | >	4 | >	7 | >	6 | >	3 | >	8 | >	15 | >	11 | >	16 | >
	Min	**0.000000E+00**	**0.000000E+00**	1.122789E+28	8.916831E+04	2.459847E+41	3.128123E+09	8.634565E+25	7.940493E+68	4.097890E+00	2.804103E+18	3.102506E+17	1.300000E−04	2.102333E+05	6.718036E+24	6.128468E+36	7.768333E+52
	Rank	**1**	**1**	12	5	14	7	11	16	4	9	8	3	6	10	13	15
	Median	**3.000000E−05**	1.200000E−04	3.302612E+37	1.913466E+13	3.748639E+78	2.215671E+29	4.350561E+39	7.786234E+91	5.175707E+06	3.865494E+27	4.781405E+24	1.570000E−03	3.771126E+09	1.150720E+40	4.068837E+59	3.556854E+86
	Rank	**1**	2	10	6	14	9	11	16	4	8	7	3	5	12	13	15
	(Std Dev)	(2.223555E−03)	(1.686395E−04)	(1.208204E+49)	(8.264839E+20)	(1.184535E+92)	(3.353170E+108)	(7.513637E+66)	(8.095452E+109)	(8.471039E+12)	(2.611829E+36)	(1.738903E+29)	(4.638997E−01)	(1.354096E+37)	(2.922128E+110)	(1.976443E+86)	(9.722655E+110)
f_3	Mean	**6.980392E−05**	7.167322E+04	2.534930E+04	1.214703E+04	1.071883E+05	1.537925E+05	3.050849E+05	1.197114E+04	6.831607E+03	3.217742E+04	8.688899E+04	3.760671E+03	3.993199E+04	1.115163E+04	1.736392E−02	1.721080E+05
	Rank|WRST	**1**	11 | >	8 | >	7 | >	13 | >	14 | >	16 | >	6 | >	4 | >	9 | >	12 | >	3 | >	10 | >	5 | >	2 | >	15 | >
	Min	**6.000000E−05**	1.002148E+04	1.106058E+04	4.176509E+03	4.989549E+04	1.143434E+05	2.216837E+05	4.166709E+03	5.685012E+02	1.988465E+04	4.609132E+04	1.346608E+03	1.914019E+04	1.902967E+03	2.600000E−04	1.241569E+05
	Rank	**1**	8	9	7	13	14	16	6	3	11	12	4	10	5	2	15
	Median	**7.000000E−05**	6.766638E+04	2.492690E+04	1.192360E+04	1.094345E+05	1.494974E+05	3.037625E+05	1.178920E+04	3.126839E+03	3.073811E+04	8.314801E+04	3.627907E+03	3.930089E+04	9.633654E+03	7.520000E−03	1.677449E+05
	Rank	**1**	11	8	7	13	14	16	6	3	9	12	4	10	5	2	15
	(Std Dev)	(1.400280E−06)	(3.226572E+04)	(7.419772E+03)	(3.673924E+03)	(2.485936E+04)	(2.531293E+04)	(4.141779E+04)	(5.041135E+03)	(9.481470E+03)	(7.208444E+03)	(1.636526E+04)	(1.416599E+03)	(1.281930E+04)	(6.720064E+03)	(2.556978E−02)	(2.151040E+04)
f_4	Mean	3.629759E+01	1.164253E+02	2.660362E+02	2.268649E+02	4.104917E+02	2.072418E+02	2.360150E+02	4.792824E+02	2.031378E+02	2.336385E+02	2.225761E+02	2.146067E+02	1.891284E+02	1.561774E+03	**4.463910E+00**	9.275554E+02
	Rank|WRST	2	3 | >	12 | >	9 | >	13 | >	6 | >	11 | >	14 | >	5 | >	10 | >	8 | >	7 | >	4 | >	16 | >	**1** | =	15 | >
	Min	**0.000000E+00**	1.467400E−01	1.465615E+02	1.339467E+02	3.022304E+02	9.841245E+01	1.470116E+02	2.389758E+02	6.637876E+01	1.545099E+02	1.572801E+02	1.921043E+02	8.961468E+01	6.723070E+02	**0.000000E+00**	3.475290E+02
	Rank	**1**	3	8	7	14	6	9	13	4	10	11	12	5	16	**1**	15
	Median	**0.000000E+00**	1.133169E+02	2.550464E+02	2.328142E+02	4.106566E+02	2.057490E+02	2.351314E+02	4.684105E+02	2.109588E+02	2.354478E+02	2.198418E+02	2.182491E+02	1.830917E+02	1.396555E+03	**0.000000E+00**	8.335064E+02
	Rank	**1**	3	12	9	13	5	10	14	6	11	8	7	4	16	**1**	15
	(Std Dev)	(7.228868E+01)	(6.355886E+01)	(5.676216E+01)	(3.581292E+01)	(4.845892E+01)	(4.402280E+01)	(3.872833E+01)	(1.120526E+02)	(4.389080E+01)	(3.910503E+01)	(3.335688E+01)	(1.800885E+01)	(3.698623E+01)	(5.943991E+02)	(1.291185E+01)	(4.459222E+02)
f_5	Mean	1.582177E+02	3.526443E+02	4.111319E+02	2.847550E+02	9.623933E+01	**7.963830E+01**	2.086288E+02	1.929256E+02	3.025913E+02	1.032958E+02	1.612806E+02	2.055261E+02	4.346096E+02	4.525968E+02	4.321024E+02	6.558473E+02
	Rank|WRST	4	11 | >	12 | >	9 | >	2 | <	**1** | <	8 | >	6 | >	10 | >	3 | <	5 | =	7 | >	14 | >	15 | >	13 | >	16 | >
	Min	1.024807E+02	2.149107E+02	2.746077E+02	2.019763E+02	**5.372778E+01**	5.968105E+01	1.343193E+02	1.343193E+02	1.940166E+02	6.865215E+01	1.243698E+02	1.412839E+02	3.452495E+02	3.088321E+02	2.169007E+02	4.318099E+02
	Rank	4	11	13	10	**1**	2	7	6	9	3	5	8	15	14	12	16
	Median	1.542184E+02	3.453419E+02	4.029563E+02	2.829404E+02	8.656137E+01	**8.133284E+01**	2.079460E+02	1.950128E+02	3.024667E+02	1.034756E+02	1.581982E+02	1.989914E+02	4.288252E+02	4.451928E+02	4.308144E+02	6.427386E+02
	Rank	4	11	12	9	2	**1**	8	6	10	3	5	7	13	15	14	16
	(Std Dev)	(2.335623E+01)	(6.605799E+01)	(6.872796E+01)	(4.347032E+01)	(4.157971E+01)	(1.044727E+01)	(3.320970E+01)	(3.481110E+01)	(6.420866E+01)	(1.867207E+01)	(2.845799E+01)	(3.879717E+01)	(4.521005E+01)	(6.090686E+01)	(1.290181E+02)	(1.016170E+02)
f_6	Mean	9.556774E+00	3.987086E+00	1.085612E+01	**1.960784E−07**	5.719406E−01	1.717647E−04	6.983352E+00	2.153132E+00	7.568627E−05	4.455849E−01	7.693569E−02	9.493810E−01	9.114804E−02	6.381051E+00	2.195025E+01	3.036892E+01
