

Research paper

An ensemble velocity learning strategy for particle swarm optimization integrating multiple local search mechanisms

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ABSTRACT

Particle Swarm Optimization (PSO) is an effective and efficient metaheuristic algorithm that has been widely applied to valuable real-world applications. Since PSO was proposed nearly three decades, many PSO-based variants have been developed. In recent years, research on ensemble complementary operators and variants aimed at enhancing the performance of PSO has maintained a certain level of enthusiasm. In this work, an ensemble velocity learning strategy is proposed and combined with two local search methods: the Improved Sine Cosine Algorithm (ISCA) and Sequential Quadratic Programming (SQP). Meanwhile, to improve the effectiveness of the ensemble velocity learning strategy, elite learning, modified inertia weight, modified acceleration coefficients, and mutation strategy are incorporated. The proposed metaheuristic algorithm, named the Ensemble Velocity Learning Strategy for Particle Swarm Optimization Integrating Multiple Local Search Mechanisms (EVLS-PSOIMLSM), is tested on the Institute of Electrical and Electronics Engineers (IEEE) Congress on Evolutionary Computation 2017 (CEC2017) benchmark functions and applied to real-world problems, including the Gear Train Design (GTD) and the Planetary Gear Train Design (PGTD) problem. Compared to 14 recently proposed PSO-based variants and 11 non-PSO algorithms, the experimental results demonstrate that EVLS-PSOIMLSM achieves superior performance. The source code of EVLS-PSOIMLSM is provided at <https://github.com/microhard1999/CODES>.

1. Introduction

A metaheuristic is a high-level problem-solving strategy that guides lower-level heuristics to efficiently explore and exploit the search space for optimal or near-optimal solutions. It is commonly used for solving complex optimization problems where traditional methods are impractical. Metaheuristics include Particle Swarm Optimization (PSO), Genetic Algorithm (GA), Ant Colony Optimization (ACO), Differential Evolution (DE), and Simulated Annealing (SA), among others. In optimization, exploration refers to the ability of a metaheuristic algorithm to search globally in the solution space, ensuring that diverse regions are investigated to avoid premature convergence to local optima. It helps in discovering promising areas in the search space. Exploitation refers to the ability of a metaheuristic algorithm to refine solutions locally, improving the best-found solutions by intensifying the search in a specific region. It ensures convergence to an optimal or near-optimal solution (Eiben and Schippers, 1998; Črepinšek et al., 2013). Metaheuristic algorithms are often used to solve function optimization problems, which involve finding the minimum or maximum of a given

function. In this work, the objective is to find the minimum value, which is defined as:

$$\min_{x \in \mathbb{D}} f(x), \quad (1)$$

where x is the decision variable (or vector of variables). $\mathbb{D} \subseteq \mathbb{R}^n$ represents the feasible domain (search space) of x .

Particle Swarm Optimization (PSO) was proposed nearly three decades ago, and many PSO-based variants have been developed (Wang et al., 2018; Leung and Wang, 2018; Houssein et al., 2021; Shami et al., 2022). PSO-based variants have also been applied to many real-world applications, such as computational finance applications (Chiam et al., 2009), solar photovoltaic systems (Khare and Rangnekar, 2013), engineering problems (Alkayem et al., 2022), electric power systems (Tiwari and Kumar, 2023), vehicle routing problems (Kuo et al., 2023), solar photovoltaic parameter estimation (Qaraad et al., 2024), etc. Researchers have investigated the components of PSO and introduced more intelligent, adaptive, robust mechanisms, operators, and coefficients to construct various variants based on PSO. Among all the

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improvements, one effective methodology is to consolidate complementary mechanisms to build an ensemble strategy, either at the algorithm level (Meng et al., 2022; Hong et al., 2024b,a) or the component level (Yang et al., 2022; Li et al., 2022; Tian et al., 2024).

In Zhang (2023), an Elite Archives-driven PSO (EAPSO) was proposed, and three categories of elite archives were constructed to design learning strategies for velocity updates in six cases. The six cases use historical best particles, promising historical best particles, and promising global best particles to guide velocity updates. In Liu et al. (2020), a modified PSO (MPSO) is proposed using an adaptive strategy. The velocity of MPSO is updated using the “inertial component”, “stochastic learning component”, and “mainstream learning component”. In Zhao and Wang (2022), an Elite-Ordinary synergistic PSO (EOPSO) was proposed. Particles are divided into elite and ordinary categories based on their fitness values. Then, humble E^d is used to calculate the velocity for ordinary particles. Observations suggest that the velocity updating strategy in EAPSO can still be improved to better balance exploration and exploitation. The current six velocity update cases in EAPSO are less efficient due to limited variation in flying speed magnitude. In MPSO, velocity updates are restricted to a single velocity, leading to less adaptive behavior. This lack of velocity variation reduces MPSO’s adaptability when handling different problems. Additionally, the learning exemplars proposed in EOPSO are insufficient to maintain particle diversity. Among all the above PSO-based variants, exploitation remains inadequate, particularly in the later stages of the evolutionary process when particles approach the global optimum. Based on these observations, we aim not only to design a more efficient and effective velocity updating strategy while significantly enhancing the exploitation capability of PSO-based variants but also to bridge the gap between PSO-based and DE-based variants, as DE-based variants rank among the top-tier metaheuristics. Taking inspiration from these studies and overcoming the drawbacks of the variants, the present study addresses the following research objectives:

- Proposing an ensemble velocity learning strategy, which consolidates complementary updating features and can better balance the exploration and exploitation.
- Integrating local search methods combined with effective triggering mechanisms and appropriate timing of intervention.
- The consolidated coefficients, mechanisms, and strategies are synergistic.

This study is to introduce an Ensemble Velocity Learning Strategy for Particle Swarm Optimization Integrating Multiple Local Search Mechanisms (EVLS-PSOIMLSM). This approach amalgamates complementary velocity updating strategies along with various coefficients, mechanisms, operators, and strategies to foster effective and efficient cooperation. The key highlights of this research are outlined below:

- An ensemble velocity learning strategy that incorporates elite learning and information derived from $Pbest$ values is proposed. The designed velocity updating formulas create three types for particle selection, ranging from excellent exploration to excellent exploitation. Particles can adjust their velocity based on the problem’s landscape.
- The modified acceleration coefficient c and inertia weight ω are combined in the velocity updating formula. They not only improve the compatibility of the proposed variant but also better balance the exploration and exploitation capabilities.
- Two local search methods, the Improved Sine Cosine Algorithm (ISCA) and Sequential Quadratic Programming (SQP), are employed in the later evolution to strongly extend the exploitation capability of the proposed variant.
- A mutation strategy is integrated and improved to enhance particle diversity in the middle and later stages of evolution while ensuring compatibility with the proposed algorithm.

- EVLS-PSOIMLSM is compared with 14 recently proposed PSO-based variants and 11 non-PSO algorithms. Two real-world applications are selected to evaluate its performance.

The structure of the remaining sections of this paper is as follows: Section 2 introduces related work, including the classic PSO, EAPSO, velocity learning strategies, and local search methods. Section 3 presents the details of the ensemble velocity learning strategy, including its coefficients, mechanisms, local search methods, and compatible improvements within the proposed algorithm. Section 4 outlines the parameter settings used for all the PSO-based variants examined in this study along with the corresponding experimental results, and this section also encompasses comparisons, analyses, and discussions. Section 5 presents the real-world applications of the proposed PSO-based variants: the gear train design and the planetary gear train design problem. Finally, Section 6 provides a summary of the study and discusses avenues for future research.

2. Related work

2.1. Classic PSO and EAPSO

The fundamental characteristics of particles in classical PSO, namely, velocity and position, are subject to continuous updates for each individual particle (Kennedy and Eberhart, 1995). The velocity $V_i(t+1)$ and position $X_i(t+1)$ of the i th particle at generation $t+1$ are recorded, and their respective formulae are presented as follows:

$$V_i^d(t+1) = V_i^d(t) + c_1 \times r_1(t) \times (Pbest_i^d(t) - X_i^d(t)) + c_2 \times r_2(t) \times (Gbest^d(t) - X_i^d(t)), \quad (2)$$

$$X_i^d(t+1) = X_i^d(t) + V_i^d(t+1), \quad (3)$$

where $r_1(t)$ and $r_2(t)$ are randomly generated from a uniform distribution over the range (0,1), $d = 1, 2, \dots, D$, D is the dimension size, $Pbest_i^d(t)$ is the i th particle’s previous best solution, $i = 1, 2, \dots, N$, N is the population size, and $Gbest^d(t)$ is the whole swarm’s best solution, which are computed as follows:

$$Pbest_i^d(t) = \minarg\{f(X_i^d(1)), f(X_i^d(2)), \dots, f(X_i^d(t))\}, \quad (4)$$

$$Gbest^d(t) = \minarg\{f(Pbest_1^d(t)), f(Pbest_2^d(t)), \dots, f(Pbest_N^d(t))\}. \quad (5)$$

2.2. Velocity strategy in EOPSO

In EOPSO (Zhao and Wang, 2022), the velocity strategy is calculated using Eqs. (6)–(9), and (10).

$$W_i(t) = \begin{cases} \frac{1}{1+f(x_i^d(t))}, & f(x_i^d(t)) \geq 0, \\ 1 + |f(x_i^d(t))|, & f(x_i^d(t)) \leq 0, \end{cases} \quad (6)$$

$$Q^d(t) = \sum_{k=1}^K \frac{W_k(t) \times Pbest_k^d(t)}{\sum_{k=1}^K W_k(t)}, \quad (7)$$

$$E_i^d(t) = Q^d(t), \quad (8)$$

$$\omega(t) = 0.9 - 0.5 \times \frac{t}{T_{max}}, \quad (9)$$

$$V_i^d(t+1) = \omega(t) \times V_i^d(t) + r(t) \times c \times (E_i^d(t) - x_i^d(t)), \quad (10)$$

where $r(t)$ is a value randomly generated in (0, 1), and c represents acceleration coefficient and is set to 1.49445, t is the current generation number, T_{max} is the maximum generation number.

2.3. Velocity learning strategy in MPSPSO

In MPSPSO (Liu et al., 2020), the velocity strategy is calculated using Eqs. (11)–(13), and (14).

$$CPbest^d(t) = \operatorname{argmin}\{f(Pbest_a^d(t)), f(Pbest_b^d(t))\}, a \neq b \in \{1, 2, \dots, N\}, \quad (11)$$

$$SPbest_i^d(t) = \begin{cases} CPbest^d(t) & f(CPbest^d(t)) < f(Pbest_i^d(t)), \\ Pbest_i^d(t) & \text{otherwise,} \end{cases} \quad (12)$$

$$Mbest^d(t) = \operatorname{mean}\{Pbest_1^d(t), Pbest_2^d(t), \dots, Pbest_N^d(t)\}, \quad (13)$$

$$V_i^d(t+1) = \omega(t) \times V_i^d(t) + r_1(t) \times c_1(t) \times (SPbest_i^d(t) - x_i^d(t)) + r_2(t) \times c_2(t) \times (Mbest^d(t) - x_i^d(t)), \quad (14)$$

where $r_1(t)$ and $r_2(t)$ are values randomly generated in (0, 1), $c_1 = c_2 = 2.0$. $\omega(t)$ is calculated using Eq. (15).

$$\omega(t) = r(t) \times \omega_{min} + \frac{t}{T_{max}} \times (\omega_{max} - \omega_{min}), \quad (15)$$

where $r(t)$ is a random number generated by the logistic chaotic (Liu et al., 2020) at t th generation, t is the current generation number, T_{max} is the maximum generation number, $\omega_{max} = 0.9$, and $\omega_{min} = 0.4$.

2.4. Velocity learning strategy in EAPSO

In EAPSO (Zhang, 2023), the particles are sorted according to their fitness values and then are evenly divided into two subpopulations, POP_o and POP_c . POP_o contains the outstanding particles with better fitness values, while POP_c contains the common particles with worse fitness values. All particles in POP_o will directly enter the population of next generation. The EAPSO builds three types of elite archives to guide the common particles (in POP_c) to learn and update their velocity. The three types of elite archives are introduced as follows:

- $A(t) = \{a_1^d(t), a_2^d(t), \dots, a_{N/2}^d(t)\}$, $A(t)$ stores the historical solutions of particles in POP_o , where $a_i^d(t)$ is the historical best solution of the i th particle in POP_o at generation t .
- $B(t) = \{b_1^d(t), b_2^d(t), \dots, b_N^d(t)\}$, $B(t)$ stores the promising historical solutions of particles in POP_c , where $b_i^d(t)$ is a historical solution of particles in POP_c at generation t .
- $C(t) = \{c_1^d(t), c_2^d(t), \dots, c_N^d(t)\}$, $C(t)$ stores promising global best solutions at generation t .

For the detailed updating strategy of $A(t)$, $B(t)$, and $C(t)$, please refer to Zhang (2023). The velocity learning strategy proposed by EAPSO follows the following rules:

Case 1: $f(a_m^d(t)) \leq f(b_r^d(t))$ and $f(a_m^d(t)) \leq f(c_q^d(t))$

$$V_i^d(t+1) = \omega(t) \times V_i^d(t) + r_1(t) \times (a_m^d(t) - x_i^d(t)) + r_2(t) \times (Gbest^d(t) - x_i^d(t)), \quad (16)$$

Case 2: $f(b_r^d(t)) \leq f(a_m^d(t))$ and $f(b_r^d(t)) \leq f(c_q^d(t))$

$$V_i^d(t+1) = \omega(t) \times V_i^d(t) + r_1(t) \times (b_r^d(t) - x_i^d(t)) + r_2(t) \times (Gbest^d(t) - x_i^d(t)), \quad (17)$$

Case 3: $f(c_q^d(t)) \leq f(a_m^d(t))$ and $f(c_q^d(t)) \leq f(b_r^d(t))$

$$V_i^d(t+1) = \omega(t) \times V_i^d(t) + r_1(t) \times (c_q^d(t) - x_i^d(t)) + r_2(t) \times (Gbest^d(t) - x_i^d(t)), \quad (18)$$

Case 4: $f(c_q^d(t)) \leq f(a_m^d(t))$ and $f(b_r^d(t)) \leq f(a_m^d(t))$

$$V_i^d(t+1) = \omega(t) \times V_i^d(t) + r_1(t) \times (c_q^d(t) - x_i^d(t)) + r_2(t) \times (b_r^d(t) - x_i^d(t)), \quad (19)$$

Case 5: $f(a_m^d(t)) \leq f(b_r^d(t))$ and $f(c_q^d(t)) \leq f(b_r^d(t))$

$$V_i^d(t+1) = \omega(t) \times V_i^d(t) + r_1(t) \times (a_m^d(t) - x_i^d(t)) + r_2(t) \times (c_q^d(t) - x_i^d(t)), \quad (20)$$

Case 6: $f(a_m^d(t)) \leq f(c_q^d(t))$ and $f(b_r^d(t)) \leq f(c_q^d(t))$

$$V_i^d(t+1) = \omega(t) \times V_i^d(t) + r_1(t) \times (a_m^d(t) - x_i^d(t)) + r_2(t) \times (b_r^d(t) - x_i^d(t)), \quad (21)$$

where $a_m^d(t)$, $b_r^d(t)$, and $c_q^d(t)$ are the randomly selected solutions from $A(t)$, $B(t)$, and $C(t)$, respectively. $\omega(t)$, $r_1(t)$, and $r_2(t)$ are values randomly generated in (0, 1).

2.5. Inertia weight

The inertia weight was proposed to balance the exploration and exploitation capabilities of PSO. A larger inertia weight can promote exploration, while a smaller inertia weight benefits exploitation (Shi and Eberhart, 1998). In EAPSO (Zhang, 2023), inertia weights are random vectors consisting of random numbers between 0 and 1, distributed uniformly. In EOPSO (Zhao and Wang, 2022), the inertia weight is calculated using Eq. (9). Similarly, in MPSPSO (Liu et al., 2020), the inertia weight is determined using Eq. (15).

2.6. SCA

In Mirjalili (2016), the SCA is proposed to update positions by using Eq. (22).

$$X_i^d(t+1) = \begin{cases} X_i^d(t) + r_1(t) \times \sin(r_2(t)) \times |r_3(t) \times Gbest^d(t) - X_i^d(t)|, & r_4(t) < 0.5, \\ X_i^d(t) + r_1(t) \times \cos(r_2(t)) \times |r_3(t) \times Gbest^d(t) - X_i^d(t)|, & r_4(t) \geq 0.5, \end{cases} \quad (22)$$

where $X_i^d(t)$ is the position of the current particle in d th dimension at t th generation; $r_1(t)$, $r_2(t)$, $r_3(t)$, and $r_4(t)$ are $2 - 2 \times \frac{t}{T_{max}}$, $2 \times \pi \times rand$, $2 \times rand$, and $rand$, respectively, and $rand$ is a random number between 0 and 1. The essence of the SCA, used as a local search method, is to generate offspring around the particles in the designated area.

3. Ensemble velocity learning strategy for PSO integrating multiple local search mechanisms

In EVLS-PSOIMLSM, a novel ensemble velocity learning strategy, hybrid local search mechanisms, and a mutation strategy are employed. This design aims to ensure sufficient exploration in the early stages and enhance exploitation in the later stages of the evolutionary process. The ensemble velocity learning strategy features a three-level flying speed that can dynamically switch between high and low based on search information derived from the problem's landscape. This allows particles to effectively cover search regions while also learning from collected information, even if some of it initially appears unhelpful. The hybrid local search mechanisms refine the best solutions, while the mutation strategy helps particles escape local optima. The pseudocode for EVLS-PSOIMLSM is presented in Algorithm 1. The source code, written in MATLAB, is available for download from the GitHub repository (<https://github.com/microhard1999/CODES>). It can also be requested from the first or corresponding author.

Algorithm 1 Pseudocode of EVLS-PSOIMLSM

```

1: Set pop size  $N = 40$ ,  $Prob_{ls} = 0.07$ ,  $Prob_{ms} = 0.7$ ,  $Mut_{dim} = 0.3366$ ,  $succ = 0$ , and all
   other parameters required.
2: for  $i : N$  do
3:    $p_i \leftarrow$  uniformly distributed  $D$  random numbers.
4: end for
5: Initialize groups A, B, and C.
6: while termination condition is not satisfied do
7:    $t = t + 1$ 
8:   Sort population by fitness values.
9:   Calculate  $E^d(t)$  using Eqs. (6) to (8), and  $SPbest$  using Eqs. (11) and (12).
10:  for  $i = \frac{N}{2} + 1 : N$  do
11:    Select guiding particles from groups A, B, and C.
12:    if  $f(x_i^d(t)) < \sum_{i=N/2+1}^N f(x_i^d(t)) \times \frac{2}{N}$  then
13:      Calculate the velocity using Eq. (23).
14:    else
15:      if  $f(b_r^d(t)) < f(a_m^d(t))$  &&  $f(b_r^d(t)) < f(c_q^d(t))$  then
16:        Calculate the velocity using Eq. (24).
17:      else if  $f(c_q^d(t)) < f(a_m^d(t))$  &&  $f(c_q^d(t)) < f(b_r^d(t))$  then
18:        Calculate the velocity using Eq. (25).
19:      end if
20:      Update the  $i^{th}$  particle using Eq. (3).
21:    end if
22:  end for
23:  Update  $curFEs$ ,  $Pbest$ , and  $Gbest$ .
24:  Update groups A, B, and C.
25:  if  $\text{rand}(0,1) < Prob_{ms}$  and  $curFEs > 0.5 \times \maxFEs$  then
26:    for  $j : D$  do
27:      if  $r_a < \sqrt{\frac{curFEs}{\maxFEs}}$  and  $r_b < Mut_{dim}$  then
28:        Apply mutation strategy using Eq. (32).
29:      end if
30:    end for
31:  end if
32:  if  $\text{rand}(0,1) < Prob_{ls}$  and  $curFEs > 0.4 \times \maxFEs$  then
33:    if  $succ == 1$  then
34:      Apply the SQP method to the  $Gbest$  to find a better solution.
35:    else
36:      Apply the ISCA to generate  $Mbest$  using Eq. (28).
37:      if  $Mbest < Gbest$  then
38:         $Gbest \leftarrow Mbest$ 
39:      end if
40:      Apply the SQP method to the  $Mbest$  to find a better solution.
41:    end if
42:    if the solution is improved then
43:       $Prob_{ls} = 0.1 \times curFEs / \maxFEs$ ,  $succ = 1$ .
44:      Replace the best particle in population with the improved solution and its
       $Pbest$ .
45:    else
46:       $Prob_{ls} = 0.03$ ,  $succ = 0$ .
47:    end if
48:  end if
49: end while

```

3.1. Novel ensemble velocity learning strategy

In EVLS-PSOIMLSM, a novel ensemble velocity learning strategy is proposed.

Type 1: $f(x_i^d(t)) < \sum_{i=1}^{N/2} f(x_i^d(t)) \times \frac{2}{N}$

$$V_i^d(t+1) = \omega(t) \times V_i^d(t) + r_1(t) \times (SPbest_{i-\frac{N}{2}}^d(t) - x_i^d(t)) + r_2(t) \times c(t) \times (Mbest^d(t) - x_i^d(t)), \quad (23)$$

Type 2: $f(x_i^d(t)) \geq \sum_{i=1}^{N/2} f(x_i^d(t)) \times \frac{2}{N}$ and $f(b_r^d(t)) < f(a_m^d(t))$ and $f(b_r^d(t)) < f(c_q^d(t))$

$$V_i^d(t+1) = \omega(t) \times V_i^d(t) + r_1(t) \times (b_r^d(t) - x_i^d(t)) + r_2(t) \times c(t) \times (E_i^d(t) - x_i^d(t)), \quad (24)$$

Type 3: $f(x_i^d(t)) \geq \sum_{i=1}^{N/2} f(x_i^d(t)) \times \frac{2}{N}$ and $f(c_q^d(t)) < f(a_m^d(t))$ and $f(c_q^d(t)) < f(b_r^d(t))$

$$V_i^d(t+1) = \omega(t) \times V_i^d(t) + r_1(t) \times (c_q^d(t) - x_i^d(t)) + r_2(t) \times c(t) \times (E_i^d(t) - x_i^d(t)), \quad (25)$$

where $r_1(t)$ and $r_2(t)$ are values randomly generated in $(0, 1)$, $i \in [\frac{N}{2} + 1, N]$. $E_i^d(t)$ is calculated using Eqs. (6), (7), and (8). $Mbest^d(t)$ =

$\text{mean}\{Pbest_1^d(t), Pbest_2^d(t), \dots, Pbest_N^d(t)\}$. $c(t)$ and $\omega(t)$ are calculated using Eqs. (26) and (27), respectively.

$$c(t) = 2.5 - 2 \times \frac{curFEs}{\maxFEs}, \quad (26)$$

$$\omega(t) = \omega_{\max} - \frac{curFEs}{\maxFEs} \times (\omega_{\max} - \omega_{\min}), \quad (27)$$

where $curFEs$ is the current function evaluation, $\maxFEs$ is the maximum function evaluation, $\omega_{\max} = 0.9$, and $\omega_{\min} = 0.4$. The motivation for the design is that the three types of velocities perform well across the spectrum from exploration to exploitation: Type 1 excels in exploration, Type 2 in exploitation, and Type 3 lies in between. Under this design, particle diversity increases as the generated step size essentially ranges from short to long. The switch conditions help particles select the proper step size based on the collected information from the problem landscape.

3.2. Hybrid local search mechanisms

In EVLS-PSOIMLSM, both the ISCA and SQP methods are employed to enhance exploitation capabilities when $curFEs$ exceeds $0.4 \times \maxFEs$. The value 0.4 is an empirical threshold derived from evaluations on the Congress on Evolutionary Computation 2017 (CEC2017) benchmark functions, where it provides the best average performance; however, this value can be adjusted based on the specific problem. Empirical studies have shown that introducing local search mechanisms to refine solutions after sufficient exploration is an effective method to improve both search efficiency and accuracy. In EVLS-PSOIMLSM, ISCA is used to search within a designated area to generate a new solution. If the new solution demonstrates improved performance, the SQP method is applied to further exploit the new solution. Additionally, $Prob_{ls}$ controls the probability of applying local search mechanisms, as excessive use of these mechanisms consumes resources. To balance the consumption and benefit, $Prob_{ls}$ is designed to regulate the utilization of local search mechanisms.

3.2.1. ISCA

ISCA is proposed in EVLS-PSOIMLSM, where Eq. (22) is replaced by Eq. (28).

$$Mbest^d(t) = \begin{cases} Gbest^d(t) + r_1(t) \times \sin(r_2(t)) \times (r_3(t) \times Gbest^d(t) - Pbest_r^d(t)), & r_4(t) < 0.5, \\ Gbest^d(t) + r_1(t) \times \cos(r_2(t)) \times (r_3(t) \times Gbest^d(t) - Pbest_r^d(t)), & r_4(t) \geq 0.5, \end{cases} \quad (28)$$

where $r_1(t)$ is replaced by $2 - 2 \times \frac{curFEs}{\maxFEs}$ and $r_2(t)$, $r_3(t)$, and $r_4(t)$ remain unchanged. The cyclic nature of the sine and cosine functions enables particles to reposition themselves around $Gbest^d(t)$, ensuring effective exploitation of the search space defined between $Gbest^d(t)$ and $r_1(t) \times \sin/\cos(r_2(t)) \times (r_3(t) \times Gbest^d(t) - Pbest_r^d(t))$. Here, $Pbest_r^d(t)$ represents a randomly selected best position encountered by a specific particle. This revised SCA expands the search region around $Gbest^d(t)$, enabling exploration in a broader space compared to the vicinity of ordinary particles. Fig. 1 illustrates the ISCA in a two-dimensional model. The motivation for this improvement is to introduce a disturbance around $Gbest^d(t)$. When particles fall into local optima, ISCA helps them escape, while the introduction of the SQP method enables effective exploitation. This collaboration enhances the efficiency of SQP, as particles trapped in local optima would otherwise hinder its performance. The cooperation between ISCA and SQP increases the success rate of SQP's application.

3.2.2. SQP

The SQP method has demonstrated exceptional exploitation capabilities in previous research (Hong et al., 2024b,a, 2023). The core

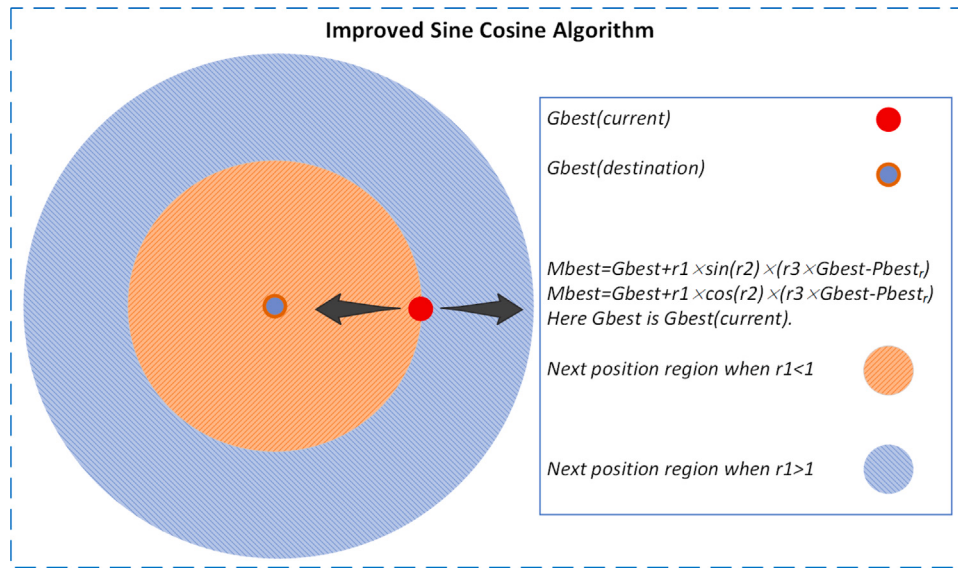


Fig. 1. Two-dimensional model of ISCA.

principle of the SQP method lies in its capability to convert a nonlinear problem into a linear one (Modares and Naghibi Sistani, 2011; Zhang et al., 2013). At its essence, the SQP method aims to determine a suitable direction and formulate a quadratic optimization problem. Nonlinear optimization problems can be formulated as follows:

$$\text{minimize } f(x), \text{ such that } \begin{cases} h(x) = 0, \\ g(x) \leq 0, \end{cases} \quad (29)$$

The Lagrangian of the aforementioned formulation can be written as follows:

$$L(x, \lambda, \mu) = f(x) + \lambda^T h(x) + \mu^T g(x), \quad (30)$$

where λ and μ represent Lagrangian multipliers. The SQP method is an iterative procedure that constructs the problem for a given iteration x_k by solving a quadratic programming sub-problem. Additionally, x_k is employed to generate a new iteration x_{k+1} . The sub-problem is established by linearizing the constraints at x_k and can be expressed as follows:

$$\text{minimize } f'(x_k)(x - x_k) + \frac{1}{2}(x - x_k)^T H f(x_k)(x - x_k), \text{ such that } \begin{cases} h(x_k) + h'(x_k)(x - x_k) = 0, \\ g(x_k) + g'(x_k)(x - x_k) \leq 0, \end{cases} \quad (31)$$

Here, $H f(x_k)$ denotes the Hessian matrix of the objective function f evaluated at $x \in \mathbb{R}^n$. Due to the strong dependence of the SQP method on the initial estimate (Costa et al., 2005), the proposed EVLS-PSOIMLSM algorithm applies the SQP method when $curFEs > 0.4 \times maxFEs$. This design allows the algorithm to perform sufficient exploration before the SQP method intervenes for exploitation.

In the proposed algorithm, the SQP method is either applied individually or combined with the ISCA method. If the SQP method does not produce a better particle, ISCA searches around the current $Gbest$ to escape local optima, after which the SQP method is applied again. This design effectively enhances the efficient use of the SQP method.

3.3. Mutation strategy

In Yuan et al. (2024), a mutation strategy was proposed using Eqs. (32) and (33).

$$X_{i,j}(t+1) = X_{i,j}(t) + \sin(rand \times \pi) \times (X_{i,j}(t) - X_{a,j}(t)), \quad (32)$$

$$K_1 = \sqrt{\frac{t}{T}}, \quad (33)$$

when $r_a < K_1$ and $r_b < 0.05$, the mutation operator, as described in Eq. (32), will be triggered. r_a and r_b are randomly generated in (0, 1). i represents the i th particle, and j represents the j th dimension. The variable t represents the current generation number, and T denotes the maximum generation number.

In EVLS-PSOIMLSM, Eq. (33) is replaced by Eq. (34).

$$K_2 = \sqrt{\frac{curFEs}{maxFEs}}, \quad (34)$$

when $\text{rand}(0,1) < Prob_{ms}$ and $curFEs > 0.5 \times maxFEs$, each dimension of the particle mutates using Eq. (32) if $r_a < K_2$ and $r_b < Mut_{dim}$, where Mut_{dim} is set to 0.3366, an empirically determined value. Observations and empirical studies indicate that setting $Prob_{mut}$ significantly above 0.3366 results in excessive diversity, destabilizing the search, while a much lower value limits the ability of convergent particles to escape local optima. However, this parameter can be adjusted based on specific problem requirements. The value 0.3366 has been fine-tuned for optimal average performance across the CEC2017 benchmark functions. r_a and r_b are randomly generated in (0, 1). The motivation behind this design is to delay the introduction of mutation, as mutation in the early generations of evolution has limited effect when the particles are sufficiently diverse. Even in the later stages of the evolutionary process, it is not necessary to perform a mutation in every generation. The mutation probability, $Prob_{ms}$, is set to 0.7, which reduces the chances of mutation to avoid excessive variation. Meanwhile, controlling the mutation rate for the dimensions of the particles ensures that their diversity remains manageable.

4. Performance evaluation

Global optimization can be represented as a pair (X, f) , where $X \in \mathbb{R}^D$ denotes a bounded subset of $\mathbb{R}^D$, and $f : X \rightarrow \mathbb{R}$ is a real-valued function defined over the D -dimensional domain X . This can be formalized as follows:

$$\forall x \in X, f(x_{\min}) \leq f(x). \quad (35)$$

The functions in the CEC2017 benchmark suite are listed in Table 1.

4.1. Parameter settings

The $maxFEs$ value was calculated as $10^4 \times D$ for all PSO-based variants tested. Therefore, the $maxFEs$ values were set to 10×10^4 , $30 \times$

Table 1
CEC2017 benchmark functions.

Types	f_n	Benchmark functions	Search range	$F(x^*)_{bias}$
Unimodal Functions	f_1	Shifted and Rotated Bent Cigar Function	$[-100, 100]^D$	100
	f_2	Shifted and Rotated Sum Diff Pow Function	$[-100, 100]^D$	200
	f_3	Shifted and Rotated Zakharov Function	$[-100, 100]^D$	300
Simple Multimodal Functions	f_4	Shifted and Rotated Rosenbrock's Function	$[-100, 100]^D$	400
	f_5	Shifted and Rotated Rastrigin's Function	$[-100, 100]^D$	500
	f_6	Shifted and Rotated Expanded Scaffer's F6 Function	$[-100, 100]^D$	600
	f_7	Shifted and Rotated Lunacek Bi-Rastrigin Function	$[-100, 100]^D$	700
	f_8	Shifted and Rotated Non-Continuous Rastrigin's Function	$[-100, 100]^D$	800
	f_9	Shifted and Rotated Levy Function	$[-100, 100]^D$	900
	f_{10}	Shifted and Rotated Schwefel's Function	$[-100, 100]^D$	1000
Hybrid Functions	f_{11}	Hybrid Function 1 (N=3)	$[-100, 100]^D$	1100
	f_{12}	Hybrid Function 2 (N=3)	$[-100, 100]^D$	1200
	f_{13}	Hybrid Function 3 (N=3)	$[-100, 100]^D$	1300
	f_{14}	Hybrid Function 4 (N=4)	$[-100, 100]^D$	1400
	f_{15}	Hybrid Function 5 (N=4)	$[-100, 100]^D$	1500
	f_{16}	Hybrid Function 6 (N=4)	$[-100, 100]^D$	1600
	f_{17}	Hybrid Function 7 (N=5)	$[-100, 100]^D$	1700
	f_{18}	Hybrid Function 8 (N=5)	$[-100, 100]^D$	1800
	f_{19}	Hybrid Function 9 (N=5)	$[-100, 100]^D$	1900
	f_{20}	Hybrid Function 10 (N=6)	$[-100, 100]^D$	2000
Composition Functions	f_{21}	Composition Function 1 (N=3)	$[-100, 100]^D$	2100
	f_{22}	Composition Function 2 (N=3)	$[-100, 100]^D$	2200
	f_{23}	Composition Function 3 (N=4)	$[-100, 100]^D$	2300
	f_{24}	Composition Function 4 (N=4)	$[-100, 100]^D$	2400
	f_{25}	Composition Function 5 (N=5)	$[-100, 100]^D$	2500
	f_{26}	Composition Function 6 (N=5)	$[-100, 100]^D$	2600
	f_{27}	Composition Function 7 (N=6)	$[-100, 100]^D$	2700
	f_{28}	Composition Function 8 (N=6)	$[-100, 100]^D$	2800
	f_{29}	Composition Function 9 (N=3)	$[-100, 100]^D$	2900
	f_{30}	Composition Function 10 (N=3)	$[-100, 100]^D$	3000

Table 2
Parameter settings for all compared algorithms.

Algorithm	Year	Default parameter settings
EVLS-PSOIMLSM	–	pop size $N = 40$ (for 10, 30, 50 and 100D), $w = 0.9 \rightarrow 0.4$, $Prob_{is} = 0.07$, $Prob_{ms} = 0.7$, $Mut_{dim} = 0.3366$.
IMPSONwithCMAR	2024	Initial subpopulation size $N_1 = 70$ for IMPSO and $N_2 = 4 + floor(3 \times \ln(D))$ for CMAR (on 10, 30, 50, and 100D), respectively; $cw = 4 \times r \times (1 - r)$, r is a random value in (0,1), $w = 0.9 \rightarrow 0.4$. The CS for 10, 30, 50, and 100D were set as 50, 100, 150, and 150, respectively.
SQPPSO-sono	2024	pop size $N = 100$ (for 10, 30, 50 and 100D), $w = 0.9 \rightarrow 0.4$, $e = \frac{D}{N} \times 0.01$.
EAPSO	2023	pop size $N = 100$ (for 10, 30, 50 and 100D).
HMPPSO	2023	pop size $N = 100$ (for 10, 30, 50 and 100D), $w = 0.9 \rightarrow 0.4$, $p = 0.5$.
SpadePSO	2023	pop size $N = 40$ (for 10, 30, 50, and 100D), $w = 0.99 \rightarrow 0.2$, $c_1 = 2.5 \rightarrow 0.5$, $c_2 = 0.5 \rightarrow 2.5$, $c = 3 \rightarrow 1.5$, $k_0 = 2$, $v_k = 6$, $n_{exp} = 5$.
ADPSO	2023	pop size $N = 60$ (for 10, 30, 50, and 100D), $w = 0.729$, $c_1 = c_2 = 1.49445$, $R = 30$, $M = 10$, $N = 6$.
PSO-sono	2022	pop size $N = 100$ (for 10, 30, 50, and 100D), $r = 0.5$, $e = \frac{D}{N} \times 0.01$, $w = 0.9 \rightarrow 0.4$.
MFCPSO	2022	pop size $N = 2 \times D$ (for 10, 30, 50, and 100D), $p = 0.05 \rightarrow 0.5$, $k = \frac{N}{20}$.
AMSEPSO	2022	pop size $N = 40$ (for 10, 30, 50, and 100D), $c_1 = 2.5 \rightarrow 1.5$, $c_2 = 1.5 \rightarrow 2.5$, $w = 0.9 \rightarrow 0.4$, $\sigma = 0.3 \times pop\ size$, $stag\ max = 5$.
EOPSO	2022	pop size $N = 40$ (for 10, 30, 50, and 100D), $c = 1.49445$, $Rmax = 0.5$, $Rmin = 0.4$, and $G = 7$.
PPSO	2022	pop size $N = 64$ (for 10, 30, 50, and 100D), ρ is set to 0.008, 0.02, 0.04 and 0.008 for 10, 30, 50, and 100D, respectively.
SDPSO	2022	pop size $N = 40$ (for 10, 30, 50, and 100D), $w = 0.9 \rightarrow 0.2$, $c_1 = 2.5 \rightarrow 0.5$, $c_2 = 0.5 \rightarrow 2.5$, $r = 4$, $LP = 200$.
AHPSO	2021	pop size $N = 60$ (for 10, 30, 50, and 100D), $w = 0.99 \rightarrow 0.29$, $c_1 = 2.5 \rightarrow 0.5$, $c_2 = 0.5 \rightarrow 2.5$.
MPSO	2020	pop size $N = 50$ (for 10, 30, 50, and 100D), r is a random value in (0,1), $cw = 4 \times r \times (1 - r)$, r is a random value in (0,1), $w = 0.9 \rightarrow 0.4$.

10^4 , 50×10^4 , and 100×10^4 for dimensions of 10, 30, 50, and 100, respectively. The parameter configurations for the proposed EVLS-PSOIMLSM algorithm and the comparison algorithms, which represent the PSO-based variants using default settings, are outlined in Table 2. These default settings, optimized by their respective developers, demonstrated optimal performance for the recently proposed PSO-based variants.

4.2. Experimental results

In Table 3, the mean, minimum, and median values ranking first are counted. In Table 4, the Friedman test rankings are evaluated for all the compared algorithms. The tables display the means, minima, medians, standard deviations, and rankings for experiments across different dimensions: 10D, 30D, 50D, and 100D (see Tables 5–7, and 8). These

Table 3

Ranking statistics based on the mean, minimum, and median values for all PSO-based variants, evaluated on the CEC2017 benchmark functions across different dimensions.

<i>D</i>	Criteria	EVLS-PSOIMLSM	IMPSOwithCMAR	SQPPSO-sono	EAPSO	HMPPSO	SpadePSO	ADPSO	PSO-sono	MFCPSO	AMSEPSO	EOPSO	PPSO	SDPSO	AHPSO	MPSO
10 <i>D</i>	Mean [1^{st}]	8	3	1	0	4	3	3	2	2	1	1	1	5	1	0
	Min [1^{st}]	10	4	8	5	2	6	5	6	3	4	1	6	7	4	3
	Median [1^{st}]	8	3	4	1	1	4	5	3	4	4	2	3	6	2	1
	TOTAL [1^{st}]	26	10	13	6	7	13	13	11	9	9	4	10	18	7	4
	Mean (> = <)	/	14 12 4	18 5 7	20 3 7	19 3 8	12 7 11	16 4 10	23 1 6	22 2 6	20 2 8	18 8 4	23 4 3	12 4 14	19 5 6	26 2 2
30 <i>D</i>	Mean [1^{st}]	11	5	0	1	1	2	0	0	5	0	3	1	1	0	0
	Min [1^{st}]	4	4	3	3	0	1	1	3	4	6	4	3	2	1	0
	Median [1^{st}]	9	7	1	2	1	0	0	0	4	1	4	2	1	0	0
	TOTAL [1^{st}]	24	16	4	6	2	3	1	3	13	7	11	6	4	1	0
	Mean (> = <)	/	14 5 11	22 4 4	24 4 2	27 0 3	20 5 5	24 1 5	27 1 2	18 4 8	25 0 5	17 5 8	26 2 2	20 5 5	21 3 6	27 1 2
50 <i>D</i>	Mean [1^{st}]	8	4	1	0	2	1	0	0	8	0	5	0	0	1	0
	Min [1^{st}]	6	5	3	3	0	0	1	1	7	4	3	1	1	0	0
	Median [1^{st}]	9	3	1	1	1	0	0	0	9	0	5	0	0	1	0
	TOTAL [1^{st}]	23	12	5	4	3	1	1	1	24	4	13	1	1	2	0
	Mean (> = <)	/	15 8 7	25 3 2	25 3 2	28 0 2	25 3 2	26 1 3	28 1 1	16 1 13	26 2 2	14 8 8	27 1 2	25 3 2	20 5 5	29 0 1
100 <i>D</i>	Mean [1^{st}]	15	1	1	0	0	1	0	0	9	0	2	0	0	1	0
	Min [1^{st}]	8	1	4	1	0	1	0	0	5	4	4	1	1	0	0
	Median [1^{st}]	11	3	1	0	0	1	0	0	11	0	2	0	0	1	0
	TOTAL [1^{st}]	34	5	6	1	0	3	0	0	25	4	8	1	1	2	0
	Mean (> = <)	/	23 4 3	27 2 1	28 1 1	29 0 1	26 2 2	26 1 3	27 1 2	16 2 12	25 3 2	19 3 8	27 2 1	27 2 1	22 2 6	30 0 0

Table 4
The Friedman test rankings were conducted on the mean values for EVLS-PSOIMLSM, IMPSOwithCMAR, SQPPSO-sono, EAPSO, HMPPSO, SpadePSO, ADPSO, PSO-sono, MFCPSO, AMSEPSO, EOPSO, PPSO, SDPSO, AHP SO, and MPSO across CEC2017 benchmark functions.

<i>D</i>	Criteria	EVLS-PSOIMLSM	IMPSOwithCMAR	SQPPSO-sono	EAPSO	HMPPSO	SpadePSO	ADPSO	PSO-sono	MFCPSO	AMSEPSO	EOPSO	PPSO	SDPSO	AHPSO	MPSO
10 <i>D</i>	Averaged Friedman ranks	5.30	6.37	7.37	10.47	8.40	4.57	6.83	8.27	10.43	8.67	9.77	10.87	4.20	7.37	10.73
	Rank	3	4	6	13	9	2	5	8	12	10	11	15	1	7	14
30 <i>D</i>	Averaged Friedman ranks	3.57	3.97	8.10	10.20	10.53	6.30	8.60	10.20	6.77	10.90	6.93	9.60	5.53	7.37	11.40
	Rank	1	2	8	11	13	4	9	12	5	14	6	10	3	7	15
50 <i>D</i>	Averaged Friedman ranks	3.23	4.20	8.33	9.43	11.03	6.63	8.03	10.43	4.67	11.07	6.57	9.87	6.73	6.80	12.97
	Rank	1	2	9	10	13	5	8	12	3	14	4	11	6	7	15
100 <i>D</i>	Averaged Friedman ranks	2.77	4.53	7.33	8.87	12.50	7.13	7.37	10.27	5.23	10.67	6.47	9.40	7.10	6.73	13.50
	Rank	1	2	8	10	14	7	9	12	3	13	4	11	6	5	15

Table 5

Experimental results of PSO variants on 10-dimensional CEC2017 benchmark functions over 51 runs, including mean, minimum, and median values, ranks, standard deviations, and Wilcoxon rank sum test results.