	Rank|WRST	13	10 | <	14 | =	**1** | <	7 | <	3 | <	12 | <	9 | <	2 | <	6 | <	4 | <	8 | <	5 | <	11 | <	15 | >	16 | >
	Min	4.768240E+00	6.779800E−01	1.373030E+00	**0.000000E+00**	1.320000E−01	7.000000E−05	4.359200E+00	1.717500E−01	2.000000E−05	9.394000E−02	4.241000E−02	2.754900E−01	4.685000E−02	9.018900E−01	1.334786E+01	1.657096E+01
	Rank	14	10	12	**1**	7	3	13	8	2	6	4	9	5	11	15	16
	Median	9.690670E+00	3.246680E+00	1.058788E+01	**0.000000E+00**	5.302900E−01	1.600000E−04	6.642650E+00	1.011210E+00	7.000000E−05	3.034800E−01	7.837000E−02	8.031700E−01	8.528000E−02	5.965370E+00	2.196148E+01	2.880767E+01
	Rank	13	10	14	**1**	7	3	12	9	2	6	4	8	5	11	15	16
	(Std Dev)	(2.287546E+00)	(2.651162E+00)	(7.521542E+00)	(1.400280E−06)	(2.845377E−01)	(9.424873E−05)	(1.655316E+00)	(2.147934E+00)	(3.976204E−05)	(3.600386E−01)	(2.429050E−02)	(5.350193E−01)	(3.432394E−02)	(2.576099E+00)	(6.714095E+00)	(8.669734E+00)
f_7	Mean	2.307448E+02	4.955669E+02	3.701225E+02	5.465561E+02	3.325638E+02	3.647875E+02	3.976993E+02	3.206608E+02	5.771796E+02	**1.973596E+02**	2.431104E+02	3.809729E+02	6.894883E+02	8.227855E+02	8.994926E+02	1.169278E+03
	Rank|WRST	2	10 | >	7 | >	11 | >	5 | >	6 | >	9 | >	4 | >	12 | >	**1** | <	3 | >	8 | >	13 | >	14 | >	15 | >	16 | >
	Min	1.929326E+02	3.199689E+02	2.601332E+02	4.014785E+02	1.927073E+02	2.330415E+02	2.910697E+02	2.357874E+02	3.611783E+02	**1.602308E+02**	2.078347E+02	2.857419E+02	4.885863E+02	5.863731E+02	4.695516E+02	7.868605E+02
	Rank	3	10	7	12	2	5	9	6	11	**1**	4	8	14	15	13	16
	Median	2.284497E+02	4.805175E+02	3.543508E+02	5.579560E+02	2.780548E+02	2.701359E+02	3.930272E+02	3.114090E+02	5.891666E+02	**1.950482E+02**	2.434030E+02	3.666023E+02	6.779558E+02	8.240909E+02	9.137908E+02	1.136016E+03
	Rank	2	10	7	11	5	4	9	6	12	**1**	3	8	13	14	15	16
	(Std Dev)	(2.070301E+01)	(6.983036E+01)	(6.251897E+01)	(4.537287E+01)	(1.645880E+02)	(2.004897E+02)	(4.984309E+01)	(4.568114E+01)	(5.788622E+01)	(1.914190E+01)	(1.925741E+01)	(4.867991E+01)	(1.085002E+02)	(1.453453E+02)	(2.675490E+02)	(2.068475E+02)
f_8	Mean	1.497118E+02	3.342649E+02	4.249040E+02	2.997863E+02	1.157485E+02	**9.125210E+01**	2.036930E+02	1.879651E+02	3.281288E+02	1.000136E+02	1.807171E+02	2.148501E+02	4.071995E+02	4.457677E+02	3.835256E+02	6.525780E+02
	Rank|WRST	4	11 | >	14 | >	9 | >	3 | <	**1** | <	7 | >	6 | >	10 | >	2 | <	5 | >	8 | >	13 | >	15 | >	12 | >	16 | >
	Min	1.074555E+02	2.298350E+02	2.726180E+02	1.751125E+02	**4.775803E+01**	5.586222E+01	1.283495E+02	1.233749E+02	1.532235E+02	6.566727E+01	1.303395E+02	1.591932E+02	2.616734E+02	2.886123E+02	2.348098E+02	4.546943E+02
	Rank	4	11	14	10	**1**	2	6	5	8	3	7	9	13	15	12	16
	Median	1.462587E+02	3.328608E+02	4.178812E+02	2.974920E+02	8.855131E+01	**7.930075E+01**	2.069511E+02	1.841427E+02	3.313201E+02	9.750590E+01	1.810839E+02	2.139156E+02	4.029941E+02	4.350042E+02	3.611676E+02	6.248297E+02
	Rank	4	11	14	9	2	**1**	7	6	10	3	5	8	13	15	12	16
	(Std Dev)	(2.573020E+01)	(5.504027E+01)	(6.280824E+01)	(4.352898E+01)	(9.949769E+01)	(7.023234E+01)	(3.035186E+01)	(4.181397E+01)	(7.398694E+01)	(1.747808E+01)	(1.850883E+01)	(2.734072E+01)	(6.104487E+01)	(9.578082E+01)	(8.168615E+01)	(1.013010E+02)
f_9	Mean	4.545567E+02	9.755471E+03	1.169924E+04	2.265956E+03	1.434079E+02	9.984075E+00	3.048022E+03	3.292471E+02	5.140730E+03	3.796778E+01	**2.585613E+00**	2.246365E+02	9.952849E+03	7.858426E+03	4.931689E+03	1.878766E+04
	Rank|WRST	7	13 | >	15 | >	8 | >	4 | <	2 | <	9 | >	6 | <	11 | >	3 | <	**1** | <	5 | <	14 | >	12 | >	10 | >	16 | >
	Min	1.488387E+02	1.826842E+03	4.598071E+03	4.907780E+02	1.877087E+01	1.087700E+00	1.549080E+03	3.702478E+01	3.275411E+02	1.437716E+01	**1.860100E−01**	2.467865E+01	4.567849E+03	2.940866E+03	1.138678E+03	1.110418E+04
	Rank	7	12	15	9	4	2	11	6	8	3	**1**	5	14	13	10	16
	Median	4.173250E+02	8.812558E+03	1.182773E+04	1.842916E+03	1.200619E+02	8.331950E+00	2.970948E+03	2.275720E+02	5.035160E+03	2.776320E+01	**1.650190E+00**	2.072900E+02	9.407558E+03	7.774940E+03	3.500594E+03	1.903113E+04
	Rank	7	13	15	8	4	2	9	6	11	3	**1**	5	14	12	10	16
	(Std Dev)	(1.718812E+02)	(6.101371E+03)	(3.075973E+03)	(1.230062E+03)	(8.104363E+01)	(6.585905E+00)	(7.081467E+02)	(3.617523E+02)	(2.591335E+03)	(3.310415E+01)	(1.942871E+00)	(1.548157E+02)	(2.574501E+03)	(2.683522E+03)	(3.955291E+03)	(4.301389E+03)

(*continued on next page*)

Table 13 (*continued*).

f_{10}	Mean	**9.156025E+03**	1.298631E+04	1.148456E+04	1.086878E+04	1.481490E+04	1.175901E+04	9.858353E+03	1.348396E+04	1.055961E+04	1.233933E+04	1.103409E+04	1.145393E+04	1.191928E+04	1.343832E+04	1.338724E+04	1.324854E+04
	Rank|WRST	**1**	11 | >	7 | >	4 | >	16 | >	8 | >	2 | >	15 | >	3 | >	10 | >	5 | >	6 | >	9 | >	14 | >	13 | >	12 | >
	Min	**6.538938E+03**	8.005343E+03	8.524951E+03	9.224837E+03	1.218431E+04	8.181814E+03	7.460481E+03	1.000302E+04	8.666965E+03	9.099727E+03	8.539686E+03	8.074462E+03	8.905579E+03	1.079895E+04	1.120610E+04	1.012336E+04
	Rank	**1**	3	6	11	16	5	2	12	8	10	7	4	9	14	15	13
	Median	**9.101697E+03**	1.318904E+04	1.150554E+04	1.098137E+04	1.478491E+04	1.088984E+04	9.863704E+03	1.330925E+04	1.063240E+04	1.262034E+04	1.119656E+04	1.126492E+04	1.176662E+04	1.325407E+04	1.299783E+04	1.318562E+04
	Rank	**1**	13	8	5	16	4	2	15	3	10	6	7	9	14	11	12
	(Std Dev)	(1.256320E+03)	(1.850099E+03)	(1.638831E+03)	(6.523160E+02)	(1.181228E+03)	(3.643281E+03)	(1.049474E+03)	(1.603962E+03)	(6.220227E+02)	(1.538934E+03)	(9.686956E+02)	(1.425155E+03)	(1.023846E+03)	(1.295717E+03)	(1.219591E+03)	(1.182541E+03)
f_{11}	Mean	9.491623E+02	4.699456E+02	1.075482E+03	7.122071E+02	8.524523E+02	1.086833E+03	1.024528E+03	3.081031E+03	9.915727E+02	7.884527E+02	8.404834E+02	9.162972E+02	**3.363121E+02**	1.380855E+03	1.152090E+03	2.627911E+03
	Rank|WRST	8	2 | <	11 | >	3 | <	6 | <	12 | >	10 | =	16 | >	9 | =	4 | <	5 | <	7 | =	**1** | <	14 | >	13 | >	15 | >
	Min	5.599224E+02	1.813726E+02	6.788197E+02	2.976447E+02	3.565594E+02	6.533217E+02	5.920445E+02	1.037498E+03	5.887809E+02	5.055068E+02	6.367993E+02	6.880692E+02	**1.642964E+02**	7.828475E+02	4.965961E+02	5.651684E+02
	Rank	7	2	13	3	4	12	10	16	9	6	11	14	**1**	15	5	8
	Median	9.526601E+02	4.366779E+02	1.083587E+03	6.919011E+02	8.570388E+02	1.073750E+03	9.668961E+02	2.872593E+03	9.484515E+02	7.936727E+02	8.490799E+02	9.089805E+02	**3.534687E+02**	1.344163E+03	1.119036E+03	2.150675E+03
	Rank	9	2	12	3	6	11	10	16	8	4	5	7	**1**	14	13	15
	(Std Dev)	(1.587717E+02)	(1.723958E+02)	(1.420554E+02)	(1.679563E+02)	(2.124502E+02)	(2.178097E+02)	(2.241185E+02)	(1.369645E+03)	(2.133388E+02)	(1.341313E+02)	(1.184207E+02)	(1.285717E+02)	(1.052809E+02)	(3.166880E+02)	(3.166141E+02)	(1.754106E+03)
f_{12}	Mean	**4.246939E+03**	2.096926E+05	5.982435E+05	1.188936E+06	2.127903E+07	7.687417E+06	1.258694E+06	2.489601E+07	2.912156E+05	9.145467E+05	2.797238E+06	1.229334E+06	5.146712E+05	9.005700E+08	5.945352E+06	4.881651E+07
	Rank|WRST	**1**	2 | >	5 | >	7 | >	13 | >	12 | >	9 | >	14 | >	3 | >	6 | >	10 | >	8 | >	4 | >	16 | >	11 | >	15 | >
	Min	**3.105501E+03**	6.753675E+04	1.470632E+05	4.028275E+05	1.845497E+06	2.021612E+06	3.472318E+05	4.398825E+05	8.357190E+04	3.000489E+05	8.689412E+05	3.947253E+05	1.577230E+05	2.312521E+07	3.710831E+03	4.652499E+06
	Rank	**1**	3	5	10	13	14	8	11	4	7	12	9	6	16	2	15
	Median	**4.210477E+03**	1.791562E+05	5.871836E+05	1.034826E+06	1.868554E+07	7.385242E+06	1.188380E+06	1.350321E+07	2.699390E+05	7.772360E+05	2.559451E+06	1.138487E+06	3.677946E+05	4.599467E+08	6.318430E+03	3.418084E+07
	Rank	**1**	3	6	8	14	12	10	13	4	7	11	9	5	16	2	15
	(Std Dev)	(5.752907E+02)	(9.540512E+04)	(2.472843E+05)	(8.782827E+05)	(1.560596E+07)	(3.295978E+06)	(6.032811E+05)	(3.345935E+07)	(1.249990E+05)	(4.888147E+05)	(1.430529E+06)	(4.441223E+05)	(3.404096E+05)	(1.080239E+09)	(2.092644E+07)	(5.618159E+07)
f_{13}	Mean	7.029313E+02	7.544694E+03	5.960905E+03	**6.256165E+02**	3.542408E+03	3.991054E+03	7.330632E+03	5.082364E+03	7.005874E+02	4.660180E+03	2.587625E+03	1.046633E+04	4.331460E+03	5.911504E+06	2.582356E+03	6.491229E+03
	Rank|WRST	3	14 | >	11 | >	**1** | <	6 | >	7 | >	13 | >	10 | >	2 | =	9 | >	5 | >	15 | >	8 | >	16 | >	4 | >	12 | >
	Min	3.417583E+02	2.174799E+02	1.028485E+03	**1.825620E+02**	2.800530E+02	4.597067E+02	2.238168E+03	3.131220E+02	2.311062E+02	2.523883E+02	4.258166E+02	3.119258E+03	2.492340E+02	3.260306E+02	9.201680E+02	5.536370E+02
	Rank	9	2	14	**1**	6	11	15	7	3	5	10	16	4	8	13	12
	Median	6.812279E+02	5.692530E+03	3.887018E+03	**6.145912E+02**	2.605295E+03	3.436618E+03	5.845393E+03	4.855137E+03	6.406998E+02	3.824292E+03	1.301581E+03	6.746389E+03	3.202415E+03	1.420007E+04	2.232655E+03	4.149339E+03
	Rank	3	13	10	**1**	6	8	14	12	2	9	4	15	7	16	5	11
	(Std Dev)	(1.869759E+02)	(7.495547E+03)	(5.015330E+03)	(2.053253E+02)	(2.944737E+03)	(3.028562E+03)	(4.606327E+03)	(3.967968E+03)	(2.415728E+02)	(4.038041E+03)	(2.903144E+03)	(8.377988E+03)	(4.330005E+03)	(2.539747E+07)	(1.200831E+03)	(6.276195E+03)
f_{14}	Mean	**3.052381E+02**	4.423983E+04	6.535471E+04	2.452276E+05	1.419304E+06	7.839681E+05	2.597231E+05	2.966516E+05	4.762368E+04	6.873780E+04	3.045669E+05	6.756358E+04	3.318408E+04	1.244297E+05	1.126131E+03	1.477856E+06
	Rank|WRST	**1**	4 | >	6 | >	10 | >	15 | >	14 | >	11 | >	12 | >	5 | >	8 | >	13 | >	7 | >	3 | >	9 | >	2 | >	16 | >
	Min	**2.040766E+02**	4.444822E+03	1.168956E+04	3.062003E+04	9.692013E+04	1.706565E+05	8.184395E+04	5.374662E+03	3.876593E+03	2.238349E+04	6.262673E+04	2.057881E+04	5.448395E+03	1.116760E+04	2.623517E+02	3.313951E+05