	Criteria	EVLS-PSOIMLSM	IMPSOwithCMAR	SQPPSO-samo	EAPSO	IMPPSO	SpadePSO	ADPSO	PSO-samo	MFCPSO	AMSPSO	EPSO	PFSO	SDPSO	AHPSO	MPSO
f_1	Mean	4.3597E-05	2.852024E-02	3.23799E-04	2.21638E+03	1.51999E+04	5.901761E+01	1.608079E+03	7.634577E+02	2.390599E+03	1.704919E+03	1.279055E+03	9.134022E+03	1.423899E+02	1.662726E+03	6.537807E+02
	Rank(WRST)	1	1.15787E-07	0.000000E+00	1.15787E-07	1.25327E-02	1.630977E+02	4.112566E-02	2.08712E+01	8.77846E-01	1.704919E+03	9.134022E+03	1.423899E+02	1.662726E+03	6.537807E+02	1.15787E-07
	Min	1.15787E-07	0.000000E+00	3.894901E-05	1.25327E-02	1.630977E+02	4.112566E-02	2.08712E+01	8.77846E-01	1.704919E+03	9.134022E+03	1.423899E+02	1.662726E+03	6.537807E+02	1.15787E-07	1.15787E-07
	Rank	2	4.57708E-07	3.604736E-04	2.007753E+03	1.630977E+02	4.112566E-02	2.08712E+01	8.77846E-01	1.704919E+03	9.134022E+03	1.423899E+02	1.662726E+03	6.537807E+02	1.15787E-07	1.15787E-07
	Median	2	4.57708E-07	3.604736E-04	2.007753E+03	1.630977E+02	4.112566E-02	2.08712E+01	8.77846E-01	1.704919E+03	9.134022E+03	1.423899E+02	1.662726E+03	6.537807E+02	1.15787E-07	1.15787E-07
	Rank	2	4.57708E-07	3.604736E-04	2.007753E+03	1.630977E+02	4.112566E-02	2.08712E+01	8.77846E-01	1.704919E+03	9.134022E+03	1.423899E+02	1.662726E+03	6.537807E+02	1.15787E-07	1.15787E-07
	(Std Dev)	(4.559246E-05)	(1.429703E-01)	(5.96887E-05)	(2.078163E+03)	(8.79403E+03)	(7.302878E+01)	(2.166852E+03)	(8.79403E+03)	(2.56699E+03)	(1.585929E+03)	(1.360752E+03)	(1.243755E+03)	(1.78123E+02)	(1.658638E+03)	(9.30056E+02)
f_2	Mean	2.4887E-06	4.924630E-05	4.631410E-06	1.561748E-05	6.843809E-01	6.07348E-05	3.615870E-05	2.055030E-05	6.370294E-02	1.919977E+01	8.658408E-01	1.197804E+05	2.012486E-05	8.660515E-06	3.061028E+02
	Rank(WRST)	1	2.15665E-08	7.748355E-07	1.65037E-07	7.77027E-09	1.360207E-01	7.113610E-07	6.53831E-08	8.53541E-08	1.130861E-07	1.300765E-06	1.263345E-06	1.26031E-07	1.26031E-07	1.26031E-07
	Min	2.15665E-08	7.748355E-07	1.65037E-07	7.77027E-09	1.360207E-01	7.113610E-07	6.53831E-08	8.53541E-08	1.130861E-07	1.300765E-06	1.263345E-06	1.26031E-07	1.26031E-07	1.26031E-07	1.26031E-07
	Rank	2	1.22321E-06	7.147830E-06	3.842420E-06	6.849870E-06	2.278742E-05	9.551085E-06	5.623612E-06	1.80759E-06	1.872211E-05	1.463555E-04	1.288828E-05	6.847755E-06	5.970113E-06	5.970113E-06
	Median	2	1.22321E-06	7.147830E-06	3.842420E-06	6.849870E-06	2.278742E-05	9.551085E-06	5.623612E-06	1.80759E-06	1.872211E-05	1.463555E-04	1.288828E-05	6.847755E-06	5.970113E-06	5.970113E-06
	Rank	2	1.22321E-06	7.147830E-06	3.842420E-06	6.849870E-06	2.278742E-05	9.551085E-06	5.623612E-06	1.80759E-06	1.872211E-05	1.463555E-04	1.288828E-05	6.847755E-06	5.970113E-06	5.970113E-06
	(Std Dev)	(1.674584E-06)	(1.597551E-04)	(3.849550E-06)	(2.404994E-05)	(1.855059E-01)	(2.884941E-05)	(1.150030E-04)	(4.150908E-05)	(4.524122E+03)	(6.578262E+01)	(2.219167E+01)	(8.551026E+00)	(2.37459E-05)	(6.849565E-06)	(4.173333E+03)
f_3	Mean	3.33258E-13	3.30522E-10	2.229154E-15	4.458307E-14	4.458307E-14	3.771325E-06	0.000000E+00	0.000000E+00	2.126098E-07	1.471023E-05	8.669458E-08	1.226035E-14	4.458307E-15	5.740216E-10	5.740216E-10
	Rank(WRST)	1	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Min	3.33258E-13	3.30522E-10	2.229154E-15	4.458307E-14	4.458307E-14	3.771325E-06	0.000000E+00	0.000000E+00	2.126098E-07	1.471023E-05	8.669458E-08	1.226035E-14	4.458307E-15	5.740216E-10	5.740216E-10
	Rank	1	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Median	10	11	1	1	1	1	1	1	1	1	1	1	1	1	1
	Rank	10	11	1	1	1	1	1	1	1	1	1	1	1	1	1
	(Std Dev)	(3.27797E-13)	(1.886217E-09)	(1.114354E-14)	(2.825403E-14)	(5.017139E-02)	(3.989579E-14)	(1.893127E-05)	(0.000000E+00)	(0.000000E+00)	(6.82324E-05)	(2.294218E-05)	(6.12883E-07)	(1.543432E-14)	(2.45988E-09)	(2.45988E-09)
f_4	Mean	2.028530E-13	2.574794E-11	4.112799E-13	8.554185E-05	2.328795E+00	2.060021E+00	1.474046E+00	2.144627E-08	2.311930E+00	4.160130E+00	4.716017E+00	4.418162E+00	2.228161E-01	3.939710E-01	1.812991E+00
	Rank(WRST)	1	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11
	Min	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11
	Rank	1	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11
	Median	1	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11
	Rank	1	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11	1.074341E-11
	(Std Dev)	(3.859560E-11)	(5.362101E-13)	(3.734492E-04)	(6.659143E-01)	(6.659143E-01)	(4.091021E-01)	(4.091021E-01)	(4.091021E-01)	(4.091021E-01)	(4.091021E-01)	(4.091021E-01)	(4.091021E-01)	(4.091021E-01)	(4.091021E-01)	(4.091021E-01)
f_5	Mean	3.511620E-03	3.280030E-03	1.042731E+01	1.28914E-01	1.28914E-01	1.28914E-01	1.28914E-01	1.28914E-01	1.28914E-01	1.28914E-01	1.28914E-01	1.28914E-01	1.28914E-01	1.28914E-01	1.28914E-01
	Rank(WRST)	1	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12
	Min	3.511620E-03	3.280030E-03	1.042731E+01	1.28914E-01	1.28914E-01	1.28914E-01	1.28914E-01	1.28914E-01	1.28914E-01	1.28914E-01	1.28914E-01	1.28914E-01	1.28914E-01	1.28914E-01	1.28914E-01
	Rank	1	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12
	Median	1	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12
	Rank	1	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12	1.050303E-12
	(Std Dev)	(1.922464E-03)	(3.145505E-03)	(9.945177E+00)	(0.945177E+00)	(0.945177E+00)	(1.617322E+00)	(1.617322E+00)	(1.617322E+00)	(1.617322E+00)	(1.617322E+00)	(1.617322E+00)	(1.617322E+00)	(1.617322E+00)	(1.617322E+00)	(1.617322E+00)
f_6	Mean	1.300470E-02	1.220195E-01	1.591649E-05	2.148323E-04	4.93791E-05	7.80238E-14	5.02749E-06	4.93791E-05	4.93791E-05	4.93791E-05	4.93791E-05	4.93791E-05	4.93791E-05	4.93791E-05	4.93791E-05
	Rank(WRST)	1	8.29551E-05	4.24097E-02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Min	8.29551E-05	4.24097E-02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank	1	8.29551E-05	4.24097E-02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Median	1	8.29551E-05	4.24097E-02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank	1	8.29551E-05	4.24097E-02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	(Std Dev)	(2.376057E-02)	(5.432511E-02)	(1.141354E-14)	(2.61662E-04)	(5.686571E-14)	(3.747025E-14)	(1.007703E-05)	(1.784153E-02)	(3.87477E-06)	(1.008062E-06)	(1.073680E-08)	(3.413530E-07)	(4.72240E-14)	(7.16462E-05)	(2.089218E-05)
f_7	Mean	1.371059E-01	1.369030E-01	2.260477E+01	2.299746E+01	2.299746E+01	2.299746E+01	2.299746E+01	2.299746E+01	2.299746E+01	2.299746E+01	2.299746E+01	2.2			

Table 5 (continued).

Criteria	EVLS-PSOIMSLM	IMPSOwithCMAR	SGPFSO-smo	EAPSO	HMPSO	SpadePSO	ADPSO	PSO-smo	MFCPSO	AMSEPSO	EOPSO	PFPSO	SPFSO	AHPSO	MFPSO	
f_1	Mean	1.331233E+01	2.462931E+01	3.757915E+00	1.603936E+02	3.568307E+01	7.153783E+01	5.305415E+00	2.859642E+00	2.330539E+02	4.037216E+00	4.753420E+00	5.184745E+01	6.121483E-01	4.024905E+01	1.144781E+02
	Min	3	4.743650E-01	1.1	14	14	14	4	1	15	1	12	12	1	13	13
	Max	1.284316E+00	2.163215E-01	2.163215E-01	1.835110E+00	1.627728E-01	1.835110E-01	1.028467E+00	1.484677E-01	1.239392E+00	1.484677E-01	1.239392E+00	1.484677E-01	1.239392E+00	1.239392E+00	1.239392E+00
	Rank	9	11	11	15	15	15	6	12	12	12	12	12	12	12	12
	Median	5	11	11	15	15	15	6	12	12	12	12	12	12	12	12
	Std (Dev)	(3.140582E-01)	(1.450374E-01)	(1.450374E-01)	(1.450374E-01)	(1.450374E-01)	(1.450374E-01)	(1.450374E-01)	(1.450374E-01)	(1.450374E-01)	(1.450374E-01)	(1.450374E-01)	(1.450374E-01)	(1.450374E-01)	(1.450374E-01)	(1.450374E-01)
	Rank	10	14	13	15	15	15	8	10	14	14	14	14	14	14	14
f_2	Mean	2.525388E+01	3.529377E+01	1.721785E+01	2.229345E+01	4.702723E+01	3.699972E+00	5.645665E+01	3.049295E+01	4.222066E+01	2.555515E+01	2.727738E+01	2.967738E+01	4.894694E+00	3.157881E+01	2.767596E+01
	Min	6.479257E+00	1.020401E+01	1.973084E-02	9.949591E-01	3.191591E+00	1.973084E-02	5.386616E-01	3.336915E+00	1.793331E+01	1.254549E+01	7.648744E+00	2.114302E+00	1.243746E-01	5.403705E+00	1.328331E+00
	Max	2.597976E+01	2.521666E+01	2.136491E+01	2.561887E+01	1.643434E+01	1.472185E+00	2.434005E+01	2.978915E+01	4.020376E+01	2.637433E+01	2.637433E+01	2.637433E+01	2.637433E+01	2.637433E+01	2.637433E+01
	Rank	10	14	13	15	15	15	8	10	14	14	14	14	14	14	14
	Median	7	13	13	15	15	15	8	10	14	14	14	14	14	14	14
	Std (Dev)	(5.498379E+00)	(1.053422E+01)	(1.197121E+01)	(2.847515E+01)	(1.746466E+00)	(1.746466E+00)	(1.746466E+00)	(1.746466E+00)	(1.746466E+00)	(1.746466E+00)	(1.746466E+00)	(1.746466E+00)	(1.746466E+00)	(1.746466E+00)	(1.746466E+00)
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
f_3	Mean	1.525945E+01	5.663935E+01	4.602855E+01	5.138144E+03	1.027559E+01	8.814988E+02	2.765670E+01	6.880232E+01	1.380953E+01	3.299133E+01	9.38136E+04	9.38136E+04	9.38136E+04	4.023338E+03	3.8469738E+02
	Min	1.525945E+01	5.663935E+01	4.602855E+01	5.138144E+03	1.027559E+01	8.814988E+02	2.765670E+01	6.880232E+01	1.380953E+01	3.299133E+01	9.38136E+04	9.38136E+04	9.38136E+04	4.023338E+03	3.8469738E+02
	Max	1.525945E+01	5.663935E+01	4.602855E+01	5.138144E+03	1.027559E+01	8.814988E+02	2.765670E+01	6.880232E+01	1.380953E+01	3.299133E+01	9.38136E+04	9.38136E+04	9.38136E+04	4.023338E+03	3.8469738E+02
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Std (Dev)	(3.960700E+00)	(4.342043E+01)	(5.058857E+01)	(6.322681E+03)	(3.478505E+00)	(8.567728E+02)	(3.497074E+01)	(5.679639E+01)	(1.202104E+04)	(2.062519E+03)	(1.349358E+04)	(6.858777E+03)	(6.858777E+03)	(5.924244E+03)	(7.249995E+02)
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
f_4	Mean	5.960522E+00	3.759367E+01	4.722148E+00	6.529438E+02	4.554386E+02	1.193727E+01	1.614668E+02	1.193727E+01	1.614668E+02	1.193727E+01	1.614668E+02	1.193727E+01	1.614668E+02	3.649535E+01	3.649535E+01
	Min	4	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Max	4	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Std (Dev)	(6.054050E+00)	(4.789596E+01)	(6.054050E+00)	(6.054050E+00)	(6.054050E+00)	(6.054050E+00)	(6.054050E+00)	(6.054050E+00)	(6.054050E+00)	(6.054050E+00)	(6.054050E+00)	(6.054050E+00)	(6.054050E+00)	(6.054050E+00)	(6.054050E+00)
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
f_5	Mean	2.055388E+01	1.079086E+01	9.592540E+00	1.452396E+01	5.210185E+00	2.195558E-01	1.496557E+00	2.256821E+01	1.010982E+02	1.258662E+01	4.282235E+01	2.076447E+01	1.167375E+00	3.727296E+01	5.127371E+01
	Min	1.613938E+00	2.273773E-13	3.121733E-01	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	3.121733E-01	0.000000E+00
	Max	2.161409E+01	6.243456E+01	5.286964E+01	2.302091E+00	5.309127E+01	2.273737E-13	9.949690E-01	2.203768E+01	1.098522E+01	1.307133E+01	1.307133E+01	1.307133E+01	1.307133E+01	1.307133E+01	1.307133E+01
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Std (Dev)	(7.363600E+00)	(2.429326E+01)	(7.363600E+00)	(7.363600E+00)	(7.363600E+00)	(7.363600E+00)	(7.363600E+00)	(7.363600E+00)	(7.363600E+00)	(7.363600E+00)	(7.363600E+00)	(7.363600E+00)	(7.363600E+00)	(7.363600E+00)	(7.363600E+00)
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
f_6	Mean	1.841734E+02	1.777931E+02	1.233800E+02	1.578594E+02	1.188368E+02	1.188368E+02	1.188368E+02	1.188368E+02	1.188368E+02	1.188368E+02	1.188368E+02	1.188368E+02	1.188368E+02	1.188368E+02	1.188368E+02
	Min	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02
	Max	2.05924952790344E+02	2.05366423025422E+02	2.05366423025422E+02	2.05366423025422E+02	2.05366423025422E+02	2.05366423025422E+02	2.05366423025422E+02	2.05366423025422E+02	2.05366423025422E+02	2.05366423025422E+02	2.05366423025422E+02	2.05366423025422E+02	2.05366423025422E+02	2.05366423025422E+02	2.05366423025422E+02
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Std (Dev)	(4.191131E+01)	(4.560716E+01)	(3.711776E+01)	(4.545119E+01)	(3.945616E+01)	(3.568312E+01)	(3.509732E+01)	(4.809513E+01)	(2.865471E+01)	(3.046893E+01)	(2.672960E+01)	(1.486286E+01)	(3.919595E+01)	(4.673815E+01)	(4.673815E+01)
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
f_7	Mean	5.909506E+01	1.009393E+02	9.902674E+01	9.766814E+01	9.902674E+01	9.902674E+01	9.902674E+01	9.902674E+01	9.902674E+01	9.902674E+01	9.902674E+01	9.902674E+01	9.902674E+01	9.902674E+01	9.902674E+01
	Min	7.90505099399760E-08	1.0000000000000000E+02	1.317217320397440E+01	1.13956226850694E+01	1.277894204956010E+01	1.11202903529873E+01	2.2070475289156E+01	1.0000000000000000E+02	4.59077834622579E+00	1.0000000000000000E+02	4.59077834622579E+00	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02
	Max	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Std (Dev)	(1.560774E+01)	(2.168016E+02)	(6.683176E+01)	(1.760243E+01)	(4.045822E+01)	(2.810253E+01)	(1.18041E+01)	(6.820025E+01)	(2.871035E+01)	(6.862196E+01)	(1.348742E+01)	(3.611958E+01)	(1.580711E+01)	(1.682222E+01)	(1.682222E+01)
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
f_8	Mean	3.049715E+02	3.055761E+02	3.108410E+02	3.244564E+02	3.392776E+02	3.392776E+02	3.392776E+02	3.392776E+02	3.392776E+02	3.392776E+02	3.392776E+02	3.392776E+02	3.392776E+02	3.392776E+02	3.392776E+02
	Min	3.0000000000000000E+02	3.0000000000000000E+02	3.0000000000000000E+02	3.0000000000000000E+02	3.0000000000000000E+02	3.0000000000000000E+02	3.0000000000000000E+02	3.0000000000000000E+02	3.0000000000000000E+02	3.0000000000000000E+02	3.0000000000000000E+02	3.0000000000000000E+02	3.0000000000000000E+02	3.0000000000000000E+02	3.0000000000000000E+02
	Max	3.049715E+02	3.055761E+02	3.108410E+02	3.244564E+02	3.392776E+02	3.392776E+02	3.392776E+02	3.392776E+02	3.392776E+02	3.392776E+02	3.392776E+02	3.392776E+02	3.392776E+02	3.392776E+02	3.392776E+02
	Rank	1	1													

statistical measures are based on 51 runs, with the best values and top rankings highlighted in bold. To better observe the experimental results, some of the data are retained with more digits. To evaluate performance, a two-sided Wilcoxon Rank Sum Test (WRST) was conducted at a significance level of 0.05, comparing EVLS-PSOIMLSM with other PSO-based variants, including IMPSOwithCMAR, SQPPSO-sono, EAPSO, HMPPSO, SpadePSO, ADPSO, PSO-sono, MFCPSO, AMSEPSO, EOPSO, PPSO, SDPSO, AHPSO, and MPPO. In the tables, the symbols “>”, “=”, and “<” indicate where EVLS-PSOIMLSM achieved significantly better results, showed no statistically significant difference, or performed significantly worse, respectively, when compared to the other PSO-based variants.

4.3. Comparison and analysis

In Table 3, EVLS-PSOIMLSM has the highest number of first rankings (8, 11, 8, and 15) for the mean value and the highest number of first rankings (8, 9, 9, and 11) for the median value across all dimensions. Additionally, EVLS-PSOIMLSM also holds the highest number of first rankings (10 and 8) for the minimum values in the 10D and 100D cases, respectively. Furthermore, AMSEPSO and MFCPSO hold the highest number of first rankings (6 and 7) for the 30D and 50D cases, respectively. In contrast, EVLS-PSOIMLSM, IMPSOwithCMAR, MFCPSO, and EOPSO hold the second-highest number of rankings (4) for the minimum value in the 30D case. EVLS-PSOIMLSM also holds the second-highest number of rankings (6) for the minimum value in the 50D case. Finally, EVLS-PSOIMLSM has the highest total number of first rankings (26, 24, and 34) for the mean, minimum, and median values across the 10D, 30D, and 100D dimensions, and holds the second-highest total number of first rankings (23), demonstrating superior performance compared to all other algorithms.

In Table 4, the Friedman test shows that EVLS-PSOIMLSM ranks first on the 30D, 50D, and 100D dimensions, with averaged Friedman ranks of 3.57, 3.23, and 2.77, respectively. However, on the lower-dimensional tests, EVLS-PSOIMLSM ranks third, with an averaged Friedman rank of 5.30. In contrast, SDPSO and SpadePSO rank first and second, with averaged Friedman ranks of 4.20 and 4.57, respectively, demonstrating their strong performance on low-dimensional tests.

Table 5 provides the 10-dimensional experimental results. EVLS-PSOIMLSM exhibits superior mean, minimum, and median values on functions f_4 , f_{12} , f_{18} , and f_{29} ; the best mean values on f_1 , f_2 , f_{13} , and f_{23} ; the best minimum values on f_3 , $f_8 - f_9$, f_{11} , f_{15} , and f_{30} ; and the best median values on f_{11} , f_{13} , and f_{23} . Both PSO-sono and MFCPSO share the best mean value on f_3 , and PSO-sono has the best mean value on f_{27} . IMPSOwithCMAR has the best mean values on f_5 , f_9 , and f_{11} , while ADPSO, MFCPSO, AMSEPSO, and AHPSO also have the best mean values on f_9 . SDPSO has the best mean values on f_6 , f_{10} , f_{16} , f_{25} , and f_{28} . PPSO, EOPSO, and SQPPSO-sono have the best mean values on f_7 , f_8 , and f_{15} , respectively. ADPSO has the best mean value on f_{14} and f_{19} . SpadePSO has the best mean values on f_{17} , f_{20} , and f_{26} . HMPPSO has the best mean values on f_{21} , f_{22} , f_{24} , and f_{30} . EVLS-PSOIMLSM has the best mean values for 8 functions, which is the highest. Meanwhile, SDPSO has the best mean values for 5 functions, which is the second-highest.

Table 6 provides the 30-dimensional experimental results. EVLS-PSOIMLSM exhibits superior mean values on f_2 , $f_4 - f_5$, f_8 , $f_{12} - f_{14}$, f_{16} , f_{18} , f_{20} , and f_{28} ; the best minimum values on f_{10} , f_{12} , f_{18} , and f_{24} ; and the best median values on $f_4 - f_5$, f_8 , $f_{12} - f_{14}$, f_{16} , f_{18} , and f_{20} . IMPSOwithCMAR has the best mean values on f_1 , f_{15} , f_{21} , and $f_{23} - f_{24}$. EAPSO, PPSO, SDPSO, and HMPPSO have the best mean values on f_3 , f_7 , f_{17} , and f_{19} , respectively. SpadePSO has the best mean values on f_6 and f_{26} . MFCPSO has the best mean values on $f_9 - f_{11}$, f_{22} , and f_{29} . EOPSO has the best mean values on f_{25} , f_{27} , and f_{30} . EVLS-PSOIMLSM has the best mean values for 11 functions, which is

the highest. Meanwhile, both IMPSOwithCMAR and MFCPSO have the best mean values for 5 functions, which is the second-highest.

Table 7 provides the 50-dimensional experimental results. EVLS-PSOIMLSM exhibits superior mean value on f_2 , $f_4 - f_5$, f_8 , f_{12} , f_{14} , f_{18} , and f_{21} ; the best minimum values on f_{12} , f_{16} , f_{18} , f_{21} , f_{25} , and f_{28} ; and the best median values on f_2 , $f_4 - f_5$, f_8 , f_{12} , f_{14} , f_{18} , f_{21} , and f_{28} . IMPSOwithCMAR has the best mean values on f_1 , f_{13} , f_{15} , f_{28} . SQPPSO-sono, SpadePSO, and AHPSO have the best mean values on f_3 , f_6 , and f_7 , respectively. MFCPSO has the best mean values on $f_9 - f_{11}$, $f_{16} - f_{17}$, $f_{23} - f_{24}$, and f_{26} . HMPPSO has the best mean values on f_{19} and f_{22} . EOPSO has the best mean values on f_{20} , f_{25} , f_{27} , and $f_{29} - f_{30}$. Both EVLS-PSOIMLSM and MFCPSO have the best mean values for 8 functions, which is the highest.

Table 8 provides the 100-dimensional experimental results. EVLS-PSOIMLSM exhibits superior mean, minimum, and median values on functions $f_{11} - f_{12}$, $f_{14} - f_{15}$, f_{18} , and f_{25} ; the best mean values on f_2 , $f_4 - f_5$, f_8 , f_{13} , f_{16} , f_{19} , f_{21} , f_{28} ; the best minimum values on f_{16} and f_{24} ; and the best median values on f_2 , $f_4 - f_5$, f_8 , and f_{28} . IMPSOwithCMAR, SQPPSO-sono, SpadePSO, and AHPSO have the best mean values on f_1 , f_3 , f_6 , and f_7 , respectively. MFCPSO has the best mean values on $f_9 - f_{10}$, f_{17} , f_{20} , $f_{22} - f_{24}$, f_{26} , and f_{29} . EOPSO has the best mean values on f_{27} and f_{30} . EVLS-PSOIMLSM has the best mean values for 15 functions, which is the highest. Meanwhile, MFCPSO has the best mean values for 9 functions, which is the second-highest.

In general, EVLS-PSOIMLSM demonstrates outstanding performance across all dimensions, particularly excelling in 100-dimensional tests. In the 50-dimensional test, both EVLS-PSOIMLSM and MFCPSO show an equivalent total number of best mean and median values, while EVLS-PSOIMLSM achieves a total of 6 best minimum values, only slightly worse than MFCPSO, which achieves 7.

4.4. Ablation test

Ablation tests are performed to assess the contribution and effectiveness of the mechanisms within EVLS-PSOIMLSM. Table 9 presents the experimental results for scenarios in which individual mechanisms of EVLS-PSOIMLSM are either utilized or removed. These mechanisms include:

- Inertia weight ω is determined using a uniform distribution between 0 and 1.
- Inertia weight is determined using a chaotic-based ω .
- Types 1-3 have been replaced by cases 1 through 6.
- Using the ISCA but without the SQP.
- Using the SQP but without the ISCA.
- Without the use of the acceleration coefficient c .
- Without a mutation strategy.
- Without elite learning, that is, using $SPbest_i^d(t)$ instead of $SPbest_{i-\frac{N}{2}}^d(t)$ in Eq. (23).