	Rank	**1**	4	8	11	14	15	13	5	3	10	12	9	6	7	2	16
	Median	**3.025766E+02**	3.958664E+04	6.558269E+04	1.917365E+05	1.094541E+06	7.391813E+05	2.260876E+05	1.032423E+05	4.023564E+04	6.333854E+04	2.647753E+05	5.985543E+04	2.997693E+04	4.726960E+04	9.442953E+02	1.131195E+06
	Rank	**1**	4	9	11	15	14	12	10	5	8	13	7	3	6	2	16
	(Std Dev)	(5.243474E+01)	(2.598430E+04)	(3.256028E+04)	(2.089463E+05)	(9.768797E+05)	(2.981630E+05)	(1.177577E+05)	(5.266913E+05)	(3.012595E+04)	(2.272331E+04)	(1.221961E+05)	(2.828793E+04)	(2.663214E+04)	(4.588788E+05)	(7.812738E+02)	(1.002252E+06)
f_{15}	Mean	**2.710290E+02**	4.526693E+03	3.336647E+03	4.512569E+02	1.289466E+03	1.625163E+03	1.775826E+03	3.415861E+03	4.606217E+02	2.748653E+03	1.575161E+03	3.178293E+03	2.559559E+03	1.392104E+04	1.544206E+03	3.688901E+03
	Rank|WRST	**1**	15 | >	12 | >	2 | >	4 | >	7 | >	8 | >	13 | >	3 | >	10 | >	6 | >	11 | >	9 | >	16 | >	5 | >	14 | >
	Min	1.707282E+02	**9.595420E+01**	3.275372E+02	2.860746E+02	1.739057E+02	2.839891E+02	2.582611E+02	5.066093E+02	2.240337E+02	2.320097E+02	2.038905E+02	4.136414E+02	2.119182E+02	5.005527E+02	3.206811E+02	2.374114E+02
	Rank	2	**1**	13	11	3	10	9	16	6	7	4	14	5	15	12	8
	Median	**2.749696E+02**	2.225905E+03	2.562736E+03	4.138292E+02	7.132057E+02	8.976788E+02	1.059406E+03	1.881596E+03	4.112913E+02	1.502111E+03	9.427705E+02	2.150164E+03	2.105578E+03	1.545527E+03	7.828558E+02	2.339424E+03
	Rank	**1**	14	16	3	4	6	8	11	2	9	7	13	12	10	5	15
	(Std Dev)	(4.598264E+01)	(5.880076E+03)	(3.498727E+03)	(1.617110E+02)	(1.592061E+03)	(1.705649E+03)	(1.922095E+03)	(4.122707E+03)	(1.856464E+02)	(2.821195E+03)	(1.730554E+03)	(3.256182E+03)	(2.756737E+03)	(6.729121E+04)	(1.962879E+03)	(4.652265E+03)
f_{16}	Mean	2.241687E+03	3.296420E+03	2.738257E+03	3.050138E+03	3.208594E+03	**1.815460E+03**	2.906981E+03	3.806357E+03	3.019598E+03	2.234547E+03	2.481079E+03	2.324994E+03	3.602928E+03	3.707932E+03	3.737924E+03	4.363287E+03
	Rank|WRST	3	11 | >	6 | >	9 | >	10 | >	**1** | <	7 | >	15 | >	8 | >	2 | =	5 | >	4 | =	12 | >	13 | >	14 | >	16 | >
	Min	1.090943E+03	1.950252E+03	1.501247E+03	2.128678E+03	1.154714E+03	**7.006742E+02**	1.390173E+03	1.773103E+03	2.007111E+03	1.138355E+03	1.170762E+03	1.239468E+03	2.234027E+03	2.607666E+03	1.861849E+03	2.865861E+03
	Rank	2	11	8	13	4	**1**	7	9	12	3	5	6	14	15	10	16
	Median	2.141588E+03	3.346539E+03	2.643052E+03	3.063843E+03	2.810798E+03	**1.755484E+03**	2.935413E+03	3.786892E+03	3.002789E+03	2.185472E+03	2.608987E+03	2.372422E+03	3.607262E+03	3.796032E+03	3.877052E+03	4.388312E+03
	Rank	2	11	6	10	7	**1**	8	13	9	3	5	4	12	14	15	16
	(Std Dev)	(6.802600E+02)	(6.583934E+02)	(5.656181E+02)	(4.149591E+02)	(1.280796E+03)	(5.658399E+02)	(5.820512E+02)	(7.693767E+02)	(5.017451E+02)	(5.294943E+02)	(4.344719E+02)	(5.850158E+02)	(7.690872E+02)	(6.012344E+02)	(6.624089E+02)	(6.265847E+02)
f_{17}	Mean	1.935815E+03	2.509598E+03	2.380294E+03	2.499166E+03	3.183538E+03	1.877182E+03	2.239101E+03	2.915022E+03	2.347233E+03	2.075379E+03	**1.811434E+03**	2.021518E+03	2.754882E+03	3.029363E+03	2.941242E+03	3.083917E+03
	Rank|WRST	3	10 | >	8 | >	9 | >	16 | >	2 | =	6 | >	12 | >	7 | >	5 | =	**1** | =	4 | =	11 | >	14 | >	13 | >	15 | >
	Min	1.216217E+03	1.273710E+03	1.475014E+03	1.745520E+03	1.310004E+03	9.603858E+02	1.334724E+03	1.753796E+03	1.514661E+03	1.051134E+03	**9.089226E+02**	1.053137E+03	1.630728E+03	1.831241E+03	1.957376E+03	1.984026E+03
	Rank	5	6	9	12	7	2	8	13	10	3	**1**	4	11	14	15	16
	Median	1.920935E+03	2.505797E+03	2.353091E+03	2.499260E+03	3.387291E+03	**1.718183E+03**	2.161338E+03	2.834980E+03	2.386422E+03	2.049472E+03	1.965745E+03	2.064117E+03	2.666503E+03	3.025941E+03	2.832895E+03	3.079973E+03
	Rank	2	10	7	9	16	**1**	6	13	8	4	3	5	11	14	12	15
	(Std Dev)	(4.107126E+02)	(5.443550E+02)	(5.086003E+02)	(3.073789E+02)	(6.925713E+02)	(5.813149E+02)	(5.284980E+02)	(5.320354E+02)	(3.633905E+02)	(3.769236E+02)	(3.622874E+02)	(5.180622E+02)	(5.739519E+02)	(5.552759E+02)	(6.007186E+02)	(5.355847E+02)
f_{18}	Mean	**1.963980E+02**	1.546744E+05	4.072708E+05	4.130087E+05	2.523096E+06	9.014840E+05	2.903047E+05	5.085486E+05	1.029519E+05	3.411825E+05	5.841477E+05	1.476794E+05	1.410954E+05	2.335894E+05	7.168210E+02	3.072176E+06
	Rank|WRST	**1**	6 | >	10 | >	11 | >	15 | >	14 | >	8 | >	12 | >	3 | >	9 | >	13 | >	5 | >	4 | >	7 | >	2 | >	16 | >
	Min	**1.358005E+02**	1.528068E+04	1.043475E+05	1.741603E+05	3.493304E+05	3.759367E+05	1.387636E+05	5.735596E+04	2.389118E+04	1.754990E+05	1.665840E+05	6.247314E+04	2.641574E+04	5.313542E+04	2.060179E+02	6.820645E+05
	Rank	**1**	3	9	12	14	15	10	7	4	13	11	8	5	6	2	16
	Median	**1.936293E+02**	1.500155E+05	3.461477E+05	3.226638E+05	2.368746E+06	9.018504E+05	2.748661E+05	3.355079E+05	9.893695E+04	3.005534E+05	5.780843E+05	1.340227E+05	1.135970E+05	2.052589E+05	7.067405E+02	2.594430E+06
	Rank	**1**	6	12	10	15	14	8	11	3	9	13	5	4	7	2	16
	(Std Dev)	(4.311886E+01)	(7.293543E+04)	(2.535679E+05)	(3.683044E+05)	(1.465049E+06)	(2.581501E+05)	(1.109491E+05)	(5.513571E+05)	(4.909578E+04)	(1.391000E+05)	(3.003642E+05)	(4.852789E+04)	(1.430739E+05)	(1.019856E+05)	(2.441248E+02)	(1.647362E+06)