The average Friedman rank of EVLS-PSOIMLSM is 2.83, which is the best. Without the SQP method, the rank drops to 7.03, the worst rank, while not using the ISCA leads to a rank drop to 6.77, the second-worst. This implies the importance and effectiveness of the cooperation between the SQP and ISCA methods. EVLS-PSOIMLSM, using an inertia weight ω of rand(0,1) or a chaotic-based ω , results in worse Friedman ranks of 5.07 and 6.00, respectively. EVLS-PSOIMLSM using the velocity learning strategy for cases 1 to 6, as described in Section 2.4, will lead to a worse Friedman rank of 5.67. EVLS-PSOIMLSM without using the acceleration coefficient c , mutation strategy, and elite learning will lead to worse Friedman ranks of 3.73, 3.97, and 3.93, respectively. In general, the ablation test demonstrates that the current ensemble velocity learning strategy, combined with the integrated mechanisms, is tuned for the best average performance.

Table 6

Experimental results of PSO variants on 30-dimensional CEC2017 benchmark functions over 51 runs, including mean, minimum, and median values, ranks, standard deviations, and Wilcoxon rank sum test results.

	Criteria	EVLS-PSOIMLSM	IMPSOwithCMAR	SQPPSO-samo	EAPSO	IMPPSO	SpadePSO	ADPSO	PSO-samo	MFCPSO	AMSPSO	IEOPSO	PPSO	SDPSO	AHPSO	MPSO
f_1	Mean	2.38863E-03	1.309628E-14	3.995877E-03	3.856516E+03	2.908436E+05	4.721052E+01	2.939427E+03	1.767905E+03	2.277145E+03	2.140239E+03	2.490888E+03	3.715470E+03	5.442955E+02	2.562180E+03	2.680032E+07
	Rank(WRST)	3	1	3	13	1	1	5	6	1	2	1	12	5	10	1
	Min	4.36381E-06	0.000000E+00	1.88074E-03	4.574832E-01	5.68909E+04	4.2	9.114321E+00	6.48349E-01	1.263196E-02	3.030137E+00	2.350732E+00	2.58310E+01	1.266632E+00	2.650497E+01	1.058497E+01
	Rank	2	1	3	5	5	5	13	8	4	1	1	10	6	11	11
	Median	3.564175E-03	1.421085E-14	4.205646E-03	1.774710E+03	1.571972E+05	1.494276E+03	1.494276E+03	1.494276E+03	1.494276E+03	1.72346E+03	1.72346E+03	1.72341E+03	3.399066E+01	1.622761E+03	1.017568E+06
	Std	(1.909140E-03)	(3.858594E-15)	(1.292903E-03)	(6.133294E+03)	(1.128870E+05)	(1.022947E+02)	(3.405095E+03)	(2.728935E+03)	(2.436051E+03)	(2.719812E+03)	(4.26718E+03)	(2.201673E+02)	(2.604599E+03)	(1.003043E+08)	
	Mean	5.98407E-06	3.728325E-05	7.394921E-06	3.596843E-05	1.168036E+11	6.823705E-04	7.544212E+04	4.752296E+16	1.997510E+11	4.299898E+15	1.101908E+12	5.442955E+02	4.271109E-04	1.143508E+00	1.118606E+16
	Rank(WRST)	4	1	2	3	10	1	8	15	11	13	12	5	1	14	1
	Min	9.612299E-08	4.87284E-07	8.178784E-07	8.178784E-07	1.25478E+08	9.43200E-08	9.43200E-08	9.43200E-08	9.43200E-08	9.43200E-08	9.43200E-08	9.43200E-08	9.43200E-08	9.43200E-08	6.355033E-05
	Rank	4	1	2	3	10	1	8	15	11	13	12	5	1	14	1
Median	4.747088E-06	2.969136E-06	4.077982E-06	1.551167E-05	1.038619E+10	3.896373E+01	1.687106E+09	1.687106E+09	1.687106E+09	1.687106E+09	1.687106E+09	1.687106E+09	2.569806E+01	7.168084E-04	8.165107E+03	
Std	(4.431802E-06)	(2.010232E-04)	(1.122434E-05)	(4.309977E-05)	(3.0807771E+11)	(9.368079E-04)	(4.760375E+05)	(3.375942E+17)	(8.886199E+11)	(1.743754E+16)	(6.83845E+12)	(3.381804E+06)	(4.540420E-04)	(8.164528E+00)	(7.72668E+16)	
f_2	Mean	1.95891E-08	4.053687E-08	9.010239E-12	1.681655E-13	1.840419E-04	2.775279E+00	2.638706E-02	3.321621E+02	1.650151E+03	7.536512E+02	2.752860E+04	1.043718E-06	3.06147E-01	1.928896E-02	5.192899E-02
	Rank(WRST)	3	4	5	2	1	6	1	13	12	15	13	5	1	11	2
	Min	6.500102E-09	0.000000E+00	5.68432E-14	5.68432E-14	6.44012E-01	1.39598E-05	2.073719E-03	1.76945E-05	1.56370E+01	1.56962E+02	6.43116E+03	5.59387E-12	6.118253E-05	4.118253E-05	4.118253E-05
	Rank	5	1	2	2	2	2	2	2	2	2	2	2	2	2	2
	Median	1.815079E-08	1.705303E-13	2.27377E-13	1.705303E-13	1.705303E-13	2.27377E-13	2.27377E-13	2.27377E-13	2.27377E-13	2.27377E-13	2.27377E-13	2.27377E-13	2.27377E-13	2.27377E-13	2.27377E-13
	Std	(8.948807E-09)	(8.948807E-09)	(8.948807E-09)	(8.948807E-09)	(8.948807E-09)	(8.948807E-09)	(8.948807E-09)	(8.948807E-09)	(8.948807E-09)	(8.948807E-09)	(8.948807E-09)	(8.948807E-09)	(8.948807E-09)	(8.948807E-09)	(8.948807E-09)
	Mean	1.563382E-01	8.952401E-02	1.646116E+01	3.118817E+01	8.974571E+01	7.129647E+01	9.149829E+01	1.184944E+02	8.684862E+01	2.23079E+02	5.200405E+01	8.718038E+01	2.07458E+01	6.823820E+01	7.887193E+01
	Rank(WRST)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Min	1.115908E-13	8.878954E-12	3.97909E-13	4.66529E-06	4.66529E-06	4.66529E-06	4.66529E-06	4.66529E-06	4.66529E-06	4.66529E-06	4.66529E-06	4.66529E-06	4.66529E-06	4.66529E-06	4.66529E-06
	Rank	2	3	1	1	1	1	1	1	1	1	1	1	1	1	1
Median	1.44845E-12	3.986424E+00	3.986424E+00	3.986424E+00	3.986424E+00	3.986424E+00	3.986424E+00	3.986424E+00	3.986424E+00	3.986424E+00	3.986424E+00	3.986424E+00	3.986424E+00	3.986424E+00	3.986424E+00	
Std	(7.815346E-01)	(1.863771E+01)	(2.542434E+01)	(3.065533E+01)	(3.065533E+01)	(3.065533E+01)	(3.065533E+01)	(3.065533E+01)	(3.065533E+01)	(3.065533E+01)	(3.065533E+01)	(3.065533E+01)	(3.065533E+01)	(3.065533E+01)	(3.065533E+01)	
f_3	Mean	1.342219E-01	1.646688E-01	6.353228E-01	6.353228E-01	1.895341E+02	4.113703E+01	9.277999E+00	1.038796E+01	1.895101E+01	5.118676E+01	2.972564E+01	2.276195E+01	3.201415E+01	2.524329E+01	6.050166E+01
	Rank(WRST)	2	3	4	5	1	2	3	4	5	6	7	8	9	10	11
	Min	6.964713E+00	6.964713E+00	6.964713E+00	6.964713E+00	1.218899E+01	1.218899E+01	1.218899E+01	1.218899E+01	1.218899E+01	1.218899E+01	1.218899E+01	1.218899E+01	1.218899E+01	1.218899E+01	1.218899E+01
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	1.29347E+01	1.492439E+01	1.687520E+01	1.687520E+01	1.895720E+01	1.895720E+01	1.895720E+01	1.895720E+01	1.895720E+01	1.895720E+01	1.895720E+01	1.895720E+01	1.895720E+01	1.895720E+01	1.895720E+01
	Std	(4.213610E+00)	(3.454522E+00)	(1.387099E+01)	(1.387099E+01)	(1.977586E+01)	(1.977586E+01)	(1.977586E+01)	(1.977586E+01)	(1.977586E+01)	(1.977586E+01)	(1.977586E+01)	(1.977586E+01)	(1.977586E+01)	(1.977586E+01)	(1.977586E+01)
	Mean	1.122886E-01	1.122886E-01	2.519144E-01	0.062936E-01	6.579174E+00	2.652936E-13	2.652936E-13	2.652936E-13	2.652936E-13	2.652936E-13	2.652936E-13	2.652936E-13	2.652936E-13	2.652936E-13	2.652936E-13
	Rank(WRST)	1	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	Min	4.340475E-01	6.42170E-01	6.42170E-01	6.42170E-01	6.42170E-01	6.42170E-01	6.42170E-01	6.42170E-01	6.42170E-01	6.42170E-01	6.42170E-01	6.42170E-01	6.42170E-01	6.42170E-01	6.42170E-01
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Median	1.066851E+00	1.066851E+00	1.066851E+00	1.066851E+00	1.066851E+00	1.066851E+00	1.066851E+00	1.066851E+00	1.066851E+00	1.066851E+00	1.066851E+00	1.066851E+00	1.066851E+00	1.066851E+00	1.066851E+00	
Std	(4.083421E-01)	(2.566406E-01)	(3.329526E-01)	(3.329526E-01)	(5.247018E-05)	(1.175661E-05)	(1.175661E-05)	(1.175661E-05)	(1.175661E-05)	(1.175661E-05)	(1.175661E-05)	(1.175661E-05)	(1.175661E-05)	(1.175661E-05)	(1.175661E-05)	
f_4	Mean	4.918156E+01	5.779798E+01	1.020008E+02	1.020008E+02	9.132368E+02	8.101739E+01	9.132368E+02	8.101739E+01	9.132368E+02	8.101739E+01	9.132368E+02	8.101739E+01	9.132368E+02	8.101739E+01	9.132368E+02
	Rank(WRST)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Min	3.87451E+01	4.340837E+01	4.340837E+01	4.340837E+01	4.340837E+01	4.340837E+01	4.340837E+01	4.340837E+01	4.340837E+01	4.340837E+01	4.340837E+01	4.340837E+01	4.340837E+01	4.340837E+01	4.340837E+01
	Rank	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	Median	4.798176E+01	5.72794E+01	9.242602E+01	9.242602E+01	2.395171E+02	8.17356E+01	9.13715E+01	8.17356E+01	9.13715E+01	8.17356E+01	9.13715E+01	8.17356E+01	9.13715E+01	8.17356E+01	9.13715E+01
	Std	(6.604428E+00)	(6.949272E+00)	(4.85392E+01)	(4.85392E+01)	(1.174761E+01)	(9.935115E+00)	(1.833805E+01)	(9.935115E+00)	(1.833805E+01)	(9.935115E+00)	(1.833805E+01)	(9.935115E+00)	(1.833805E+01)	(9.935115E+00)	(1.833805E+01)
	Mean	1.414245E+01	1.574376E+01	7.912821E+01	6.825406E+01	1.932800E+02	4.748954E+01	3.67155E+01	3.283508E+01	1.876106E+01	3.287357E+01	2.257190E+01	3.452140E+01	2.278531E+01	5.965722E+01	9.12244E+01
	Rank(WRST)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Min	6.964713E+00	7.95972E+00	2.487396E+01	3.780840E+01	1.687230E+02	2.188990E+01	1.79052E+01	1.293477E+01	8.963831E+00	4.974795E+00	6.365056E+00	1.29346E+01	1.691400E+01	6.949991E+00	2.686030E+01
	Rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Median	1.392943E+01	1.591934E+01	7.561677E+01	1.950265E+02	4.612512E+01	3.581849E+01	3.084371E+01	1.820113E+01	7.337861E+01	1.555000E+01	2.188990E+01	5.289703E+01	2.08941E+01	5.776437E+01	9.95256E+01	
Std	(3.858693E+00)	(3.841650E+00)	(4.016763E+01)	(1.897494E+01)	(1.032652E+01)	(1.198594E+01)	(1.198594E+01)	(1.198594E+01)	(1.198594E+01)	(1.198594E+01)	(1.198594E+01)	(1.198594E+01)	(1.198594E+01)	(1.198594E+01)	(1.198594E+01)	
f_5	Mean	6.641871E-01	5.471623E-01	6.299172E+00	6.982466E-01	9.097006E+01	3.967138E+00	4.208920E+01	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank(WRST)	6	5	1	4	1	2	3	4	5	6	7	8	9	10	11
	Min	8.952825E-02	2.501110E-12	2.685848E-01	1.136868E-13	1.995505E+01	1.79055E-01	1.45818E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank	10														

Table 6 (continued).

Criteria	EVLS-PSOIMLM	IMPSOwithCMAR	QPPSO-smo	EAPSO	IMPPSO	SpadePSO	ADPSO	PSO-smo	MFCPSO	AMSEPSO	IOPSO	PPSO	SDPSO	AHPSO	MFPSO	
f_n	Mean	3.158981E+02	5.244432E+02	6.971780E+02	8.817022E+02	1.250111E+03	5.456284E+02	5.598639E+02	5.984500E+02	4.234305E+02	6.104293E+02	4.081239E+02	7.114449E+02	4.022516E+02	4.484738E+02	8.267855E+02
	Min	1.092100E+01	4.1	1.307854E+01	3.116229E+02	1.307854E+01	1.494777E+02	1.368327E+02	1.143802E+02	1.639743E+02	2.839424E+01	1.081359E+02	1.779602E+01	1.2	1.241013E+01	1.366422E+02
	Rank	3	1	1	14	15	12	10	11	8	2	2	1	4	9	5
	Median	3.963333E+02	5.110146E+02	4.93896E+02	8.43266E+02	1.277463E+03	5.851651E+02	5.865406E+02	6.003456E+02	4.830818E+02	6.19886E+02	7.396673E+02	5.35836E+02	6.253505E+02	6.253505E+02	9
	Rank	1	6	11	13	15	9	10	2	3	8	2	12	5	7	12
	(Std Dev)	1.1756092E+02	2.223345E+02	2.8244088E+02	2.857440E+02	1.628310E+02	1.708233E+02	1.266686E+02	1.237241E+02	1.283825E+02	1.253616E+02	1.217338E+02	1.725996E+02	1.725996E+02	1.984501E+02	12.280564E+02
	Mean	1.854709E+02	1.489481E+02	1.526202E+02	4.014314E+02	4.082312E+02	1.022394E+02	1.018623E+02	1.906644E+02	1.641415E+02	1.480750E+02	1.242274E+02	2.265448E+02	6.459355E+01	1.525480E+02	2.068775E+02
f_n	Min	4.107626E+01	2.65964E+01	4.94844E+00	4.255179E+01	2.409129E+02	3.369743E+01	3.790844E+01	5.50022E+01	1.520067E+01	1.520067E+01	1.229633E+01	3.541354E+01	1.3	3.912701E+02	4.997237E+01
	Rank	10	3	1	3	14	2	2	13	13	2	2	1	14	14	12
	Median	1.811463E+02	1.51631E+02	1.128179E+02	3.11234E+02	4.452323E+02	2.568895E+01	8.077157E+01	1.584763E+02	1.750386E+02	1.358996E+02	4.011679E+02	6.85932E+02	7.497263E+01	1.346521E+02	2.950575E+02
	Rank	11	8	5	1	15	2	15	10	10	10	12	12	1	12	10
	(Std Dev)	7.623994E+01	8.813144E+01	1.165133E+02	12.93974E+02	8.132406E+01	7.900377E+01	6.304187E+01	1.021478E+02	1.611695E+01	6.431356E+01	6.872615E+01	1.791887E+02	1.382396E+01	6.826455E+01	1.683635E+02
	Mean	2.850882E+01	1.507411E+02	1.410343E+02	2.483700E+04	4.084952E+01	4.471887E+04	7.309664E+04	4.515197E+04	1.12303E+05	2.63362E+05	2.213464E+05	1.59279E+05	9.184992E+04	6.191747E+04	2.242971E+04
	Min	1	2	2	1	2	1	2	1	1	1	1	1	1	1	1
f_n	Rank	1	4	4	1	1	1	1	1	1	1	1	1	1	1	1
	Median	2.741230E+01	5.705452E+01	2.328010E+01	3.801337E+01	3.820944E+01	2.928992E+01	5.192588E+02	4.573321E+04	1.119303E+04	4.573321E+04	1.388181E+04	2.398977E+04	5.473403E+03	1.664209E+04	2.879636E+03
	Rank	1	4	4	1	1	1	1	1	1	1	1	1	1	1	1
	(Std Dev)	1.543335E+00	7.384888E+01	6.398644E+01	1.768875E+04	4.22964E+01	4.944957E+04	9.375848E+04	7.016345E+04	1.930038E+05	1.132461E+05	1.062107E+05	9.476055E+04	8.765005E+04	5.243335E+04	1.795235E+04
	Mean	8.633388E+01	8.391664E+01	9.673197E+01	1.845693E+02	3.678813E+01	1.845693E+02	4.522383E+01	3.889264E+01	5.253444E+01	5.728503E+01	3.673620E+01	3.838160E+02	6.159366E+01	6.159366E+01	3.055462E+01
	Min	2.807586E+01	1.542972E+01	1.458513E+01	4.404233E+01	2.526871E+01	2.250451E+01	2.036300E+01	3.474219E+01	4.131733E+01	4.533762E+01	1.458594E+01	7.616975E+01	1.615361E+01	1.203134E+01	1.243779E+02
	Rank	3	1	1	1	2	2	3	4	11	11	1	14	14	14	1
f_n	Median	8.318246E+01	7.620713E+01	7.584292E+01	2.064223E+02	3.655888E+01	8.326619E+01	2.064223E+02	4.586096E+01	3.63687E+01	3.63687E+01	1.612113E+02	2.569932E+02	3.209088E+01	2.959152E+01	2.959152E+01
	Rank	3	1	1	1	2	2	3	4	11	11	1	14	14	14	1
	(Std Dev)	3.256288E+01	7.384888E+01	8.716853E+01	1.338623E+04	1.338623E+04	1.338623E+04	1.338623E+04	6.683767E+03	4.221813E+03	5.962953E+03	4.450031E+03	1.899913E+03	7.572660E+02	7.572660E+02	1.339092E+02
	Mean	1.199925E+02	1.908656E+02	1.320188E+02	1.295408E+02	1.320188E+02	1.320188E+02	1.295408E+02	1.320188E+02	1.985417E+02	1.490723E+02	1.535253E+02	3.081724E+02	1.424955E+02	2.125303E+02	2.571932E+02
	Min	3.411971E+01	2.36059E+01	4.353192E+01	2.880271E+01	2.900112E+01	3.580652E+01	3.580652E+01	3.801376E+01	1.580566E+02	4.453241E+00	2.669322E+01	1.548017E+02	3.799904E+01	8.091044E+01	3.573161E+01
	Rank	1	2	1	2	2	2	2	2	1	1	1	1	1	1	1
	Median	9.938751E+01	2.033717E+02	1.895012E+02	1.305524E+02	1.375071E+02	1.610482E+02	1.610482E+02	1.907807E+02	1.331074E+02	1.610478E+02	2.569932E+02	1.612113E+02	2.569932E+02	2.118902E+02	2.118902E+02
f_n	Rank	1	9	9	12	12	12	12	12	12	12	12	12	12	12	12
	(Std Dev)	6.972506E+01	8.925321E+01	1.077600E+02	1.181874E+02	1.514475E+02	6.822467E+01	6.822467E+01	9.383794E+01	3.278380E+01	8.154846E+01	5.141704E+01	1.147693E+02	1.730335E+01	4.915314E+01	1.319092E+02
	Mean	2.138411E+02	2.670576E+02	2.670576E+02	2.670576E+02	2.670576E+02	2.670576E+02	2.670576E+02	2.338781E+02	2.266635E+02	2.722777E+02	2.238336E+02	2.256419E+02	2.275581E+02	2.631308E+02	2.631308E+02
	Min	1.00000000161626E+02	1.0000000282989E+02	1.2172857949206814E+02	1.835809531648806E+02	1.16357874900074971E+02	2.20639217769120E+02	2.116334221747367E+02	2.120624919275983E+02	2.071960433472043E+02	2.07063382426237E+02	2.116462894946018E+02	1.0000000000000000E+02	1.0000000000000000E+02	2.33120619086989E+02	2.33120619086989E+02
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	2.168953E+02	2.704845E+02	2.628005E+02	2.830533E+02	2.815751E+02	2.336741E+02	2.786685E+02	2.336741E+02	2.786685E+02	2.786685E+02	2.234406E+02	2.274875E+02	2.274875E+02	2.631308E+02	2.631308E+02
	(Std Dev)	2.418879E+01	3.347630E+01	3.437280E+01	3.437280E+01	3.437280E+01	3.437280E+01	3.437280E+01	3.437280E+01	3.437280E+01	3.437280E+01	3.437280E+01	3.437280E+01	3.437280E+01	3.437280E+01	3.437280E+01
f_n	Mean	6.593500E+02	1.511318E+02	1.868797E+02	2.040298E+02	1.129655E+02	1.001927E+02	1.673459E+02	1.000000E+02	3.84919E+02	1.000000E+02	1.002890E+02	1.002890E+02	1.002890E+02	1.002890E+02	1.002890E+02
	Min	1.000000030554505E+02	1.00000000747545E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	1.00000000855868E+02	1.00000007501960E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	(Std Dev)	8.219996E+02	3.651801E+02	6.189598E+02	1.842636E+03	1.320140E+03	1.001927E+02	1.673459E+02	1.000000E+02	3.84919E+02	1.000000E+02	1.002890E+02	1.002890E+02	1.002890E+02	1.002890E+02	1.002890E+02
	Mean	3.739848E+02	3.707611E+02	4.193591E+02	4.171884E+02	5.155092E+02	3.947210E+02	3.875144E+02	3.917359E+02	3.736346E+02	4.301122E+02	3.897554E+02	3.817511E+02	3.897554E+02	3.891642E+02	4.373698E+02
f_n	Min	3.604158E+02	3.546706E+02	3.602505E+02	3.738795E+02	3.630101E+02	3.630101E+02	3.630101E+02	3.630101E+02	3.630101E+02	3.630101E+02	3.630101E+02	3.630101E+02	3.630101E+02	3.630101E+02	3.630101E+02
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	3.740744E+02	3.682952E+02	4.158822E+02	4.106166E+02	5.155092E+02	3.947210E+02	3.875144E+02	3.917359E+02	3.736346E+02	4.301122E+02	3.897554E+02	3.817511E+02	3.897554E+02	3.891642E+02	4.373698E+02
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	(Std Dev)	8.737670E+00	1.013339E+01	2.326302E+01	5.674904E+01	1.237305E+01	1.237305E+01	1.237305E+01	1.237305E+01	1.237305E+01	1.237305E+01	1.237305E+01	1.237305E+01	1.237305E+01	1.237305E+01	1.237305E+01
	Mean	4.241272E+02	4.194184E+02	4.913758E+02	4.903473E+02	5.10774E+02	4.585191E+02	4.552501E+02	4.529645E+02	4.402515E+02	5.006866E+02	4.515733E+02	4.541826E+02	4.320838E+02	4.753491E+02	4.988780E+02
	Min	1.0000000000000000E+02	1.00000015995811E+02	1.1	4.50839698182945E+02											

Table 7

Experimental results of PSO variants on 50-dimensional CEC2017 benchmark functions over 51 runs, including mean, minimum, and median values, ranks, standard deviations, and Wilcoxon rank sum test results.

	Criteria	EVLS-PSOIMLSM	IMPSOwithCMAR	SQPPSO-somo	EAPSO	IMPPSO	SpadePSO	ADPSO	PSO-somo	MFPSO	AMSPSO	ICPSO	PPSO	SDPSO	AHPSO	MPSO
f ₁	Mean	7.305756e-03	1.421085E-14	1.492106e-02	5.549533e+03	1.162201E+08	2.581034E+02	2.963022E+03	3.360690E+03	2.831076E+03	3.289099E+03	2.133955E+03	3.643616E+03	3.199551E+02	3.065201E+03	1.448386E+09
	Rank(WRST)	1	1	3	1	14	1	8	11	1	1	6	1	5	9	15
	Min	1.145816E-04	1.421085E-14	1.125628E-02	1.891912E+01	1.254835E+07	1.637178E+01	2.462969E+00	3.420332E-02	2.281500E-04	2.406154E-01	1.624629E-02	2.094175E-01	5.159964E-00	2.298137E+01	5.729391E+00
	Rank	2	1	1	12	15	1	10	10	1	1	6	1	6	11	14
	Median	7.224686E-03	1.530829E-04	2.070726E+03	1.721778E+07	1.212325E+03	1.506543E+03	1.184472E+03	1.368011E+03	1.877346E+03	1.582415E+03	2.106716E+03	1.607175E+03	1.7716E+02	1.807171E+03	1.918033E+09
	(Std Dev)	(3.829534E-03)	(0.000000E+00)	(1.289461E-03)	(6.582562E+03)	(1.082516E+08)	(4.529499E+02)	(3.436126E+03)	(4.131681E+03)	(5.878189E+03)	(3.736550E+03)	(2.506675E+03)	(3.977236E+03)	(5.977236E+03)	(1.350781E+09)	
	Mean	8.390439E-06	8.873053E-06	1.007372E-05	3.075456E-05	5.942413E+32	2.952057E+01	1.079167E+14	1.620203E+38	2.442275E+26	2.887288E+38	8.423998E+36	1.314448E+17	1.266192E+01	1.253377E+15	1.614009E+36
	Rank(WRST)	1	2	3	4	1	6	7	14	15	15	9	9	13	12	15
	Min	4.141345E-07	3.043878E-06	1.296625E-06	1.118792E-09	1.030603E+21	5.106740E-04	8.152350E+03	3.293374E+15	1.448288E+15	5.578814E+09	4.842770E+01	3.272074E+05	8.053534E-06	2.298137E+01	6.560844E+05
	Rank	2	4	5	6	1	15	12	13	12	12	10	10	14	11	14
Median	7.259796E-07	7.757073E-06	2.697468E-06	2.045401E-05	1.863333E+27	6.499621E+01	7.730205E+09	4.641650E+22	1.871370E+18	6.046654E+29	8.497761E+08	1.070632E+13	2.403333E+01	1.774615E+02	1.774615E+02	
(Std Dev)	(8.046668E-06)	(4.959120E-06)	(9.629779E-06)	(4.293446E-05)	(4.186165E+33)	(5.308530E+01)	(5.980385E+14)	(1.154395E+39)	(2.046425E+39)	(6.015938E+37)	(3.696545E+17)	(2.617592E+01)	(8.965182E+15)	(1.132343E+37)		
f ₂	Mean	3.211592E-07	5.101328E-07	5.715055E-09	1.162047E+00	2.271783E+03	2.772689E+01	2.138276E+01	1.141846E+02	1.719241E+04	1.854148E+04	1.762548E+04	7.323617E+04	9.784911E-01	5.938086E+02	3.896739E+02
	Rank(WRST)	2	3	1	1	1	1	1	8	8	8	8	8	1	1	1
	Min	1.950048E-07	5.604338E-14	7.355338E-11	2.758484E-12	1.343911E+02	2.010667E+00	4.333641E+00	1.305255E+01	8.030406E+03	1.959088E+03	9.098461E+03	2.384641E-03	1.230040E+01	1.070484E+02	3.076491E+00
	Rank	4	2	2	2	2	2	2	9	9	9	9	9	15	10	10
	Median	3.258635E-07	8.275297E-07	2.923666E-06	5.363035E-06	2.024800E+03	2.349486E+01	6.867796E+01	1.767396E+04	1.778696E+04	1.778696E+04	1.778696E+04	1.778696E+04	2.954546E-01	5.337996E+02	2.437437E+01
	(Std Dev)	(6.434398E-08)	(4.688673E-07)	(8.927626E-09)	(8.254714E+00)	(1.932404E+03)	(2.304175E+01)	(1.599671E+01)	(1.319808E+02)	(2.750197E+03)	(2.750197E+03)	(2.750197E+03)	(2.750197E+03)	(1.856554E-00)	(5.938086E+02)	(5.938086E+02)
	Mean	5.418651E-11	2.077121E+01	3.973509E+01	3.880577E+01	1.802154E+02	6.467359E+01	1.342418E+02	2.244000E+02	1.254295E+02	2.494854E+02	1.002541E+02	1.082302E+02	6.477297E+01	7.374902E+01	4.079900E+02
	Rank(WRST)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Min	9.633811E-13	3.115908E-12	7.390444E-13	1.719224E-04	6.599396E+01	2.991948E+01	1.912958E+02	6.811523E+01	9.893836E+01	2.022488E+02	2.038213E+01	3.734169E-01	1.575456E-05	1.084369E+02	6.940713E+01
	Rank	2	3	1	1	2	3	4	5	6	7	8	9	10	11	12
Median	2.444276E-12	2.851271E+01	3.851271E+01	3.746848E+01	1.917823E+02	7.946848E+01	1.208424E+02	2.228523E+02	1.084041E+02	2.529664E+02	8.834229E+01	1.007771E+02	6.478230E+01	7.263751E+01	1.079999E+02	
(Std Dev)	(8.869691E+00)	(2.659908E+01)	(4.864261E+01)	(4.659929E+01)	(6.712059E+01)	(5.087268E+01)	(5.493947E+01)	(1.319808E+02)	(2.750197E+03)	(2.750197E+03)	(2.750197E+03)	(2.750197E+03)	(1.856554E-00)	(5.938086E+02)	(5.938086E+02)	
f ₃	Mean	2.920496E-01	1.437855E-01	1.810622E-02	1.608282E-02	3.642902E+12	1.932092E+01	6.963173E-01	4.122477E-01	1.186903E+02	5.013956E+01	5.385346E-01	8.043575E-01	4.325574E-01	1.150039E+02	9.1
	Rank(WRST)	1	2	3	4	1	2	3	4	5	6	7	8	9	10	11
	Min	1.391945E-01	2.884064E-01	2.27758E-01	2.27758E-01	4.775799E+01	4.673305E+01	4.673305E+01	2.242556E+01	9.949991E-01	1.951397E+01	3.880337E-01	2.785884E-01	2.785884E-01	2.785884E-01	2.785884E-01
	Rank	2	3	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	2.686388E-01	3.880339E-01	1.840667E+02	1.646808E+02	9.253101E+01	6.698202E+01	5.666725E-01	3.834547E+01	1.516318E+02	3.076120E+01	6.069241E-01	7.860133E-01	7.860133E-01	7.860133E-01	7.860133E-01
	(Std Dev)	(8.537699E-06)	(1.240199E-01)	(8.12227E+01)	(4.23897E+01)	(1.792595E+01)	(1.955757E+01)	(1.548528E+01)	(1.773396E+01)	(1.194751E+01)	(8.065123E+01)	(5.343031E+01)	(1.016706E+01)	(2.243982E+01)	(1.019546E+01)	(2.243982E+01)
	Mean	3.36812E-00	3.100738E-00	1.677134E+00	6.038981E-02	1.155031E+02	4.748097E-13	8.367981E-01	5.102702E-01	1.930501E-06	2.135423E-06	8.480219E-01	1.172754E-05	5.589851E-02	2.776444E+00	1.6
	Rank(WRST)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Min	1.481496E+00	1.903075E+00	6.022340E-01	1.000000E-00	7.305688E+00	2.27377E-13	1.867075E-05	5.347456E-09	2.171419E-11	1.680111E-07	4.779648E-02	1.000000E-00	8.3961342E-03	5.87237E-02	5.87237E-02
	Rank	3	4	5	1	6	1	1	1	1	1	1	1	1	1	1
Median	3.492500E+00	3.008539E+00	1.475384E+00	1.000000E-00	1.126900E+01	4.547474E-13	3.127032E-01	9.972026E-01	1.948080E-06	7.234169E-04	1.031271E-06	4.889552E-08	1.676700E+00	1.676700E+00	1.676700E+00	
(Std Dev)	(9.782146E-01)	(8.954569E-01)	(9.046118E-01)	(2.186014E-01)	(1.766767E+00)	(1.152807E-13)	(5.093682E-01)	(1.082932E-01)	(1.994699E-06)	(2.840874E-03)	(2.854064E-06)	(7.341110E-01)	(1.765935E-02)	(5.338722E-02)	(5.338722E-02)	
f ₄	Mean	1.068914E+02	1.199797E+02	1.797835E+02	1.486967E+02	1.713277E+02	1.183830E+02	1.494903E+02	1.015710E+02	2.856524E+02	1.397294E+02	1.075962E+02	1.811031E+02	1.075962E+02	1.075962E+02	1.075962E+02
	Rank(WRST)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Min	8.901624E+01	9.309458E+01	7.873345E+01	4.417854E+02	1.220121E+02	1.04521E+02	8.513106E+01	6.850650E+01	6.638437E-01	7.984808E+01	8.948525E-01	1.131385E+02	6.945898E+01	1.529855E+02	1.529855E+02
	Rank	2	3	4	1	2	3	4	5	6	7	8	9	10	11	12
	Median	1.032956E+02	1.154974E+02	1.513606E+02	4.811564E+02	1.722477E+02	1.965711E+02	1.145010E+02	9.804327E+01	2.919703E+02	1.079919E+02	1.066075E-01	1.914701E+02	8.701400E-01	2.282144E+02	2.282144E+02
	(Std Dev)	(1.333646E-01)	(1.806667E+01)	(7.452697E+01)	(2.846094E+01)	(2.128938E+01)	(1.591720E+01)	(3.249133E+01)	(2.009062E+01)	(6.415435E+01)	(1.369977E+01)	(1.295512E+01)	(8.217197E+01)	(1.369977E+01)	(1.431359E+01)	(1.431359E+01)
	Mean	2.780032E-01	4.188506E-01	1.731806E+02	3.596327E+02	8.862574E+01	6.679047E+01	3.983398E+01	1.426242E+02	5.350338E+01	6.057338E+01	8.119342E+01	4.499863E+01	1.541333E+02	1.541333E+02	1.541333E+02
	Rank(WRST)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Min	1.492439E+01	2.487397E+01	4.079330E+01	9.651082E+01	3.219497E+02	5.571765E+01	3.482255E+01	2.496691E+01	1.293447E-01	1.690407E+01	4.178826E+01	2.586891E+01	9.014392E+01	9.014392E+01	9.014392E+01
	Rank	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Median	2.686388E-01	4.079330E+01	3.576404E+02	8.357642E+01	3.566724E+01	3.681513E+01	3.566724E+01	1.729769E+02	3.084420E+01	6.168737E-01	1.677068E+01	1.437781E+01	1.437781E+01	1.437781E+01	1.437781E+01	
(Std Dev)	(7.084246E+00)	(9.439533E+00)	(7.219682E+01)	(3.200800E+01)	(1.640917E+01)	(2.313751E+01)	(1.497744E+01)	(2.064494E+01)	(1.179959E+01)	(6.012072E+01)	(9.366649E+00)	(1.979919E+01)	(1.487343E+01)	(1.337046E+01)	(1.337046E+01)	
f ₅	Mean	1.379289E+01	2.984579E+01	4.318207E+01	1.641761E+01	7.252972E+01	2.249734E+02	2.276034E+02	8.247869E-14	1.789200E+00	5.098308E-01	1.648547E+02	2.426176E+00	8.088437E+02	8.088437E+02	8.088437E+02
	Rank(WRST)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Min	8.753887E-12	6.952825E-02	5.989037E+00	1.739261E+00	3.967405E+02	7.166269E+00	3.435426E+01	8.124370E-01	0.						

Table 7 (continued).

f_n	Criteria	ENVIS-PSOIMLM	IMPSOwithCMAR	SOPSO-somo	EAPSO	HMPSO	SpadePSO	ADPSO	PSO-somo	MPCPSO	AMSPSO	IOPSO	PPSO	SDPSO	AHPDSO	MPDSO
f_{10}	Mean	5.360188E+02	8.749776E+02	1.267315E+03	1.396872E+03	2.685409E+03	1.071580E+03	9.166223E+02	1.443018E+03	4.625890E+02	1.351194E+03	5.955517E+02	1.096764E+03	9.210356E+02	7.113881E+02	1.397896E+03
	Rank(WRST)	5	5	10	12	15	11	13	14	1	3	9	10	7	13	8
	Min	8.576387E+00	4.740232E+02	2.367911E+02	5.231605E+02	2.302386E+03	4.388670E+02	4.731083E+02	2.570502E+02	2.531172E+02	1.398350E+02	7.479303E+02	4.151693E+02	2.479303E+02	1.860772E+02	1.516937E+02
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	5.021398E+02	1.878485E+02	1.439232E+03	1.720727E+03	2.503743E+03	1.082043E+03	1.129272E+02	1.584256E+03	4.248518E+02	1.325584E+03	5.504015E+02	1.066233E+03	5	5	8.440296E+02
	(Std Dev)	(2.232004E+02)	(2.562385E+02)	(4.859395E+02)	(4.661882E+02)	(2.220780E+02)	(2.210084E+02)	(2.222808E+02)	(3.546319E+02)	(1.397955E+02)	(2.649806E+02)	(2.441816E+02)	(3.523326E+02)	(2.958573E+02)	(4.360450E+02)	(4.360450E+02)
f_{11}	Mean	8.124051E+02	8.325708E+02	8.267629E+02	1.095312E+03	1.768856E+02	8.192251E+02	9.19448E+02	1.101292E+03	4.662655E+02	1.066655E+03	1.097071E+02	5.936957E+02	7.564163E+02	7.510525E+02	1.097071E+02
	Rank(WRST)	8	8	7	12	12	12	11	11	2	4	3	10	10	13	13
	Min	2.761540E+02	3.738985E+02	1.818294E+02	3.607280E+02	3.943414E+02	2.787902E+02	3.640077E+02	2.230535E+02	4.499007E+02	3.548991E+02	4.761773E+02	3.100491E+02	3.599539E+02	6.188006E+02	6.188006E+02
	Rank	3	3	1	10	10	10	8	8	1	2	11	12	11	7	7
	Median	3.141142E+02	8.167427E+02	4.434342E+02	1.102033E+03	1.756883E+03	8.911426E+02	8.911426E+02	1.254006E+03	4.659997E+02	1.059383E+03	6.346688E+02	8.227966E+02	6.057893E+02	7.236681E+02	1.104955E+02
	(Std Dev)	(2.572326E+02)	(2.110702E+02)	(3.886638E+02)	(3.192036E+02)	(1.465958E+02)	(1.919970E+02)	(2.907536E+02)	(2.412864E+02)	(2.491056E+02)	(2.491056E+02)	(2.890157E+02)	(1.943703E+02)	(2.380403E+02)	(2.380403E+02)	(2.380403E+02)
f_{12}	Mean	3.527883E+01	1.928761E+02	1.877466E+02	2.167635E+04	9.98192E+01	1.617434E+05	1.490972E+05	4.395224E+05	7.67078E+05	7.680388E+05	9.88763E+05	2.896205E+05	4.736600E+04	8.482217E+04	9.62884E+04
	Rank(WRST)	1	2	3	1	2	1	2	1	2	1	2	1	1	2	1
	Min	2.366797E+01	4.000263E+02	5.729138E+01	5.830751E+03	7.480019E+01	3.094176E+04	2.655111E+04	3.874208E+04	3.710637E+04	1.987771E+05	1.826295E+05	7.969909E+03	1.647257E+04	5.699661E+03	5.699661E+03
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	1.343781E+01	1.945388E+02	2.241023E+04	8.787536E+04	9.760229E+01	9.834870E+04	1.836995E+05	4.231187E+05	5.78177E+05	2.660188E+05	4.221956E+04	7.258031E+04	6.991854E+04	6.991854E+04	6.991854E+04
	(Std Dev)	(7.835296E+00)	(8.888484E+01)	(6.129847E+01)	(1.096411E+04)	(1.132004E+01)	(1.930991E+05)	(1.738107E+05)	(1.120542E+06)	(2.870117E+05)	(3.993336E+05)	(5.928695E+05)	(2.890157E+02)	(2.707818E+04)	(3.369048E+04)	(6.658191E+04)
f_{13}	Mean	9.842577E+01	2.270757E+02	1.237428E+02	1.318789E+04	1.237428E+02	5.423435E+02	1.309519E+04	1.138241E+04	1.095404E+04	1.42628E+04	1.42628E+04	1.005418E+03	1.426306E+04	1.426306E+04	1.426306E+04
	Rank(WRST)	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Min	5.202147E+01	2.711269E+01	5.600172E+01	2.943192E+01	5.987896E+01	4.576315E+01	1.670533E+01	1.462106E+02	4.317725E+01	3.138657E+01	3.138657E+01	3.368548E+01	9.389714E+01	1.806239E+01	1.806239E+01
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	9.130088E+01	1.531041E+02	1.117028E+02	1.318789E+04	1.237428E+02	5.423435E+02	1.309519E+04	1.138241E+04	1.095404E+04	1.42628E+04	1.42628E+04	1.005418E+03	1.426306E+04	1.426306E+04	1.426306E+04
	(Std Dev)	(2.807327E+01)	(3.003236E+02)	(4.453786E+01)	(1.325041E+04)	(6.260651E+00)	(8.636200E+02)	(6.197581E+03)	(8.962277E+03)	(5.345737E+03)	(6.884609E+03)	(6.76247E+03)	(7.148714E+03)	(1.776253E+03)	(2.351856E+03)	(5.586719E+03)
f_{14}	Mean	3.837568E+02	5.255028E+02	1.046090E+03	9.227323E+02	1.096836E+03	5.255028E+02	4.965729E+02	6.410338E+02	3.119788E+02	7.057659E+02	1.091371E+02	5.55807E+02	4.264037E+02	8.246358E+02	8.157251E+02
	Rank(WRST)	2	3	3	1	1	1	1	1	1	1	1	1	1	1	1
	Min	1.067029E+02	2.444332E+02	3.777622E+02	3.112665E+02	3.112665E+02	7.146175E+02	1.576565E+02	1.576565E+02	5.930919E+01	2.572195E+02	1.071849E+02	1.552470E+02	1.835422E+02	2.216242E+02	2.216242E+02
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	3.844150E+02	5.699450E+02	8.880335E+02	1.111024E+03	1.111024E+03	5.071011E+02	6.313616E+02	6.313616E+02	3.265138E+02	7.488303E+02	2.243458E+02	3.398373E+02	3.730049E+02	8.824728E+02	8.824728E+02
	(Std Dev)	(1.680360E+02)	(1.741841E+02)	(3.403616E+02)	(3.424803E+02)	(1.327768E+02)	(2.048110E+02)	(7.148626E+02)	(2.415021E+02)	(9.323733E+01)	(2.084081E+02)	(2.039406E+02)	(2.539478E+02)	(1.91587E+02)	(1.57707E+02)	(2.944978E+02)
f_{15}	Mean	2.290709E+02	2.438148E+02	3.513209E+02	3.314771E+02	2.849528E+02	2.712274E+02	2.685606E+02	2.685606E+02	2.391882E+02	2.482045E+02	2.60575E+02	2.711775E+02	2.479754E+02	2.479754E+02	2.479754E+02
	Rank(WRST)	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2
	Min	1.000000E+02	2.297297E+02	2.245444E+02	2.826222E+02	2.520358E+02	2.303314E+02	2.249598E+02	2.249598E+02	2.199556E+02	2.218858E+02	2.358075E+02	2.358075E+02	2.287575E+02	2.287575E+02	2.287575E+02
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	2.297237E+02	2.415411E+02	3.56328E+02	2.712248E+02	2.561861E+02	2.241187E+02	2.267942E+02	2.267942E+02	2.379742E+02	2.371280E+02	2.600431E+02	2.685481E+02	2.472689E+02	2.472689E+02	2.472689E+02
	(Std Dev)	(2.041406E+01)	(1.206278E+01)	(3.131350E+01)	(3.131350E+01)	(1.612505E+01)	(2.028010E+01)	(9.115678E+01)	(1.847605E+01)	(1.006341E+01)	(5.803663E+01)	(4.950688E+01)	(1.285911E+01)	(1.682502E+01)	(1.241945E+01)	(2.770324E+01)
f_{16}	Mean	3.656652E+03	3.030234E+03	6.181686E+03	1.948185E+03	2.517646E+03	3.471740E+03	3.77985E+03	6.548555E+03	3.471740E+03	6.548555E+03	2.775257E+03	2.626304E+03	1.309848E+03	1.309848E+03	1.309848E+03
	Rank(WRST)	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Min	2.5620479281542E+03	4.00000057447592E+02	1.0000000000000000E+02	1.9634320380276E+03	1.463199917109105E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02	1.0000000000000000E+02
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	3.621614291320764E+03	1.4972020524921E+03	6.688290437861342E+03	1.861545504069659E+03	1.61401087104681E+03	1.04622613709247E+03	1.61401087104681E+03	1.61401087104681E+03	1.61401087104681E+03	1.61401087104681E+03	1.61401087104681E+03	1.61401087104681E+03	1.61401087104681E+03	1.61401087104681E+03	1.61401087104681E+03
	(Std Dev)	(6.173803E+02)	(2.510917E+03)	(9.921364E+03)	(3.124664E+01)	(2.412664E+01)	(2.250999E+03)	(1.862263E+03)	(7.622652E+03)	(1.862263E+03)	(1.826560E+03)	(3.128884E+03)	(2.554850E+03)	(2.331000E+03)	(2.204351E+03)	(2.217772E+03)
f_{17}	Mean	4.734732E+02	4.760232E+02	5.71719E+02	5.697323E+02	7.990936E+02	4.960729E+02	5.208437E+02	4.960729E+02	4.560237E+02	5.92425E+02	4.90246E+02	5.32380E+02	5.135125E+02	5.05419E+02	6.42839E+02
	Rank(WRST)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Min	4.392654E+02	4.426186E+02	4.641880E+02	4.760606E+02	4.737462E+02	4.680877E+02	4.74731E+02	4.379356E+02	4.264335E+02	4.379356E+02	4.335694E+02	4.797198E+02	4.645737E+02	4.480077E+02	5.124621E+02
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	4.708001E+02	4.730454E+02	5.65529E+02	5.789789E+02	5.302121E+02	4.988112E+02	5.179647E+02	4.670894E+02	4.330698E+02	4.87673E+02	5.257998E+02	5.119978E+02	5.06581E+02	5.33605E+02	6.403721E+02
	(Std Dev)	(1.970381E+01)	(1.711030E+01)	(7.781068E+01)	(2.29356E+01)	(2.373768E+01)	(1.978696E+01)	(2.424732E+01)	(3.30858E+01)	(2.424732E+01)	(7.321170E+01)	(3.88240E+01)	(3.269326E+01)	(2.71131E+01)	(2.545117E+01)	(6.403721E+02)
f_{18}	Mean	5.755525E+02	5.359665E+02	6.415290E+02	6.254736E+02	8.437769E+02	6.619033E+02	5.743471E+02	5.398201E+02	5.88282E+02	5.837099E+02	5.95431E+02	5.837099E+02	6.525299E+02	6.897957E+02	6.897957E+02
	Rank(WRST)	3	3	1	1	1	1	1	1	1	1	1	1	1	1	1
	Min	5.260618E+02	5.179178E+02	5.275424E+02	5.773142E+02	5.471857E+02	5.293658E+02	5.326240E+02	5.199002E+02	5.007654E+02	5.199002E+02	5.199002E+02	5.199002E+02	5.380435E+02	5.380435E+02	5.380435E+02
	Rank	4	4	2	2	2	2	2	2	2	2	2	2	2	2	2
	Median	5.349364E+02	5.346399E+02	6.272577E+02	6.458581E+02	6.627666E+02	6.627666E+02	5.721259E+02	5.721259E+02	5.929299E+02	6.774775E+02	5.52491E+02	5.912657E+02	7.77736E+02	7.77736E+02	7.77736E+02

Table 8

Experimental results of PSO variants on 100-dimensional CEC2017 benchmark functions over 51 runs, including mean, minimum, and median values, ranks, standard deviations, and Wilcoxon rank sum test results.