(*continued on next page*)

Table 13 (*continued*).

	Mean	1.299479E+03	6.179678E+03	3.187112E+03	**2.354223E+02**	1.259194E+03	1.727089E+03	2.253689E+03	2.738716E+03	3.350084E+02	4.193815E+03	1.071904E+03	7.059472E+03	2.614868E+03	1.565694E+04	7.126450E+02	3.634971E+03
	Rank|WRST	6	14 | >	11 | >	**1** | <	5 | <	7 | =	8 | =	10 | =	2 | <	13 | =	4 | <	15 | >	9 | =	16 | =	3 | <	12 | =
	Min	5.419722E+02	**4.991080E+01**	2.013344E+02	1.385790E+02	6.028750E+01	1.705472E+02	2.258367E+02	2.179620E+02	9.780360E+01	1.826873E+02	1.546732E+02	3.732059E+02	1.766378E+02	1.439062E+02	2.606811E+02	1.322059E+02
f_{19}	Rank	16	**1**	11	5	2	8	13	12	3	10	7	15	9	6	14	4
	Median	1.247585E+03	3.129182E+03	1.964370E+03	**2.092495E+02**	6.851393E+02	1.089975E+03	1.475098E+03	1.649574E+03	2.686425E+02	1.854518E+03	8.941984E+02	2.801601E+03	1.541998E+03	1.183961E+03	4.306047E+02	1.404254E+03
	Rank	8	16	14	**1**	4	6	10	12	2	13	5	15	11	7	3	9
	(Std Dev)	(3.717389E+02)	(7.713394E+03)	(3.299801E+03)	(9.644997E+01)	(1.481895E+03)	(1.742057E+03)	(2.386114E+03)	(3.291869E+03)	(2.788003E+02)	(7.140637E+03)	(9.968567E+02)	(9.135703E+03)	(3.212946E+03)	(9.918280E+04)	(9.220962E+02)	(5.161984E+03)
	Mean	**9.669799E+02**	2.771318E+03	2.089969E+03	2.171879E+03	2.852494E+03	1.577711E+03	1.956728E+03	2.346435E+03	2.170303E+03	1.949220E+03	1.695160E+03	2.307008E+03	2.423352E+03	2.693095E+03	2.518449E+03	2.857013E+03
	Rank|WRST	**1**	14 | >	6 | >	8 | >	15 | >	2 | >	5 | >	10 | >	7 | >	4 | >	3 | >	9 | >	11 | >	13 | >	12 | >	16 | >
	Min	**4.133975E+02**	1.711712E+03	9.698344E+02	1.224681E+03	1.276120E+03	5.456228E+02	1.028154E+03	1.479550E+03	1.399116E+03	8.283515E+02	1.199661E+03	1.473900E+03	1.357568E+03	1.422247E+03	1.492949E+03	1.301869E+03
f_{20}	Rank	**1**	16	4	7	8	2	5	14	11	3	6	13	10	12	15	9
	Median	**9.715813E+02**	2.697845E+03	2.096279E+03	2.212459E+03	2.897948E+03	1.434473E+03	1.927868E+03	2.218883E+03	2.126444E+03	1.940868E+03	1.653430E+03	2.330935E+03	2.512911E+03	2.615614E+03	2.333954E+03	2.912289E+03
	Rank	**1**	14	6	8	15	2	4	9	7	5	3	10	12	13	11	16
	(Std Dev)	(3.057151E+02)	(6.073304E+02)	(4.361261E+02)	(3.571738E+02)	(5.754680E+02)	(6.074562E+02)	(5.074802E+02)	(5.016335E+02)	(3.329637E+02)	(5.171570E+02)	(3.478097E+02)	(4.534124E+02)	(5.121307E+02)	(5.026461E+02)	(5.008093E+02)	(6.182096E+02)
	Mean	**3.044532E+02**	5.598480E+02	5.973953E+02	5.484248E+02	3.631213E+02	3.695485E+02	4.696431E+02	4.539149E+02	5.601605E+02	3.441315E+02	4.117522E+02	4.257006E+02	6.793177E+02	7.252071E+02	6.168691E+02	8.635586E+02
	Rank|WRST	**1**	10 | >	12 | >	9 | >	3 | >	4 | >	8 | >	7 | >	11 | >	2 | >	5 | >	6 | >	14 | >	15 | >	13 | >	16 | >
	Min	**2.792985E+02**	4.302537E+02	4.991648E+02	4.580709E+02	2.848632E+02	2.895606E+02	3.797519E+02	3.708891E+02	4.144837E+02	3.074542E+02	3.609484E+02	3.719531E+02	5.475823E+02	5.397130E+02	4.689651E+02	6.906699E+02
f_{21}	Rank	**1**	10	13	11	2	3	8	6	9	4	5	7	15	14	12	16
	Median	**3.060234E+02**	5.611310E+02	5.956805E+02	5.554356E+02	3.445218E+02	3.213685E+02	4.671392E+02	4.475105E+02	5.695503E+02	3.391396E+02	4.127871E+02	4.215593E+02	6.584748E+02	7.194585E+02	6.186922E+02	8.778804E+02
	Rank	**1**	10	12	9	4	2	8	7	11	3	5	6	14	15	13	16
	(Std Dev)	(1.158012E+01)	(5.082459E+01)	(4.568848E+01)	(5.195265E+01)	(9.084503E+01)	(1.367787E+02)	(3.481704E+01)	(3.849718E+01)	(6.643213E+01)	(2.397524E+01)	(2.181295E+01)	(3.102729E+01)	(7.128606E+01)	(7.722684E+01)	(6.513965E+01)	(9.040776E+01)
	Mean	9.710973E+03	1.418216E+04	1.371162E+04	1.163088E+04	1.537947E+04	1.104081E+04	9.484935E+03	1.421154E+04	1.165962E+04	**8.784887E+03**	1.236005E+04	1.272828E+04	1.333329E+04	1.368769E+04	1.537742E+04	1.565683E+04
	Rank|WRST	3	12 | >	11 | >	5 | >	15 | >	4 | >	2 | <	13 | >	6 | >	**1** | =	7 | >	8 | >	9 | >	10 | >	14 | >	16 | >
	Min	**1.000000E+02**	9.945811E+03	**1.000000E+02**	**1.000000E+02**	1.020106E+04	**1.000000E+02**	**1.000000E+02**	1.117412E+04	**1.000000E+02**	**1.000000E+02**	9.896317E+03	**1.000000E+02**	**1.000000E+02**	1.788580E+02	1.233578E+04	**1.000000E+02**
f_{22}	Rank	**1**	13	**1**	**1**	14	**1**	**1**	15	**1**	**1**	12	**1**	**1**	11	16	**1**
	Median	**1.021458E+04**	1.426949E+04	1.425071E+04	1.222001E+04	1.542567E+04	1.127193E+04	1.149125E+04	1.406479E+04	1.197805E+04	1.178913E+04	1.230523E+04	1.406123E+04	1.351700E+04	1.460632E+04	1.588028E+04	1.652940E+04
	Rank	**1**	12	11	6	14	2	3	10	5	4	7	9	8	13	15	16
	(Std Dev)	(3.739195E+03)	(1.579898E+03)	(3.094333E+03)	(3.015284E+03)	(1.464679E+03)	(2.909620E+03)	(5.157093E+03)	(1.287870E+03)	(2.452216E+03)	(6.037403E+03)	(1.004693E+03)	(3.992177E+03)	(2.304083E+03)	(4.682444E+03)	(1.328577E+03)	(3.399536E+03)