	Criteria	EVLS-PSOIMLSM	IMPSOwithCMAR	SQPPSO-somo	EAPSO	HMPPSO	SpadePSO	ADPSO	PSO-somo	MCPFSO	AMSEPSO	EDPSO	PPSO	SDPSO	AHPSO	MPSO
f_1	Mean	6.769721E-02	4.402579E-14	1.340605E-01	1.008176E+04	3.269358E+10	2.376824E+02	4.226754E+03	5.755840E+03	4.904690E+03	6.952018E+03	5.853760E+03	5.974347E+02	5.800646E+03	5.800646E+03	1.857905E+10
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Min	5.774842E-04	2.842171E-14	3.839932E-02	1.209746E+01	2.349424E+10	1.283007E-01	7.774116E-01	1.230507E-01	1.231395E+00	1.231395E+00	4.300881E+01	5.1E-5	5.1E-5	5.1E-5	8.897216E+08
	Rank	2	2.842171E-14	3	3	3	3	3	3	3	3	3	8	8	8	12
	Median	7.247130E-02	2.842171E-14	1.390739E-01	7.352249E+03	1.196032E+10	2.822666E+03	4.660000E+03	5.758272E+03	4.660000E+03	6.037205E+03	4.339562E+03	2.391603E+02	7.20831E+07	7.20831E+07	1.68730E+10
	Rank	3	2.842171E-14	3	3	3	3	3	3	3	3	3	8	8	8	12
	(Std Dev)	(3.470968E-02)	(3.961286E-14)	(1.274480E-02)	(1.072330E+04)	(5.609933E+09)	(5.339798E+02)	(4.793116E+03)	(5.823401E+03)	(5.974013E+03)	(7.291819E+03)	(5.968029E+03)	(6.621767E+03)	(6.621767E+03)	(6.896261E+03)	(1.038382E+10)
	Mean	5.179986E-06	6.744081E-05	8.106206E-06	1.353877E-04	22.778232E-08	1.268696E+15	1.041849E+32	4.316677E+106	1.652127E+59	1.207887E+85	3.094037E+64	5.704983E+55	2.971020E+12	5.302748E+35	2.097403E+115
	Rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Min	3.572801E-07	1.599992E-06	7.1E-5	2.768088E-07	3.20253E+80	6.19688E+28	8.1E-5	1.552764E+46	5.752570E+43	4.688835E+27	6.257856E+27	4.688835E-03	2.22071E+17	2.22071E+17	2.95311E+35
f_2	Mean	3.66137E-06	2.576805E-05	5.473022E-06	7.791368E-05	1.3048593E+41	2.116182E+10	10	1.14574E+89	2.344010E+53	1.236437E+75	3.997193E+65	9.948723E+40	7.520831E+07	7.520831E+07	9.074664E+49
	Rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	(Std Dev)	(4.027875E-06)	(1.311683E-04)	(1.020135E-05)	(1.379523E-04)	(9.554149E+98)	(7.832763E+15)	(7.431950E+52)	(1.299463E+107)	(1.098666E+60)	(7.146602E+85)	(2.208411E+26)	(3.964533E+56)	(1.394311E+13)	(3.604147E+36)	(1.419397E+116)
	Mean	4.087182E-05	6.725868E-05	5.660306E-05	8.409562E+04	1.106157E+05	1.200501E+04	1.627232E+03	1.183718E+04	1.664043E+05	1.300237E+05	1.551453E+05	3.016962E+05	3.269300E+04	3.269300E+04	1.498011E+04
	Rank	3	3	1	10	10	10	10	10	10	10	10	10	10	10	10
	Median	4.089413E-05	6.701429E-05	5.042296E-05	6.560373E+04	1.095288E+05	1.189818E+04	1.510134E+03	1.114886E+04	1.645698E+05	1.3076768E+05	1.512326E+05	3.079536E+05	5.90925E+04	5.90925E+04	7.296651E+04
	(Std Dev)	(4.370729E-06)	(3.248088E-06)	(3.248088E-06)	(4.245211E+04)	(1.879431E+04)	(3.845964E+03)	(6.175612E+02)	(6.455023E+03)	(1.385041E+04)	(2.074984E+04)	(2.496102E+04)	(5.069629E+04)	(6.466027E+03)	(6.466027E+03)	(8.625806E+03)
	Mean	2.34957E-10	6.296349E+01	6.086151E+01	1.388159E+02	2.151292E+03	2.319898E+02	2.568496E+02	5.064697E+02	2.392558E+02	2.267167E+02	2.138654E+02	2.400943E+02	2.022647E+02	2.241478E+02	2.796430E+03
	Rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Min	2.72848E-12	1.10272E-11	1.70353E-12	4.117430E+00	1.102816E+03	1.833598E+02	2.037902E+02	8.097265E+01	2.038030E+02	1.348159E+02	1.544736E+02	1.631951E+01	1.544736E+02	1.544736E+02	8.941156E+02
f_3	Mean	7.068132E-01	1.290776E-02	3.891045E-02	1.524767E+02	7.600794E-02	2.900093E-02	1.263390E-02	4.791677E+02	1.717962E+02	4.027080E+02	1.021408E+02	1.896076E+02	1.22367E+02	9.95772E+01	4.859419E+02
	Rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Min	5.173786E-01	2.795632E-01	7.661179E-01	2.099360E+02	6.040336E-02	1.810822E+02	1.064605E+02	1.104404E+02	6.618179E+01	4.477315E+01	5.870273E+01	1.631730E+02	5.870273E+01	1.631730E+02	8.707991E+02
	Rank	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
	Median	6.964710E-01	1.293458E+02	4.119114E+02	3.512193E+02	7.664152E+02	2.974920E+02	1.790922E+02	1.790922E+02	1.089896E+02	4.641306E+02	7.995056E+02	1.902029E+02	5.372898E+02	5.372898E+02	7.995056E+02
	(Std Dev)	(9.980606E-00)	(2.581038E+01)	(2.005813E+02)	(5.022227E+01)	(5.314778E+01)	(4.874997E+01)	(2.637578E+01)	(3.193695E+01)	(3.174484E+02)	(3.174484E+02)	(1.012919E+02)	(3.462626E+01)	(1.399990E+01)	(1.399990E+01)	(7.167760E+01)
	Mean	9.119392E-01	1.249875E-01	7.750505E-08	3.291789E+00	3.254139E+01	1.39970E+00	1.391090E+00	1.099969E+00	1.099969E+00	7.747499E-02	7.088408E-04	7.136868E-04	7.332495E-02	7.136868E-04	1.277185E+01
	Rank	12	13	1	13	13	13	13	13	13	13	13	13	13	13	13
	Min	1.715116E+00	5.570996E+00	1.738630E+00	5.894447E+01	2.474660E+01	6.660956E-01	9.075002E-02	1.405107E-06	1.405107E-06	1.074331E-02	1.761205E-05	1.669871E-01	2.1057187E-06	2.1057187E-06	4.763925E+02
	f_4	Mean	3.999226E+02	3.560639E+02	4.918396E+02	4.937018E+02	5.465982E+02	4.691795E+02	4.691795E+02	3.16437E+02	2.61627E+02	6.998006E+02	3.352426E+02	5.786418E+02	3.352426E+02	1.987389E+02
Rank		9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
Min		2.399138E+02	2.786960E+02	3.709962E+02	3.319848E+02	1.403533E+03	3.299778E+02	2.418506E+02	1.862972E+02	1.862972E+02	1.559741E+02	2.340667E+02	3.032799E+02	3.447969E+02	1.628769E+02	5.344328E+02
Rank		9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
Median		3.887994E+02	3.476593E+02	4.639373E+02	4.886410E+02	1.674948E+02	4.729845E+02	3.027320E+02	2.460131E+02	2.732882E+02	2.732882E+02	3.957378E+02	5.890061E+02	3.957378E+02	1.949789E+02	5.823295E+02
(Std Dev)		(4.560368E+01)	(4.378256E+01)	(1.964937E+02)	(6.502421E+01)	(1.591250E+02)	(4.506713E+01)	(7.339635E+01)	(3.879639E+01)	(4.42547E+02)	(3.879639E+01)	(1.640683E+02)	(4.871916E+01)	(5.93386E+01)	(2.17335E+01)	(1.391456E+01)
Mean		7.760651E-01	1.270815E-02	3.841593E+02	3.473683E+02	7.117538E+02	3.003538E+02	1.58024E+02	1.872248E+02	1.141264E+02	4.276593E+02	1.141438E+02	2.016837E+02	3.271221E+02	1.012159E+02	4.040310E+02
Rank		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Min		5.770761E-01	9.253113E+01	6.865215E+01	2.517257E+02	5.55577E+02	1.940166E+02	9.054124E+01	9.863665E+01	5.930408E+01	3.880406E+01	5.074394E+01	1.4942437E+02	5.074394E+01	5.870556E+02	3.486631E+02
f_5		Mean	7.362695E-01	1.213849E+02	4.069539E+02	3.532090E+02	6.970778E+02	3.124159E+02	1.570233E+02	1.900372E+02	1.107744E+02	4.685915E+02	7.871953E+01	3.372898E+02	1.494948E+02	1.004988E+02
	Rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Median	7.362695E-01	1.213849E+02	4.069539E+02	3.532090E+02	6.970778E+02	3.124159E+02	1.570233E+02	1.900372E+02	1.107744E+02	4.685915E+02	7.871953E+01	3.372898E+02	1.494948E+02	1.004988E+02	4.040310E+02
	(Std Dev)	(1.170049E+01)	(2.517147E+01)	(2.182517E+02)	(5.480828E+01)	(7.833308E+01)	(5.786975E+01)	(2.379621E+01)	(4.302556E+01)	(3.603266E+01)	(1.491306E+02)	(1.207318E+02)	(6.853579E+01)	(6.160202E+01)	(9.041130E+01)	4.040310E+02
	Mean	2.140990E+02	2.749883E+02	3.163773E+02	8.245887E+03	1.834048E+04	2.286206E+03	8.37224E+02	2.780125E+02	4.181704E-08	3.164843E+02	1.217895E+02	4.890457E+03	4.380012E+01	4.380012E+01	1.758939E+02
	Rank	12	13	1	13	13	13	13	13	13	13	13	13	13	13	13
	Min	9.894262E+00	3.043077E+02	4.943172E+01	1.817988E+03	8.788055E+03	3.145440E+02	2.791428E+01	1.932876E-12	1.103828E+02	2.240473E+00	1.759138E+03	6.968739E+02	1.163877E+01	1.163877E+01	3.587771E+03
	Rank	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
	Median	2.069654E+02	6.984743E+02	2.704890E+02	7.539589E+03	2.907284E+03	8.427430E+02	1.767388E+02	1.767388E+02	2.927560E+01	9.781568E+00	2.897222E+03	4.684521E+03	3.202555E+01	7.674262E+03	1.758939E+02
	f_6	Mean	1.494227E+02	(2.37324E+02)	(1.674605E+02)	(4.636716E+03)	(6.692194E+03)	(1.218534E+03)	(2.662933E+02)	(2.988583E+02)	(2.475419E-08)	(1.436177E+01)	(7.485132E+00)	(6.730086E+02)	(2.909756E+01)	(3.202555E+01)
Rank		2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Min		8.349467E+03	1.1650005E+04	1.264538E+04	1.297493E+04	7.2325										

Table 8 (continued).

	Criteria	EVLS-PSO/MLSM	IMPSOwithCMAR	SQPPSO-smo	EAPSO	HMPPSO	SpadePSO	ADPSO	PSO-smo	MCPSSO	AMSPSO	EOPSO	PPSO	SOPSO	AHPSO	MPSO
f_1	Mean	1.322085E+03	2.646776E+03	3.133587E+03	3.472058E+03	6.611877E+03	3.086465E+03	2.818817E+03	3.511316E+03	1.325345E+03	4.640995E+03	1.900038E+03	2.675912E+03	2.962867E+03	2.309411E+03	3.768100E+03
	Rank(WRST)	1	5	5	10	1	1	1	12	1	3	3	6	8	13	1
	Min	1.210000E+03	1.863207E+03	1.863207E+03	1.702533E+03	4.407714E+03	1.588136E+03	1.453834E+03	1.223005E+03	1.782874E+03	2.444837E+03	7.107308E+02	1.064388E+03	1.588722E+03	4.116288E+03	2.378300E+03
	Rank	1	6	6	11	15	12	2	10	2	10	5	8	8	13	13
	Median	2	2.771728E+03	3.170501E+03	3.410657E+03	5.118739E+03	3.187304E+03	2.847320E+03	3.473979E+03	1.302308E+03	4.685576E+03	1.804471E+03	2.650835E+03	3.051166E+03	2.232042E+03	3.814330E+03
	Rank	1	6	6	11	15	12	2	10	2	10	5	8	8	13	13
	(Std Dev)	2	(3.767258E+02)	(6.053601E+02)	(7.066495E+02)	(5.727995E+02)	(3.563951E+02)	(5.581807E+02)	(5.369071E+02)	(2.779446E+02)	(5.996940E+02)	(6.433925E+02)	(5.962322E+02)	(5.381897E+02)	(6.772021E+02)	(5.881897E+02)
f_2	Mean	2.342222E+03	2.294146E+03	2.622225E+03	2.649966E+03	4.540452E+03	2.445519E+03	2.649464E+03	2.917941E+03	1.108872E+03	2.382934E+03	1.951772E+03	2.289729E+03	2.218626E+03	2.111665E+03	2.057922E+03
	Rank(WRST)	7	5	1	11	15	1	1	12	1	1	6	13	6	13	1
	Min	1.583555E+03	1.419371E+03	1.028766E+03	1.257198E+03	3.366655E+03	1.781005E+03	1.870646E+03	1.795454E+03	5.513295E+02	2.347024E+03	1.197037E+03	1.459785E+03	1.346048E+03	1.016992E+03	2.038879E+03
	Rank	9	2	2	3	15	12	12	11	1	1	4	4	4	12	13
	Median	2	2.499488E+03	2.249326E+03	2.578348E+03	3.219105E+03	2.420646E+03	2.363624E+03	2.962710E+03	1.088215E+03	3.388403E+03	1.555894E+03	2.347095E+03	2.232329E+03	2.156686E+03	1.343493E+03
	Rank	9	2	2	3	15	12	12	11	1	1	4	4	4	12	13
	(Std Dev)	7	(3.840661E+02)	(4.729418E+02)	(9.108531E+02)	(8.303368E+02)	(2.916107E+02)	(4.474466E+02)	(2.707398E+02)	(3.777033E+02)	(4.835613E+02)	(4.960206E+02)	(4.051571E+02)	(3.817623E+02)	(5.767001E+02)	(6.425866E+02)
f_3	Mean	8.884293E+01	1.75643E+02	2.176949E+02	1.786496E+02	7.232188E+02	4.086811E+02	2.043590E+02	6.022006E+02	8.627206E+02	2.71374E+06	8.401293E+05	2.613298E+05	1.880796E+05	2.828616E+05	2.828616E+05
	Rank(WRST)	1	2	1	1	4	1	1	14	1	1	1	1	1	1	1
	Min	3.840148E+01	1.229768E+02	1.428072E+02	2.890242E+02	4.905630E+02	1.710577E+05	9.635311E+04	6.151704E+04	5.151704E+04	5.846612E+05	2.828027E+05	2.571326E+04	1.194989E+05	1.112613E+05	1.194989E+05
	Rank	1	2	2	6	4	4	4	1	1	1	1	1	1	1	1
	Median	1	1.717379E+02	2.224057E+02	1.866178E+02	7.092861E+02	3.014754E+05	1.792265E+02	1.792265E+02	1.792265E+02	2.424757E+06	8.195619E+05	2.618143E+05	1.015483E+05	2.409986E+05	2.409986E+05
	Rank	1	2	2	6	4	4	4	1	1	1	1	1	1	1	1
	(Std Dev)	2	(3.625410E+01)	(5.020961E+01)	(8.725408E+01)	(1.575300E+02)	(3.14169E+05)	(7.980477E+04)	(6.768112E+05)	(1.488510E+06)	(2.426661E+06)	(6.498980E+04)	(5.336687E+04)	(1.966798E+05)	(4.133651E+05)	(4.133651E+05)
f_4	Mean	2.110274E+02	2.675598E+02	2.461761E+02	4.694677E+02	2.171749E+02	2.487357E+02	2.793357E+02	2.524569E+02	2.192357E+02	1.485007E+02	1.999781E+02	3.033744E+02	3.454542E+02	1.526567E+04	1.526567E+04
	Rank(WRST)	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1
	Min	1.381799E+02	8.573962E+01	1.370885E+02	1.443415E+02	1.349177E+02	1.656348E+02	2.419084E+02	8.352176E+01	9.827034E+01	1.982840E+02	3.109678E+02	1.335304E+02	2.338857E+02	1.912275E+02	1.912275E+02
	Rank	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2
	Median	2	2.110706E+02	1.627050E+02	2.298705E+02	2.011412E+03	2.175415E+02	1.391427E+02	1.479549E+02	7.807604E+02	1.309527E+02	1.123285E+02	2.328105E+02	1.472045E+03	1.122045E+03	1.122045E+03
	Rank	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2
	(Std Dev)	4	(2.872793E+02)	(6.656475E+01)	(6.235061E+01)	(6.235061E+01)	(1.195071E+02)	(2.579794E+02)	(3.136477E+02)	(2.392489E+02)	(1.634112E+02)	(1.590371E+02)	(3.037623E+02)	(2.020793E+02)	(5.394302E+02)	(6.240270E+02)
f_5	Mean	1.469259E+02	2.152225E+03	3.164052E+03	2.711806E+03	3.854111E+03	2.153087E+03	2.082195E+03	2.429828E+03	1.027568E+03	2.928090E+03	1.398495E+03	2.613298E+03	1.818266E+03	2.458096E+03	2.458096E+03
	Rank(WRST)	1	8	14	12	9	1	1	1	1	1	1	1	1	1	1
	Min	5.254891E+02	1.195248E+03	1.195248E+03	1.254175E+03	3.186957E+03	1.292345E+03	1.335372E+03	1.338409E+03	5.939589E+02	1.900832E+03	9.972722E+02	1.217311E+03	6.680031E+02	1.337964E+03	1.337964E+03
	Rank	1	6	13	12	4	1	1	1	1	1	1	1	1	1	1
	Median	1	1.474626E+03	2.181097E+03	3.096700E+03	3.808262E+03	2.669904E+03	2.967906E+03	2.476776E+03	1.015393E+03	2.955638E+03	1.309527E+02	1.797154E+03	1.735130E+03	2.433646E+03	2.433646E+03
	Rank	1	6	13	12	4	1	1	1	1	1	1	1	1	1	1
	(Std Dev)	3	(3.605730E+02)	(3.761406E+02)	(6.885912E+02)	(6.885912E+02)	(3.525534E+02)	(4.164557E+02)	(4.800760E+02)	(2.36494E+02)	(3.790655E+02)	(2.392489E+02)	(3.748742E+02)	(5.593946E+02)	(5.912493E+02)	(4.796476E+02)
f_6	Mean	3.238187E+02	3.639796E+02	5.877116E+02	5.987172E+02	1.315996E+02	3.824310E+02	4.551523E+02	3.263298E+02	6.440988E+02	3.764675E+02	4.587463E+02	3.459757E+02	4.796757E+02	4.796757E+02	4.796757E+02
	Rank(WRST)	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1
	Min	2.955769E+02	3.253764E+02	3.198814E+02	4.531070E+02	7.033444E+02	3.348196E+02	3.857263E+02	2.79787E+02	2.681958E+02	2.802691E+02	4.055825E+02	2.958905E+02	5.900773E+02	5.900773E+02	5.900773E+02
	Rank	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2
	Median	3.241274E+02	3.585656E+02	5.944196E+02	5.492508E+02	1.814077E+02	4.499074E+02	4.499074E+02	3.201130E+02	6.709696E+02	3.240488E+02	5.736031E+02	3.641242E+02	5.736031E+02	5.736031E+02	5.736031E+02
	Rank	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2
	(Std Dev)	1	(1.372536E+01)	(2.558048E+01)	(1.943572E+02)	(3.326103E+02)	(4.064999E+01)	(6.066923E+01)	(2.131168E+01)	(3.299402E+01)	(1.509337E+02)	(1.526863E+02)	(4.312377E+01)	(2.828252E+01)	(4.796476E+02)	(4.796476E+02)
f_7	Mean	9.325807E+03	1.060152E+04	1.486568E+04	1.444116E+04	4.794351E+04	1.190507E+04	1.264482E+04	1.477927E+04	1.547097E+04	1.106342E+04	9.985203E+03	1.617710E+04	6.541337E+03	1.544911E+04	1.544911E+04
	Rank(WRST)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Min	6.87288836301093E+03	1.00000015631207E+04	1.16319667466821E+04	1.163094407677123E+04	1.567061692114045E+03	1.000000000000000E+04	9.90484022861726E+03	1.085370343913608E+04	1.246344236309544E+04	1.000000299474850E+02	1.000000000000000E+02	1.000000000000000E+02	1.000000000000000E+02	3.20336743077506E+02	3.20336743077506E+02
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	1	1.23779E+04	1.482188E+04	1.251308E+04	2.598506E+03	1.256311E+04	1.459522E+04	1.459522E+04	1.534168E+04	1.091586E+04	1.218542E+04	1.212490E+04	1.767682E+04	1.615712E+04	1.615712E+04
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	(Std Dev)	1	(1.144378E+03)	(4.936790E+03)	(1.404533E+03)	(1.360158E+03)	(7.310818E+03)	(2.562157E+03)	(1.228449E+03)	(1.263786E+03)	(4.207777E+03)	(1.907890E+03)	(6.442612E+03)	(3.958030E+03)	(1.359830E+03)	(1.359830E+03)
f_8	Mean	6.547021E+02	7.017594E+02	9.201001E+02	8.258171E+02	1.238104E+02	7.768986E+02	6.553944E+02	8.137392E+02	5.961321E+02	6.689554E+02	8.171555E+02	7.92597E+02	7.954578E+02	1.074589E+03	1.074589E+03
	Rank(WRST)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Min	6.192214E+02	6.332271E+02	6.342800E+02	7.212026E+02	1.062404E+03	7.015109E+02	5.958873E+02	7.127612E+02	5.696975E+02	5.955623E+02	6.620210E+02	8.224264E+02	6.726871E+02	8.352607E+02	8.352607E+02
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Median	6.543881E+02	6.901917E+02	8.715183E+02	8.200019E+02	7.810453E+02	6.569089E+02	8.026927E+02	8.026927E+02	6.219017E+02	6.057489E+02	7.950777E+02	7.950777E+02	7.950777E+02	7.950777E+02	7.950777E+02
	Rank	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 9

The mean best values and standard deviations of EVLS-PSOIMLSM, employing various mechanisms, were computed over 51 runs for a 50-dimensional test on CEC2017 benchmark functions.