	Mean	6.644672E+02	8.229085E+02	9.256755E+02	7.810940E+02	**6.610763E+02**	8.040219E+02	9.275248E+02	8.345288E+02	7.862620E+02	8.665998E+02	8.135397E+02	7.367721E+02	8.547323E+02	1.073509E+03	8.685679E+02	1.308985E+03
	Rank|WRST	2	8 | >	13 | >	4 | >	**1** | =	6 | >	14 | >	9 | >	5 | >	11 | >	7 | >	3 | >	10 | >	15 | >	12 | >	16 | >
	Min	6.255332E+02	7.386669E+02	7.322370E+02	7.217548E+02	**6.161436E+02**	6.737825E+02	7.894672E+02	7.454090E+02	7.074549E+02	6.777439E+02	7.414942E+02	6.747321E+02	7.576940E+02	8.706891E+02	7.639751E+02	9.959379E+02
f_{23}	Rank	2	9	8	7	**1**	3	14	11	6	5	10	4	12	15	13	16
	Median	6.653771E+02	8.184361E+02	9.238846E+02	7.833088E+02	**6.580034E+02**	8.060795E+02	9.293329E+02	8.312565E+02	7.924666E+02	8.458510E+02	8.128962E+02	7.336848E+02	8.523969E+02	1.052287E+03	8.557617E+02	1.272653E+03
	Rank	2	8	13	4	**1**	6	14	9	5	10	7	3	11	15	12	16
	(Std Dev)	(1.836327E+01)	(5.475395E+01)	(8.198523E+01)	(2.946696E+01)	(2.110250E+01)	(6.137325E+01)	(6.831650E+01)	(4.721912E+01)	(2.801895E+01)	(1.246574E+02)	(3.513063E+01)	(3.347120E+01)	(5.990176E+01)	(1.582435E+02)	(7.145884E+01)	(1.991108E+02)
	Mean	1.067654E+03	1.213028E+03	1.197361E+03	1.292176E+03	**1.044164E+03**	1.239268E+03	1.264117E+03	1.229927E+03	1.277210E+03	1.750477E+03	1.095556E+03	1.083815E+03	1.404019E+03	1.492304E+03	1.324893E+03	2.022375E+03
	Rank|WRST	2	6 | >	5 | >	11 | >	**1** | <	8 | >	9 | >	7 | >	10 | >	15 | >	4 | >	3 | >	13 | >	14 | >	12 | >	16 | >
	Min	9.970698E+02	1.095681E+03	1.075448E+03	1.194192E+03	**9.843727E+02**	1.011868E+03	1.120025E+03	1.096136E+03	1.151789E+03	1.124661E+03	1.031350E+03	1.018605E+03	1.251320E+03	1.194473E+03	1.178761E+03	1.627586E+03
f_{24}	Rank	2	7	6	13	**1**	3	9	8	11	10	5	4	15	14	12	16
	Median	1.065871E+03	1.216779E+03	1.197443E+03	1.296290E+03	**1.034655E+03**	1.201553E+03	1.252960E+03	1.194541E+03	1.285035E+03	1.643991E+03	1.090560E+03	1.083432E+03	1.386991E+03	1.444459E+03	1.308004E+03	1.979633E+03
	Rank	2	8	6	11	**1**	7	9	5	10	15	4	3	13	14	12	16
	(Std Dev)	(3.489757E+01)	(6.186875E+01)	(5.380583E+01)	(3.749902E+01)	(3.325854E+01)	(1.529611E+02)	(8.632293E+01)	(1.091485E+02)	(4.575649E+01)	(4.492204E+02)	(3.728692E+01)	(3.408858E+01)	(7.473348E+01)	(2.127750E+02)	(1.299738E+02)	(2.494277E+02)
	Mean	7.269874E+02	7.530810E+02	7.574444E+02	7.531074E+02	1.055265E+03	7.624201E+02	8.107970E+02	1.151985E+03	7.416585E+02	7.854404E+02	7.623352E+02	7.302654E+02	7.646014E+02	1.782852E+03	**6.748131E+02**	1.466246E+03
	Rank|WRST	2	5 | >	7 | >	6 | >	13 | >	9 | >	12 | >	14 | >	4 | =	11 | >	8 | >	3 | =	10 | >	16 | >	**1** | <	15 | >
	Min	6.222516E+02	6.224503E+02	**5.655670E+02**	6.202112E+02	9.572026E+02	6.693541E+02	6.120263E+02	9.739218E+02	6.382995E+02	6.376878E+02	6.072248E+02	5.870758E+02	6.149157E+02	1.147868E+03	5.984525E+02	1.032836E+03
f_{25}	Rank	8	9	**1**	7	13	12	5	14	11	10	4	2	6	16	3	15
	Median	7.147079E+02	7.622250E+02	7.485033E+02	7.602441E+02	1.053802E+03	7.635938E+02	8.158202E+02	1.164659E+03	7.433641E+02	8.008218E+02	7.688796E+02	7.296715E+02	7.756539E+02	1.753056E+03	**6.789324E+02**	1.412712E+03
	Rank	2	7	5	6	13	8	12	14	4	11	9	3	10	16	**1**	15
	(Std Dev)	(5.943349E+01)	(5.572859E+01)	(6.050414E+01)	(5.595644E+01)	(4.388144E+01)	(4.077483E+01)	(8.250289E+01)	(9.548665E+01)	(5.312653E+01)	(6.268132E+01)	(5.736663E+01)	(5.869115E+01)	(6.092289E+01)	(3.728594E+02)	(5.789273E+01)	(2.946669E+02)
	Mean	5.259708E+03	6.800547E+03	**2.796457E+03**	7.074986E+03	4.582461E+03	4.380775E+03	7.069580E+03	5.613912E+03	7.032478E+03	2.974099E+03	4.983282E+03	5.427933E+03	9.069079E+03	1.459456E+04	7.226740E+03	1.385014E+04
	Rank|WRST	6	9 | >	**1** | <	12 | >	4 | <	3 | <	11 | >	8 | =	10 | >	2 | <	5 | <	7 | =	14 | >	16 | >	13 | >	15 | >
	Min	3.000001E+02	5.370989E+03	**3.000000E+02**	1.389043E+03	3.942841E+03	3.317904E+03	5.670229E+03	4.561158E+03	**3.000000E+02**	**3.000000E+02**	3.036107E+02	4.683774E+03	**3.000000E+02**	2.124574E+03	5.853261E+03	**3.000000E+02**
f_{26}	Rank	6	14	**1**	8	11	10	15	12	**1**	**1**	7	13	**1**	9	16	**1**
	Median	5.339353E+03	6.936609E+03	**3.000000E+02**	7.407845E+03	4.591729E+03	4.056292E+03	6.976187E+03	5.543484E+03	7.358852E+03	4.025535E+03	5.083468E+03	5.344004E+03	8.587290E+03	1.598898E+04	7.158684E+03	1.431132E+04
	Rank	6	9	**1**	13	4	3	10	8	12	2	5	7	14	16	11	15
	(Std Dev)	(1.540013E+03)	(6.048676E+02)	(4.292933E+03)	(1.326235E+03)	(2.825049E+02)	(1.326714E+03)	(6.781498E+02)	(5.575364E+02)	(1.604280E+03)	(1.967389E+03)	(7.419165E+02)	(5.152357E+02)	(2.896206E+03)	(4.842120E+03)	(7.623426E+02)	(4.333061E+03)

(*continued on next page*)

Table 13 (*continued*).

	Mean	8.874994E+02	7.464626E+02	7.639827E+02	7.775312E+02	7.731675E+02	**5.096867E+02**	9.670027E+02	1.110445E+03	7.916897E+02	7.317767E+02	7.227145E+02	6.366901E+02	8.540341E+02	1.000459E+03	9.611440E+02	1.686011E+03
	Rank|WRST	11	5 | <	6 | <	8 | <	7 | <	**1** | <	13 | >	15 | >	9 | <	4 | <	3 | <	2 | <	10 | <	14 | >	12 | >	16 | >
	Min	7.026448E+02	6.310963E+02	6.584999E+02	7.174116E+02	7.095285E+02	**4.992444E+02**	8.128783E+02	8.798982E+02	7.142213E+02	6.390137E+02	6.462791E+02	5.852119E+02	7.056498E+02	7.359965E+02	7.023805E+02	1.082604E+03
f_{27}	Rank	8	3	6	12	10	**1**	14	15	11	4	5	2	9	13	7	16
	Median	8.766401E+02	7.449026E+02	7.583715E+02	7.739611E+02	7.737564E+02	**5.000237E+02**	9.582217E+02	1.123494E+03	7.954166E+02	7.231863E+02	7.203786E+02	6.389803E+02	8.548044E+02	9.956065E+02	9.512780E+02	1.632262E+03
	Rank	11	5	6	8	7	**1**	13	15	9	4	3	2	10	14	12	16
	(Std Dev)	(7.375709E+01)	(4.609171E+01)	(5.405686E+01)	(3.315784E+01)	(3.199009E+01)	(3.913236E+01)	(9.284606E+01)	(1.079144E+02)	(3.843071E+01)	(7.721467E+01)	(3.562761E+01)	(2.136648E+01)	(7.036403E+01)	(1.139919E+02)	(1.203416E+02)	(3.297723E+02)
	Mean	**4.594768E+02**	5.117495E+02	6.256084E+02	5.661646E+02	8.037634E+02	5.531038E+02	5.966730E+02	1.236508E+03	5.483428E+02	5.709389E+02	5.708589E+02	5.319600E+02	5.428186E+02	2.578854E+03	4.936113E+02	1.899783E+03
	Rank|WRST	**1**	3 | =	12 | >	8 | >	13 | >	7 | >	11 | >	14 | >	6 | >	10 | >	9 | >	4 | >	5 | >	16 | >	2 | =	15 | >
	Min	**3.000000E+02**	4.123037E+02	4.944657E+02	5.022441E+02	7.407060E+02	5.008770E+02	5.353256E+02	7.714880E+02	4.783479E+02	5.149757E+02	5.265581E+02	4.792021E+02	4.783330E+02	1.175714E+03	3.000025E+02	9.977732E+02
f_{28}	Rank	**1**	3	7	9	13	8	12	14	5	10	11	6	4	16	2	15
	Median	5.186428E+02	5.081206E+02	5.991714E+02	5.640794E+02	7.965364E+02	5.543534E+02	5.899971E+02	1.222913E+03	5.487747E+02	5.641984E+02	5.616070E+02	5.268148E+02	5.426484E+02	2.481295E+03	**5.041180E+02**	1.822062E+03
	Rank	3	2	12	9	13	7	11	14	6	10	8	4	5	16	**1**	15
	(Std Dev)	(1.188810E+02)	(4.065894E+01)	(7.622714E+01)	(3.081412E+01)	(3.851793E+01)	(2.839327E+01)	(3.031675E+01)	(2.127955E+02)	(2.432281E+01)	(3.610340E+01)	(2.395913E+01)	(2.295091E+01)	(3.393050E+01)	(8.183224E+02)	(5.247819E+01)	(5.952889E+02)
	Mean	3.576865E+03	3.016040E+03	3.173332E+03	2.788957E+03	2.912891E+03	2.348806E+03	3.387520E+03	3.326787E+03	2.945813E+03	2.446979E+03	**2.249054E+03**	2.344684E+03	3.403517E+03	3.658635E+03	4.133176E+03	4.197733E+03
	Rank|WRST	13	8 | <	9 | <	5 | <	6 | <	3 | <	11 | <	10 | <	7 | <	4 | <	**1** | <	2 | <	12 | <	14 | =	15 | >	16 | >
	Min	2.918134E+03	1.893605E+03	2.195028E+03	2.025145E+03	1.752116E+03	**1.102997E+03**	2.316237E+03	2.196021E+03	2.032908E+03	1.557648E+03	1.465764E+03	1.321818E+03	1.759583E+03	2.556858E+03	2.373039E+03	3.087135E+03
f_{29}	Rank	15	7	10	8	5	**1**	12	11	9	4	3	2	6	14	13	16
	Median	3.558939E+03	3.009926E+03	3.159587E+03	2.777226E+03	2.785818E+03	**1.811996E+03**	3.380428E+03	3.387132E+03	2.931271E+03	2.449920E+03	2.193249E+03	2.349141E+03	3.388467E+03	3.717925E+03	4.017928E+03	4.208391E+03
	Rank	13	8	9	5	6	**1**	10	11	7	4	2	3	12	14	15	16
	(Std Dev)	(3.631130E+02)	(5.817658E+02)	(4.561278E+02)	(3.171680E+02)	(7.677502E+02)	(1.225630E+03)	(4.653844E+02)	(5.786672E+02)	(4.083424E+02)	(3.725187E+02)	(4.407016E+02)	(4.084528E+02)	(5.272704E+02)	(5.028808E+02)	(6.693745E+02)	(5.073675E+02)
	Mean	6.014818E+03	6.706797E+03	5.410149E+04	5.723414E+03	1.697454E+04	**3.421253E+03**	1.009331E+04	1.558838E+05	5.560417E+03	7.612688E+03	6.083851E+03	8.625665E+03	7.039910E+03	3.498785E+06	6.002578E+03	1.500260E+06
	Rank|WRST	5	7 | =	13 | >	3 | =	12 | >	**1** | <	11 | >	14 | >	2 | <	9 | =	6 | =	10 | =	8 | =	16 | >	4 | <	15 | >
	Min	4.850095E+03	2.333671E+03	6.819248E+03	3.064373E+03	7.201169E+03	**4.129267E+02**	4.262776E+03	1.268794E+04	2.583724E+03	2.780741E+03	2.990264E+03	3.928463E+03	2.566455E+03	3.862692E+03	2.713673E+03	3.446987E+04
f_{30}	Rank	12	2	13	8	14	**1**	11	15	4	6	7	10	3	9	5	16
	Median	5.799981E+03	5.009443E+03	3.717948E+04	5.242274E+03	1.422597E+04	**2.507654E+03**	9.561023E+03	7.528077E+04	4.634252E+03	6.148183E+03	5.803831E+03	6.793722E+03	4.959387E+03	1.958513E+05	4.617084E+03	2.392544E+05
	Rank	7	5	13	6	12	**1**	11	14	3	9	8	10	4	15	2	16
	(Std Dev)	(8.637362E+02)	(4.149065E+03)	(5.427302E+04)	(2.170464E+03)	(8.574379E+03)	(2.989637E+03)	(4.174967E+03)	(1.784902E+05)	(2.455499E+03)	(4.575813E+03)	(2.129423E+03)	(5.000071E+03)	(4.406379E+03)	(8.294886E+06)	(3.529247E+03)	(6.440677E+06)