	Criteria	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSOIMLSM	EVLS-PSO
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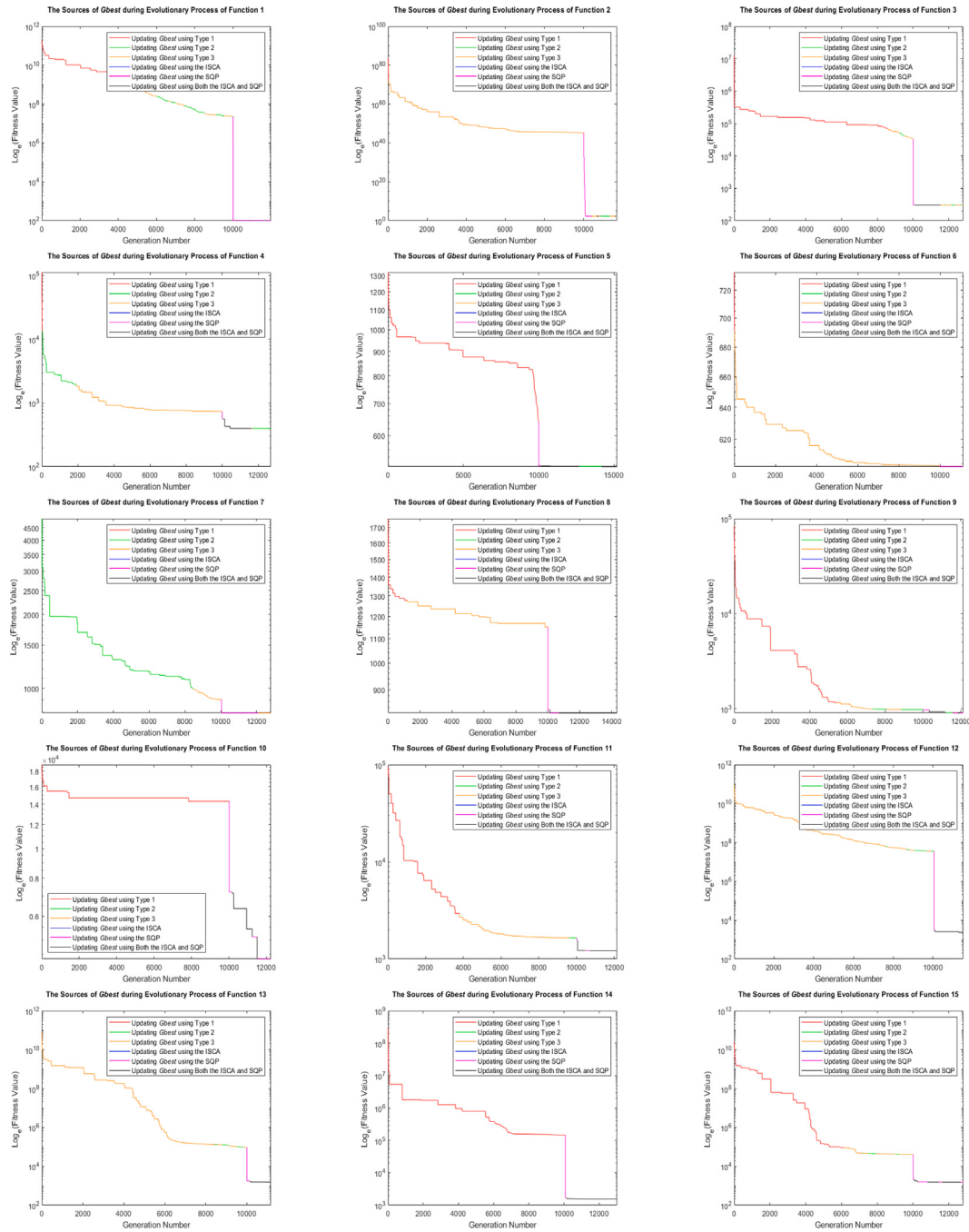


Fig. 2. Sources of the best particles in 50-dimensional experiments over the evolutionary process of an individual run on the $f_1 - f_{15}$ functions.

4.5. Search behavior of EVLS-PSOIMLSM

Figs. 2 and 3 illustrate the progression of the best particles by source across different functions in individual 50-dimensional runs. The search patterns are described as follows:

- The combination of Types 1 to 3 alternately contributes the best particles during the early and middle generations of the evolution. Then, the SQP method results in a sudden drop in performance. Finally, the ISCA combined with the SQP methods, contributes the best particles from the later generations until the end.
- The combination of Types 1 to 3 alternately contributes the best particles during the early and middle generations of the evolution.

Then, the SQP method contributes the best particles from the later generations until the end.

- The combination of Types 1 to 3 alternately contributes the best particles during the early and middle generations of the evolution. Then, the SQP method results in a sudden drop in performance. Finally, the SQP method and the ISCA combined with the SQP methods alternately contribute the best particles from the later generations until the end.
- The combination of Types 1 to 3 alternately contributes the best particles during the early and middle generations of the evolution. Then, the SQP method results in a sudden drop in performance. Finally, Types 2 and 3, the SQP method, and the ISCA combined with the SQP methods alternately contribute the best particles from the later generations until the end.

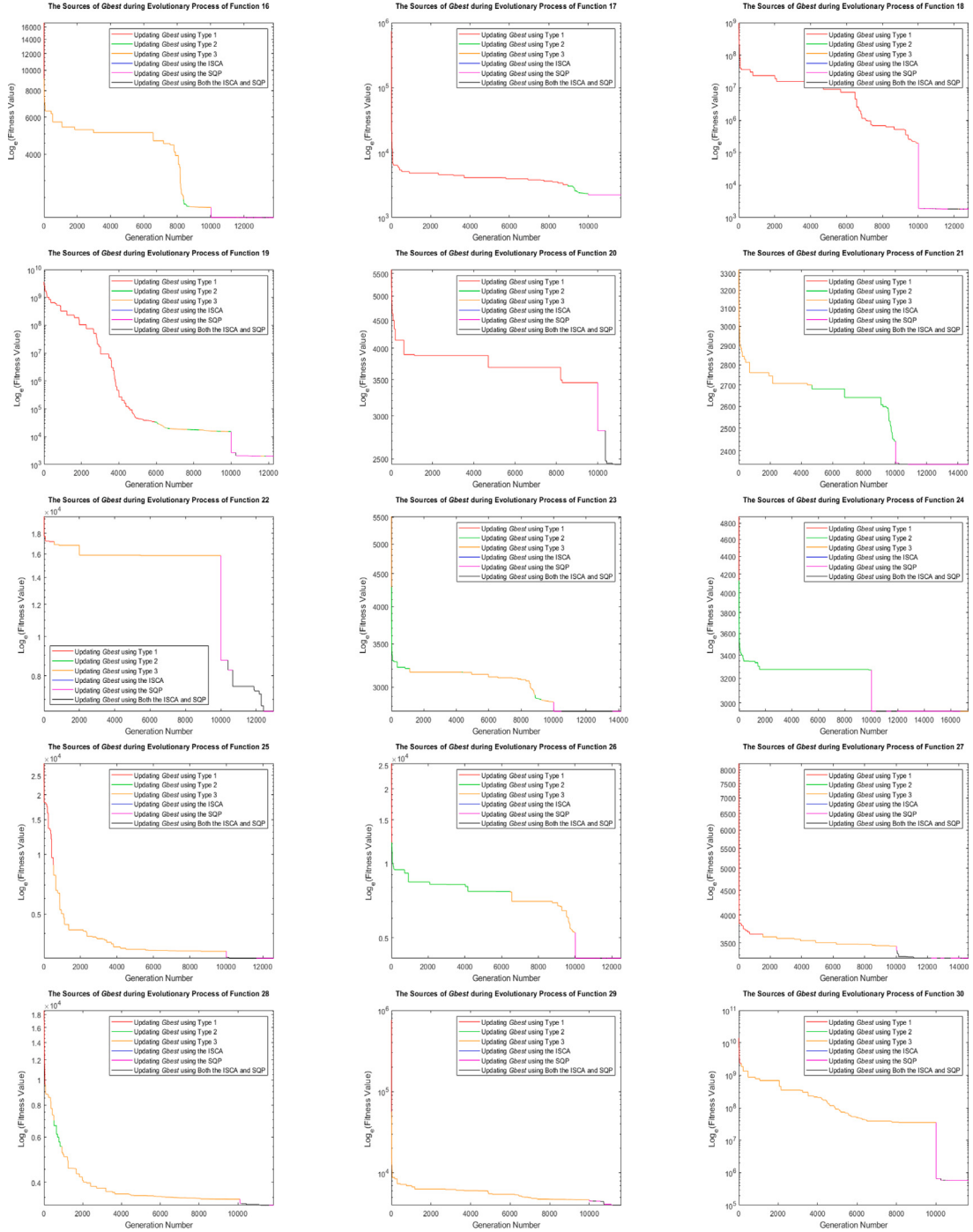


Fig. 3. Sources of the best particles in 50-dimensional experiments over the evolutionary process of an individual run on the $f_{16} - f_{30}$ functions.

From the above search behavior, it is evident that the proposed EVLS-PSOIMLSM demonstrates significant intelligent evolution. It can automatically adjust its search behavior and preferentially choose the search strategies or combinations according to the benchmark functions to fit their landscapes. The ISCA method does not independently contribute the best particle. However, when it cooperates with the SQP method, its disturbances enable the SQP method to evolve more effectively and contribute the best particle to the proposed algorithm.

Figs. 4 and 5 illustrate the 2-dimensional particle evolution of EVLS-PSOIMLSM using different coefficients and mechanisms. The orange star signifies the global optimum. It is evident that using an inertia

weight ω with $\text{rand}(0,1)$ or a chaotic-based approach causes EVLS-PSOIMLSM to lose population diversity in the early generations, leading to premature convergence to local optima. When applied to f_4 , EVLS-PSOIMLSM with cases 1 to 6 results in particles being more dispersed compared to Types 1 to 3. Conversely, on f_{10} , EVLS-PSOIMLSM with cases 1 to 6 causes particles to cluster more closely than with Types 1 to 3. Both cases 1 to 6 and Types 1 to 3 demonstrate adaptability across different benchmark functions. However, the experimental results in Table 9 for f_4 and f_{10} show that Types 1 to 3 achieve better mean values and have better averaged Friedman ranks across all functions, indicating that the proposed Types 1 to 3 strike a superior balance between exploration and exploitation. EVLS-PSOIMLSM without using the

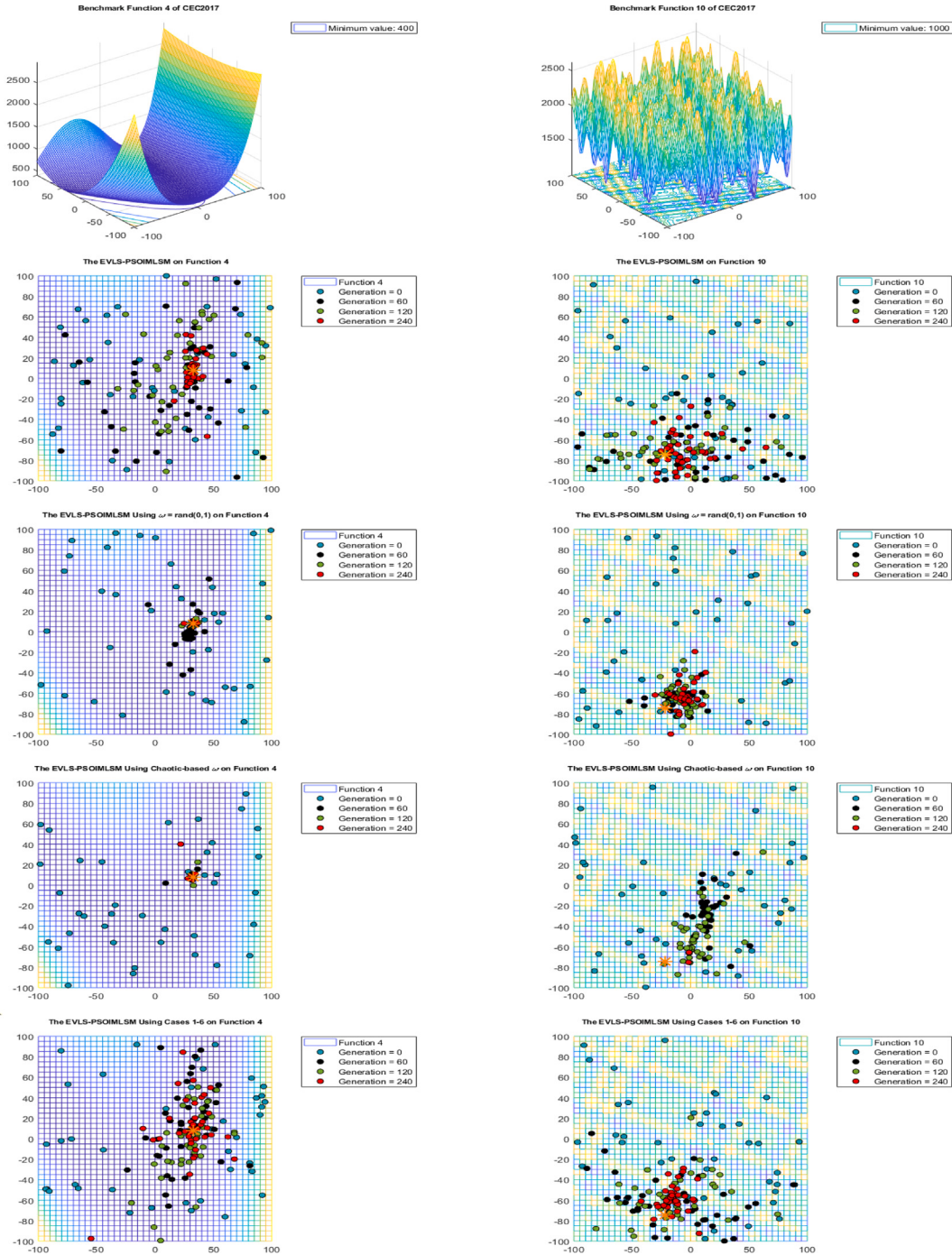


Fig. 4. Plots illustrating the 2-dimensional particle evolution across different generations (0, 60, 120, and 240) and the results of ablation tests on functions 4 and 10.

ISCA or SQP methods results in particles being more dispersed, which is expected, as both methods are employed to enhance the local search capability. EVLS-PSOIMLSM without the acceleration coefficient c results in particles clustering more on f_4 and dispersing more on f_{10} . The experimental results in Table 9 show that EVLS-PSOIMLSM performs better on f_4 but worse on f_{10} . This observation suggests a potential future direction to design an ensemble strategy for the acceleration coefficient. EVLS-PSOIMLSM without the mutation strategy results in the particles being far away from the global optimum, illustrating that the mutation strategy can effectively prevent the algorithm from falling

into local optima. EVLS-PSOIMLSM without elite learning results in the particles becoming more dispersed, illustrating that elite learning can effectively guide the evolution process.

The convergence characteristic graphs comparing EVLS-PSOIMLSM with PSO-based variants on 50-dimensional tests are presented in Figs. 6 and 7. EVLS-PSOIMLSM maintains a high convergence rate not only in the early stages of the evolution process but also in the later stages, in most cases, demonstrating impressive performance. Notably, EVLS-PSOIMLSM does not exhibit an outstanding convergence rate for f_{22} , f_{26} , and f_{27} . This suggests that there is potential for further improvement in EVLS-PSOIMLSM on these functions in future work.

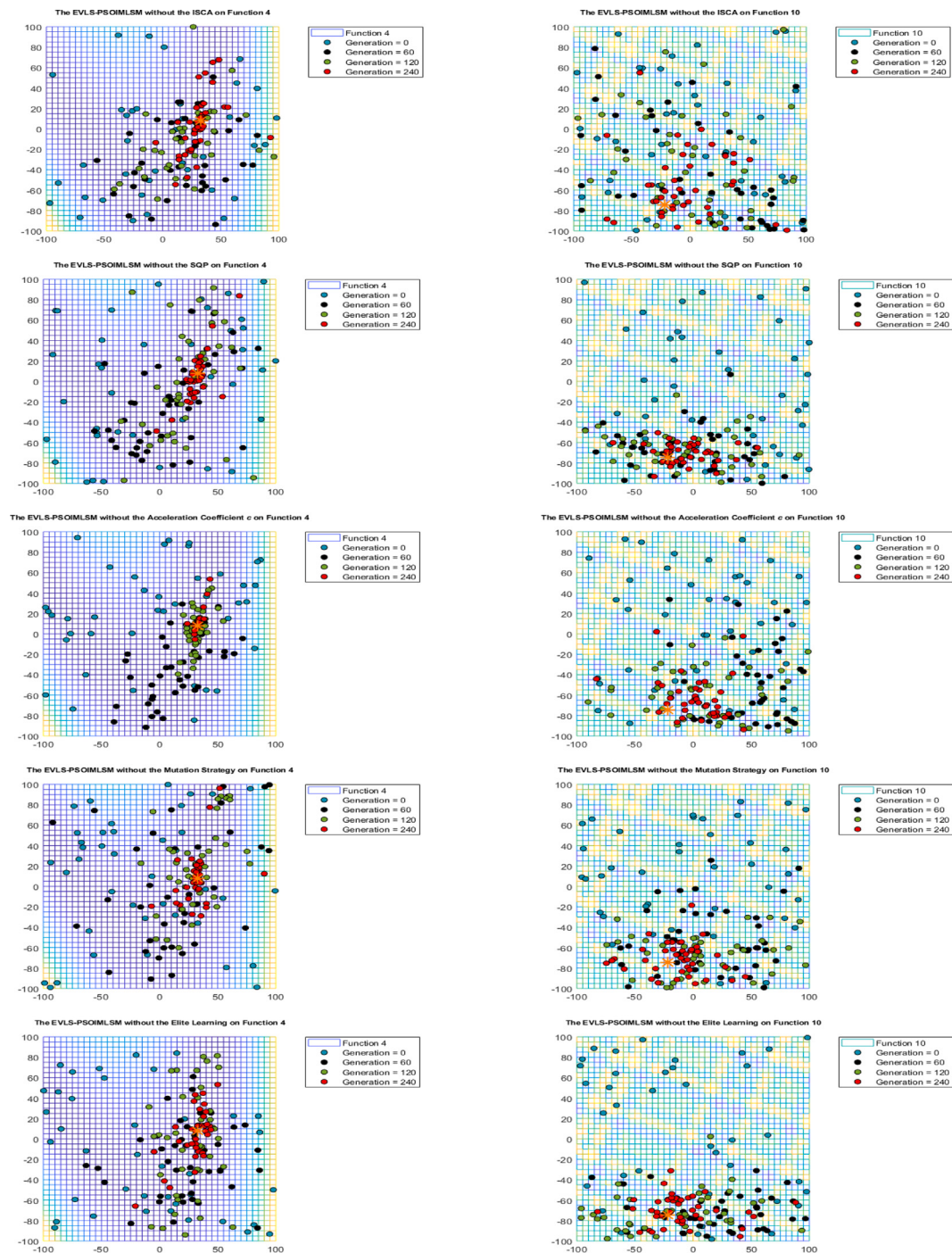


Fig. 5. Plots illustrating the 2-dimensional particle evolution across different generations (0, 60, 120, and 240) and the results of ablation tests on functions 4 and 10.

4.6. EVLS-PSOIMLSM vs. non-PSO algorithms

To more effectively evaluate the performance of EVLS-PSOIMLSM against various non-PSO algorithms, 11 recently proposed metaheuristic and classic algorithms were selected: IVY Algorithm (IVYA) (Ghasemi et al., 2024), Black Kite Algorithm (BKA) (Wang et al., 2024b), Artificial Protozoa Optimizer (APO) (Wang et al., 2024a), Partial Reinforcement Optimizer (PRO) (Taheri et al., 2024), Attraction-Repulsion Optimization Algorithm (AROA) (Cymerys and Oszust, 2024), Sinh Cosh Optimizer (SCHO) (Bai et al., 2023), Grey Prediction Evolution Algorithm (GPEA) (Hu et al., 2020), Water Wave Optimization

(WWO) (Zheng et al., 2019), EBOwithCMAR (Kumar et al., 2017), LSHADE-cnEpSin (Awad et al., 2017b), LSHADE-SPACMA (Mohamed et al., 2017), and Covariance Matrix Adaptation Evolution Strategy (CMA-ES) (Hansen and Ostermeier, 2001). The experimental results are presented in Tables 10 to 13. These tables present both the individual rankings and the averaged Friedman rankings. The averaged Friedman ranking of EVLS-PSOIMLSM in the 10-dimensional test is 6.30, worse than APO (5.00), PRO (4.60), AROA (5.63), EBOwithCMAR (1.83), LSHADE-cnEpSin (3.13), and SHADE-SPACMA (2.96). The averaged Friedman ranking of EVLS-PSOIMLSM in the 30-dimensional test is 4.56, worse than EBOwithCMAR (1.90), LSHADE-cnEpSin (2.16), and

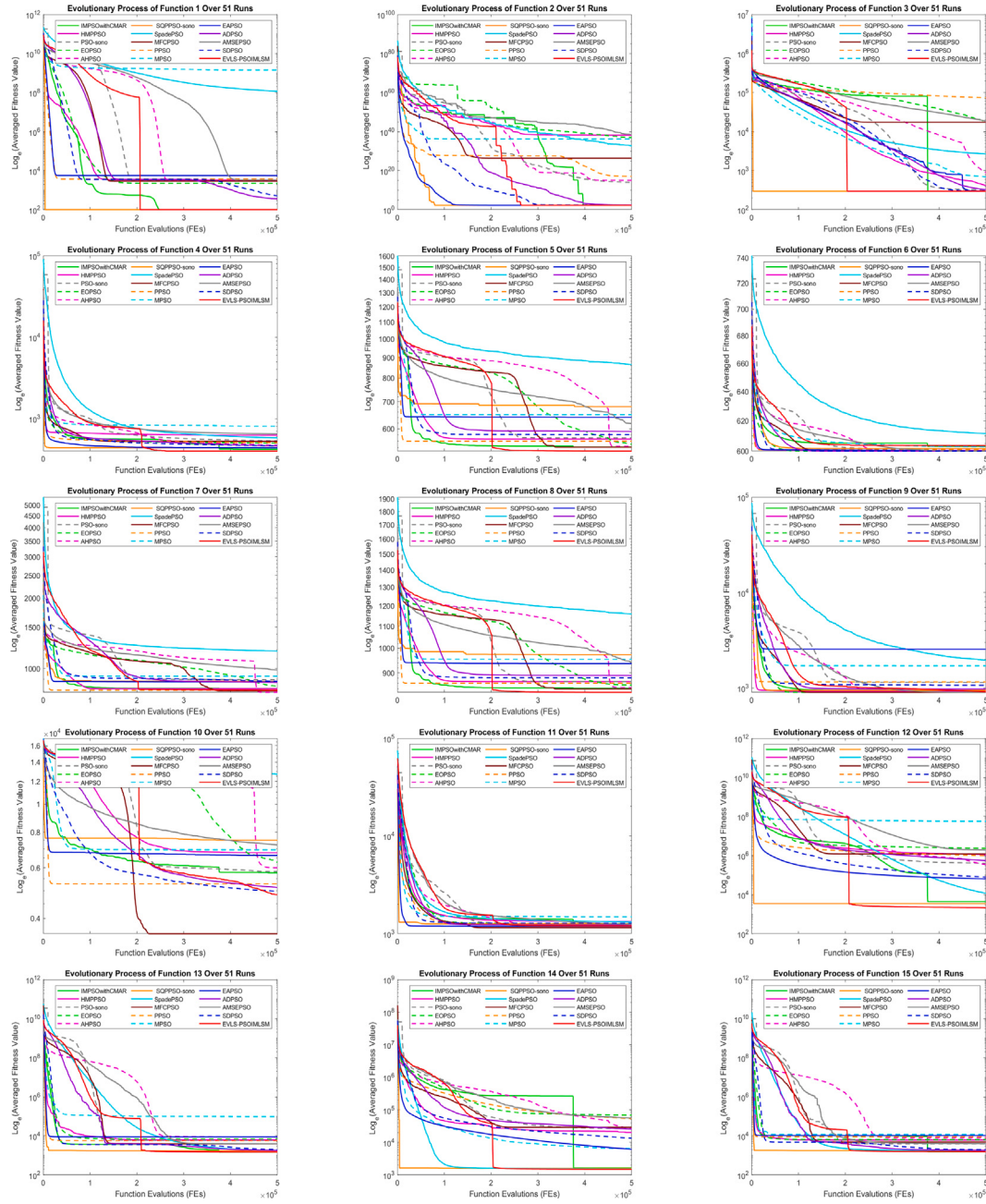


Fig. 6. The proposed EVLS-PSOIMLSM and all compared PSO-based variants were subjected to evolutionary processes in 50-dimensional experiments. These processes were averaged over 51 runs and evaluated on the $f_1 - f_{15}$ functions of the CEC2017 benchmark. The fitness values and function evaluations (FEs) are depicted on the Y- and X-axes, respectively.