5. Real-world application and computational complexity

5.1. Long-term transmission network expansion planning problem

The GECCO 2023 and IEEE CEC 2023 competition provides a Direct Current (DC) model, a nonconvex mixed-integer nonlinear programming problem, which is used to formulate the TNEP problem. The objective is to find the optimal expansion plan that identifies the transmission lines needed for installation in the electrical system, ensuring adequate operation within a predefined planning horizon at the lowest investment cost [21]. To ensure consistency in the comparison, we follow the instructions and constraints provided by [21]. That is, minimize v in Eq. (26).

$$v = \sum_{ij\in\Omega_B} C_{ij}n_{ij} + \gamma \sum_{i\in\Omega_N} r_i \tag{26}$$

Subject to:

$$D_i = \sum_{ji\in\Omega_B} f_{ji} - \sum_{ij\in\Omega_B} f_{ij} + G_i + r_i,\ \forall i \in \Omega_N \tag{27}$$

$$f_{ij} = (N^o_{ij} + n_{ij})\frac{(\theta_i - \theta_j)}{X_{ij}},\ \forall ij \in \Omega_B \tag{28}$$

$$|f_{ij}| \le \overline{F}_{ij}(N^o_{ij} + n_{ij}),\ \forall ij \in \Omega_B \tag{29}$$

$$0 \le r_i \le D_i,\ \forall i \in \Omega_N \tag{30}$$

$$0 \le n_{ij} \le \overline{N}_{ij},\ \forall ij \in \Omega_B \tag{31}$$

$$n_{ij} \in \mathbb{Z},\ \forall ij \in \Omega_B \tag{32}$$

Here, Ω_N represents the set of buses, and Ω_B represents the set of branches (or corridors). C_{ij} is the cost of constructing a line on corridor ij, γ is the load shedding cost, G_i is the active power generation at bus i, D_i is the active demand at bus i, N^o_{ij} is the number of existing lines on corridor ij, X_{ij} is the reactance of a line on corridor ij, $\overline{F}_{ij}$ is the capacity of a line on corridor ij, and $\overline{N}_{ij}$ is the maximum number of lines that can be constructed on corridor ij. n_{ij} is the number of lines constructed on corridor ij, r_i is the load shedding at bus i, f_{ij} is the active power flow on corridor ij, and θ_i is the voltage phase angle at bus i. For more details, please refer to [21].

According to the instructions in [21], to make a fair comparison, the $maxFEs$ is set to 2×10^4 for all compared PSO-based variants. The experimental results are given in Table 7. PSO-BHF ranks 2nd among all 16 state-of-the-art PSO-based variants, illustrating very competitive performance.

5.2. Computational complexity of algorithms

The computational complexity of the PSO-BHF algorithm was evaluated in accordance with the criteria specified in the CEC2017 benchmark competition, as suggested by [42]. All experiments were carried out on the following system: • CPU: Intel Core i7-8750 h @ 2.20 GHz; • RAM: 16 GB; • OS: Windows 10; • Software: MATLAB 2020b. The computational complexity of the PSO-BHF for various dimensions is presented in Table 8, where T_0 denotes the time required to execute the following statements:

$x = 0.55;$
$for\ i = 1 : 1000000$
$x = x + x;\ x = x/2;\ x = x * x;$
$x = sqrt(x);\ x = log(x);\ x = exp(x);\ x = x/(x + 2);$
end

T_1 represents the time taken to perform benchmark function f_{18} individually with 2×10^5 evaluations in D dimensions. T_2 is the PSO-BHF algorithm runtime on function f_{18}, using 2×10^5 evaluations, in D dimensions. The time ($\hat{T}_2$) corresponds to the averaged mean values of T_2, obtained from five runs. As depicted in Table 8, T_1, $\hat{T}_2$, and $((\hat{T}_2 - T_1)/T_0)$ demonstrate linear scaling concerning the number of dimensions.

To ensure a fair comparison of computational complexity, Table 9 displays computational complexity comparisons between the PSO-BHF algorithm and various PSO-based variants, all of which were evaluated in 30 dimensions. The same system and procedure were consistently applied to calculate the computational complexity for each algorithm under comparison. In Table 9, while the computational complexity of PSO-BHF may not be the best, it does outperform EAPSO, PSO-ELPM, and HIDMS-PSO in this regard. Considering that the performance of the algorithm has been significantly improved, such computational complexity is acceptable.

6. Conclusions

This study introduces a PSO-based hybrid algorithm that uses an opposition-based learning mechanism, a success history intelligent optimizer mechanism, CMA-ES, and the sequential quadratic programming method. The opposition-based learning mechanism is responsible for escaping local optima owing to the extension of search regions. The success history intelligent optimizer mechanism effectively prevents premature convergence, and it does this by guiding the search using information from the three best particles. The sequential quadratic programming method intervenes in subsequent generations of the evolutionary process, and its purpose is to reinforce the exploitation capability around the current best particle. To better control the timing of the CMA-ES intervention, a probability calculation for using CMA-ES is proposed. Ablation experiments were conducted to validate the effectiveness of each individual mechanism and the probability calculation for using CMA-ES. The experimental results illustrate that PSO-BHF performs better when integrated with these mechanisms than when using individual mechanisms alone. Furthermore, the integration of this framework is effective due to the collaboration among different mechanisms, and these ideas can be considered for application in other evolutionary algorithms. All PSO-based variants were tested on CEC2017 benchmark functions with varying dimensions. PSO-BHF was also compared with 11 non-PSO algorithms. The experimental results consistently demonstrate the outstanding performance of PSO-BHF in most cases. In real-world applications of the long-term TNEP problem, PSO-BHF ranks second among the selected PSO-based variants, illustrating its robust and flexible ability. The current experimental results on the CEC2017 benchmark functions and the TNEP problem use the same parameter settings. It is suggested that, when applying the method to other real-world problems, slight adjustments may be needed for the timing of the SQP method intervention and the population size. The Friedman rank, in which PSO-BHF ranks fourth in the 10-dimensional test, suggests room for improvement in low-dimensional tests. Additionally, PSO-BHF does not demonstrate outstanding performance on a few benchmark functions, indicating that the current integrated mechanisms do not account for all search situations. Although PSO-BHF does not surpass the performance of DE-based variants, which sit in the top tier of metaheuristics, it has significantly narrowed the gap and shown promise in catching up. Potential future research directions for PSO-BHF include integrating novel mechanisms, refining the timing of the triggering mechanism, combining with opposition-based learning variants, enhancing global search mechanisms, developing a dynamic ensemble strategy using sequential quadratic programming and interior-point methods, and applying these techniques to real-world optimization challenges.

CRediT authorship contribution statement

Libin Hong: Writing – review & editing, Writing – original draft, Supervision, Software, Project administration, Methodology, Investigation, Formal analysis, Conceptualization. **Guodong Wang:** Methodology, Investigation, Conceptualization. **Ruibin Bai:** Writing – review & editing.

Declaration of competing interest

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Data availability

Data will be made available on request.

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