SHADE-SPACMA (2.83). The averaged Friedman ranking of EVLS-PSOIMLSM in the 50-dimensional test is 4.13, worse than EBOwithCMAR (2.13), LSHADE-cnEpSin (2.06), and SHADE-SPACMA (2.30). The averaged Friedman ranking of EVLS-PSOIMLSM in the 100-dimensional test is 3.83, worse than EBOwithCMAR (2.90), LSHADE-cnEpSin (2.43), and SHADE-SPACMA (2.53). Overall, EVLS-PSOIMLSM performs worse than the DE-based variants (EBOwithCMAR, LSHADE-cnEpSin, and SHADE-SPACMA), which is consistent with the results of the CEC competition (Awad et al., 2017a; Song et al., 2019) in recent years. As highlighted in recent CEC competitions, DE-based variants are in the first tier and demonstrate highly competitive performance among all metaheuristics. However, with the increase in tested dimensions, the rankings of EVLS-PSOIMLSM improve, showing more competitive performance. The experimental results not only highlight the gap between EVLS-PSOIMLSM and the first tier of metaheuristics but also

demonstrate the notable progress of EVLS-PSOIMLSM achieved through our dedicated efforts.

5. Real-world applications

The particle swarm optimization has been applied to many real-world applications (Kuo et al., 2023; Qaraad et al., 2024; Saini et al., 2021; Bai et al., 2021; Bey et al., 2024). This study applies EVLS-PSOIMLSM to the Gear Train Design (GTD) problem (Rizk-Allah, 2018; Pathak and Srivastava, 2022) and the Planetary Gear Train Design (PGTD) problem (Zhang et al., 2018). For both the GTD and PGTD problems, the $maxFEs$ is set to 2.5×10^4 for all the compared PSO-based variants. The best simulation results obtained from 20 independent runs of the algorithms are shown in Tables 14 to 17. Table 14 presents a

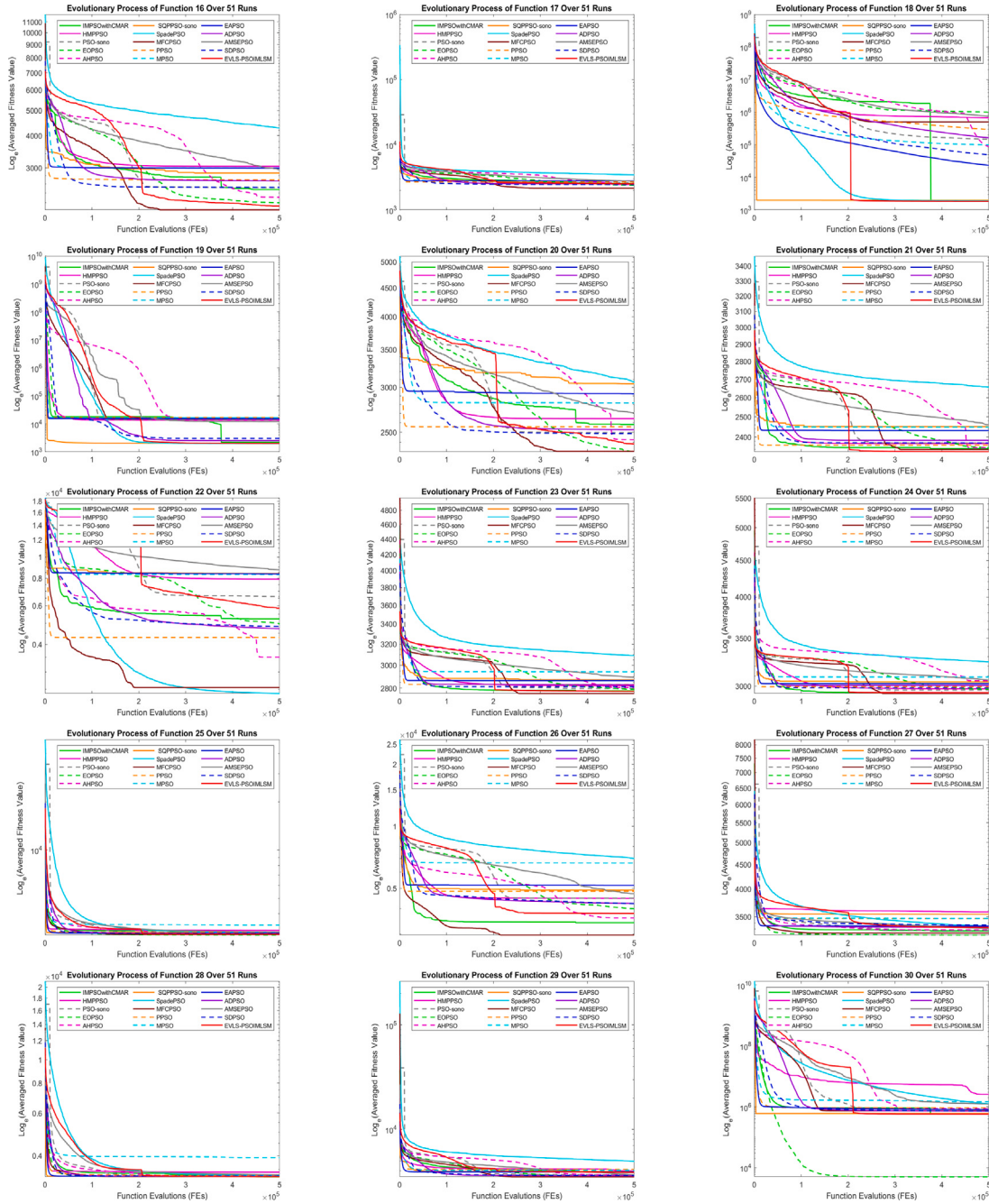


Fig. 7. The proposed EVLS-PSOIMLSM and all compared PSO-based variants were subjected to evolutionary processes in 50-dimensional experiments. These processes were averaged over 51 runs and evaluated on the $f_{16} - f_{30}$ functions of the CEC2017 benchmark. The fitness values and Function Evaluations (FEs) are depicted on the Y- and X-axes, respectively.

comparison of PSO-based variants on the GTD problem, where EVLS-PSOIMLSM, SQPPSO-sono, EAPSO, HMPPSO, SpadePSO, ADPSO, PSO-sono, AMSEPSO, EOPSO, PPSO, SDPSO, and AHPSO achieve the best value of $2.700857\text{E}-12$. Similarly, Table 15 compares non-PSO algorithms on the GTD problem, where EVLS-PSOIMLSM, BKA, PRO, AROA, WWO, EBOwithCMAR, LSHADE-cnEpSin, and LSHADE-SPACMA also achieve the best value of $2.700857\text{E}-12$. Table 16 presents a comparison of PSO-based variants on the PGTD problem, where EVLS-PSOIMLSM, ADPSO, and AMSEPSO achieve the best value of $5.262806\text{E}-01$. Table 17 compares non-PSO algorithms on the PGTD problem, where EVLS-PSOIMLSM reports a value of $5.262806\text{E}-01$, which is worse than APO, EBOwithCMAR, and LSHADE-cnEpSin, but better than LSHADE-SPACMA, a DE-based variant.

5.1. Gear train design problem

The main objective of the gear train design problem is to optimize the gear ratio in a compound gear train (Rizk-Allah, 2018; Pathak and Srivastava, 2022). This is achieved by determining the appropriate or optimal number of gear teeth. The problem involves four gears (A, B, C, and D) as design variables, represented as $\eta_A(x_1)$, $\eta_B(x_2)$, $\eta_C(x_3)$, and $\eta_D(x_4)$. This involves minimizing $f(x)$ as defined in Eq. (36).

$$f(x) = \left(\frac{1}{6.931} - \frac{x_2 x_3}{x_1 x_4} \right)^2, \quad (36)$$

Subject to:

$$12 \leq x_i \leq 60, \text{ where } i = 1, 2, 3, \text{ and } 4, \text{ and } x_i \text{ is an integer.}$$

Table 10

Experimental results of ELVS-PSOIMLSM versus non-PSO algorithms on 10-dimensional test functions over 51 runs, including mean values, ranks, standard deviations, and Wilcoxon rank sum test results.

f_i	Criteria	ELVS-PSOIMLSM	DVA 2024	NBA 2023	APD 2024	PBO 2024	ABDA 2024	SGD 2023	OPFA 2020	WVO 2019	ESOWithCMAR 2017	LSHADE-csElites 2017	LSHADE-SPACMA 2017	CMA-ES 2005
f_1	Mean	4.359734E-05	2.416598E+03	3.590360E+07	1.350355E-04	1.462532E-07	1.649295E+03	2.120271E+08	3.058161E+01	1.517295E+03	0.000000E+00	0.000000E+00	0.000000E+00	2.901393E+06
	Rank(WRST Std Dev)	5 (4.559246E-05)	12 (2.068235E+03)	10 (2.269623E+08)	6 (1.229662E+08)	4 (3.231184E-07)	8 (1.798304E-04)	13 (5.372555E+08)	7 (1.291825E+02)	8 (1.857352E+03)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)	2.624515E+06
f_2	Mean	2.488726E-06	8.518316E-03	1.927694E+06	1.600875E-07	3.682548E-06	1.185261E-04	3.975462E+06	1.894353E-02	1.878439E+01	0.000000E+00	0.000000E+00	0.000000E+00	6.867121E+07
	Rank(WRST Std Dev)	5 (1.874588E-06)	12 (3.844906E-02)	10 (2.877944E+06)	4 (3.530804E-07)	6 (3.103108E-06)	3 (3.388389E-04)	12 (5.372555E+08)	9 (5.920396E-02)	10 (1.878439E+01)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)	13 (2.299416E+08)
f_3	Mean	3.322585E-13	5.290307E+00	4.461834E+02	5.435415E-05	2.407486E-13	1.366322E-06	2.935267E+03	8.811348E-02	1.568651E+03	0.000000E+00	0.000000E+00	0.000000E+00	1.506064E+04
	Rank(WRST Std Dev)	5 (3.272729E-13)	9 (5.290307E+00)	10 (3.754842E+01)	10 (1.200745E-04)	4 (1.297490E+03)	6 (1.200745E-04)	12 (2.324005E-06)	8 (1.362666E-01)	11 (1.176345E+03)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)	13 (2.058335E+04)
f_4	Mean	2.028303E-13	4.114928E+00	7.584055E+00	4.642466E+00	2.066774E-02	6.899223E-02	5.221779E+01	3.949328E-01	3.276054E+00	0.000000E+00	0.000000E+00	0.000000E+00	8.267636E-07
	Rank(WRST Std Dev)	4 (1.926832E-13)	10 (9.573714E+00)	12 (1.115806E+01)	11 (1.215806E+01)	7 (1.315656E-02)	7 (1.315656E-02)	12 (2.324005E-06)	8 (1.362666E-01)	9 (1.176345E+03)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)	13 (2.058335E+04)
f_5	Mean	3.511620E+00	2.850431E+01	3.572387E+01	1.358298E+00	4.701698E+00	6.251517E+00	3.398122E+01	1.915789E+01	1.990691E+01	3.901800E-02	1.699552E+00	1.835436E+00	7.608513E-01
	Rank(WRST Std Dev)	6 (1.926444E+00)	11 (1.487358E+01)	11 (3.572387E+01)	3 (1.358298E+00)	7 (4.701698E+00)	7 (4.701698E+00)	12 (2.324005E-06)	8 (1.362666E-01)	9 (1.176345E+03)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)	13 (2.058335E+04)
f_6	Mean	1.500470E-02	1.069927E+00	2.452908E+01	1.703039E-12	5.555246E-08	5.011020E-04	1.541423E+01	3.366018E+00	3.730742E+00	0.000000E+00	0.000000E+00	0.000000E+00	5.338600E-11
	Rank(WRST Std Dev)	8 (2.376057E-02)	12 (4.712431E+00)	10 (8.454742E+00)	4 (1.703039E-12)	4 (5.555246E-08)	4 (5.011020E-04)	12 (2.324005E-06)	12 (2.324005E-06)	11 (1.176345E+03)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)	13 (2.058335E+04)
f_7	Mean	1.371059E+01	7.707227E+01	8.263008E+01	5.263800E+01	1.160544E+01	1.649800E+01	3.329490E+01	3.247440E+01	4.079920E+01	1.065320E+01	1.197792E+01	1.099920E+01	1.596960E+01
	Rank(WRST Std Dev)	5 (1.844105E+00)	13 (7.707227E+01)	13 (8.263008E+01)	5 (5.263800E+01)	5 (1.160544E+01)	5 (1.649800E+01)	8 (3.329490E+01)	9 (3.247440E+01)	10 (4.079920E+01)	1 (1.065320E+01)	1 (1.197792E+01)	1 (1.099920E+01)	7 (1.596960E+01)
f_8	Mean	2.536170E+00	2.227678E+01	2.076210E+01	1.297935E+00	4.779705E+00	2.808640E+01	1.725444E+01	2.124778E+01	1.750637E+00	7.51874E-02	1.750637E+00	1.750637E+00	6.828150E-01
	Rank(WRST Std Dev)	1 (1.231721E+00)	12 (7.271684E+00)	12 (7.271684E+00)	7 (1.297935E+00)	7 (4.779705E+00)	12 (2.808640E+01)	8 (1.725444E+01)	9 (2.124778E+01)	5 (1.750637E+00)	1 (7.51874E-02)	1 (1.750637E+00)	1 (1.750637E+00)	13 (2.058335E+04)
f_9	Mean	1.755456E-03	5.100925E-02	2.030740E-02	0.000000E+00	2.754566E-03	2.009918E-07	2.234676E-02	3.483826E-01	8.408187E-01	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank(WRST Std Dev)	8 (2.536170E+00)	12 (7.271684E+00)	12 (7.271684E+00)	1 (0.000000E+00)	7 (1.297935E+00)	12 (2.808640E+01)	8 (1.725444E+01)	9 (2.124778E+01)	5 (1.750637E+00)	1 (7.51874E-02)	1 (1.750637E+00)	1 (1.750637E+00)	13 (2.058335E+04)
f_{10}	Mean	2.229518E+02	1.246431E+03	7.909000E+02	1.397136E+02	1.391408E+02	2.480574E+02	8.011507E+02	1.153335E+03	7.916407E+02	2.861868E+01	2.664251E+01	2.861868E+01	1.876107E+01
	Rank(WRST Std Dev)	1 (1.757004E+02)	13 (3.337201E+02)	12 (3.303057E+02)	12 (3.303057E+02)	12 (3.303057E+02)	12 (3.303057E+02)	12 (3.303057E+02)	12 (3.303057E+02)	12 (3.303057E+02)	12 (3.303057E+02)	12 (3.303057E+02)	12 (3.303057E+02)	12 (3.303057E+02)
f_{11}	Mean	1.463175E+00	3.617948E+01	4.733068E+01	2.121261E+00	1.801644E+00	1.364300E+00	2.228498E+00	1.033558E+01	2.660421E+01	0.000000E+00	0.000000E+00	0.000000E+00	5.792312E+00
	Rank(WRST Std Dev)	6 (1.369331E+00)	12 (8.281491E+01)	12 (4.733068E+01)	5 (2.121261E+00)	5 (1.801644E+00)	5 (1.364300E+00)	5 (2.228498E+00)	5 (1.033558E+01)	5 (2.660421E+01)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)	13 (2.058335E+04)
f_{12}	Mean	5.788918E+01	1.586777E+02	5.500336E+04	2.170799E+02	1.022914E+04	3.373235E+02	5.308360E+05	1.289226E+02	2.997197E+04	1.190214E+02	1.056350E+02	1.190214E+02	2.864944E+06
	Rank(WRST Std Dev)	6 (5.252721E+01)	12 (2.274906E+05)	12 (5.500336E+04)	10 (2.170799E+02)	10 (1.022914E+04)	10 (3.373235E+02)	12 (5.308360E+05)	8 (1.289226E+02)	9 (2.997197E+04)	2 (1.190214E+02)	2 (1.056350E+02)	2 (1.190214E+02)	13 (2.058335E+04)
f_{13}	Mean	6.240939E+00	7.515556E+03	5.493845E+02	5.815300E+00	7.599919E+00	4.732862E+00	1.100262E+04	3.306316E+01	7.760459E+03	3.565903E+00	4.411754E+02	4.411754E+02	1.175459E+04
	Rank(WRST Std Dev)	6 (2.802006E+00)	12 (7.717826E+03)	12 (5.493845E+02)	5 (5.815300E+00)	5 (7.599919E+00)	5 (4.732862E+00)	12 (1.100262E+04)	8 (3.306316E+01)	9 (7.760459E+03)	1 (3.565903E+00)	1 (4.411754E+02)	1 (4.411754E+02)	13 (2.058335E+04)
f_{14}	Mean	2.335492E+01	4.040818E+03	7.905789E+01	1.549323E-01	1.658276E+00	2.223276E+00	1.899747E+03	1.259488E+01	6.400506E+01	1.349808E-02	9.754501E-02	2.734333E-01	2.734333E-01
	Rank(WRST Std Dev)	8 (3.947598E+00)	13 (9.359511E+03)	12 (7.905789E+01)	12 (1.549323E-01)	12 (1.658276E+00)	12 (2.223276E+00)	12 (1.899747E+03)	10 (1.259488E+01)	9 (6.400506E+01)	1 (1.349808E-02)	1 (9.754501E-02)	1 (2.734333E-01)	13 (2.058335E+04)
f_{15}	Mean	7.588029E+00	2.401859E+02	2.211112E+02	6.317890E-01	1.140345E+02	3.021890E-01	3.439870E+03	6.787605E+00	1.266840E+02	1.321946E-01	6.787605E+00	6.787605E+00	6.074787E+03
	Rank(WRST Std Dev)	6 (5.333736E+00)	12 (2.502588E+03)	12 (2.211112E+02)	9 (6.317890E-01)	9 (1.140345E+02)	9 (3.021890E-01)	12 (3.439870E+03)	10 (6.787605E+00)	10 (1.266840E+02)	1 (1.321946E-01)	1 (6.787605E+00)	1 (6.787605E+00)	13 (2.058335E+04)
f_{16}	Mean	1.531233E+00	2.943433E+02	1.156442E+02	8.978195E-01	3.100784E+02	2.475175E+02	9.472900E+01	1.146646E+02	9.472900E+01	1.358842E-01	7.357842E-01	7.357842E-01	1.766031E+01
	Rank(WRST Std Dev)	6 (1.426833E-01)	12 (3.534918E+02)	11 (1.156442E+02)	4 (8.978195E-01)	4 (3.100784E+02)	4 (2.475175E+02)	12 (9.472900E+01)	12 (1.146646E+02)	12 (9.472900E+01)	1 (1.358842E-01)	1 (7.357842E-01)	1 (7.357842E-01)	13 (2.058335E+04)
f_{17}	Mean	2.525388E+01	6.809615E+01	5.994761E+01	1.373956E+00	1.273158E+01	1.157115E+01	9.927708E+01	2.990607E+01	5.507226E+01	1.797855E+01	6.584966E-01	6.584966E-01	9.501173E-02
	Rank(WRST Std Dev)	5 (5.498378E+00)	12 (5.599642E+01)	12 (5.994761E+01)	5 (1.373956E+00)	5 (1.273158E+01)	5 (1.157115E+01)	12 (9.927708E+01)	8 (2.990607E+01)	9 (5.507226E+01)	1 (1.797855E+01)	1 (6.584966E-01)	1 (6.584966E-01)	13 (2.058335E+04)
f_{18}	Mean	1.552945E+01	1.139422E+04	3.408730E+02	4.067304E-01	1.477113E+04	1.477113E+04	8.002476E+03	7.553333E+01	2.348225E+00	2.348225E+00	5.115350E+00	5.115350E+00	8.889377E+04
	Rank(WRST Std Dev)	7 (9.397000E+00)	12 (1.225648E+04)	12 (3.408730E+02)	12 (4.067304E-01)	12 (1.477113E+04)	12 (1.477113E+04)	12 (8.002476E+03)	10 (7.553333E+01)	10 (2.348225E+00)	1 (2.348225E+00)	1 (5.115350E+00)	1 (5.115350E+00)	13 (2.058335E+04)
f_{19}	Mean	8.509622E+00	1.225648E+04	3.408730E+02	4.067304E-01	1.477113E+04	1.477113E+04	8.002476E+03	7.553333E+01	2.348225E+00	2.348225E+00	5.115350E+00	5.115350E+00	8.889377E+04
	Rank(WRST Std Dev)	7 (9.397000E+00)	12 (1.225648E+04)	12 (3.408730E+02)	12 (4.067304E-01)	12 (1.477113E+04)	12 (1.477113E+04)	12 (8.002476E+03)	10 (7.553333E+01)	10 (2.348225E+00)	1 (2.348225E+00)	1 (5.115350E+00)	1 (5.115350E+00)	13 (2.058335E+04)
f_{20}	Mean	2.055380E+01	1.189305E+02	9.198600E+01	3.780725E-01	1.488051E-01	2.118997E+01	1.193118E+02	1.236144E+02	2.765929E-01	1.775103E-01	3.605232E-01	3.605232E-01	5.392867E+01
	Rank(WRST Std Dev)	7 (2.463600E+00)	12 (1.681458E+02)	12 (9.198600E+01)	10 (3.780725E-01)	10 (1.488051E-01)	10 (2.118997E+01)	12 (1.193118E+02)	12 (1.236144E+02)	12 (2.765929E-01)	1 (1.775103E-01)	1 (3.605232E-01)	1 (3.605232E-01)	13 (2.058335E+04)
f_{21}	Mean	1.841734E+02	1.948256E+02	1.609870E+02	1.270239E+02	1.043391E+02	2.284528E+02	1.289515E+02	1.052708E+02	1.425046E+02	1.425046E+02	1.052708E+02	1.052708E+02	2.002612E+02
	Rank(WRST Std Dev)	4 (1.911311E+01)	12 (3.533535E+04)	12 (1.609870E+02)	12 (1.270239E+02)	12 (1.043391E+02)	12 (2.284528E+02)	12 (1.289515E+						

Table 11

Experimental results of ELVS-PSOIMLSM versus non-PSO algorithms on 30-dimensional test functions over 51 runs, including mean values, ranks, standard deviations, and Wilcoxon rank sum test results.

f_i	Criteria	ELVS-PSOIMLSM	IWA 2024	NBA 2024	APD 2024	PBO 2024	ABDA 2024	SG20 2017	OPFA 2020	WVO 2019	ESOWHOEMAR 2017	LSHADE-csflgfm 2017	LSHADE-SPACMA 2001	CMA-ES 2001
f_1	Mean	3.288639E-03	9.172952E+04	5.434949E+09	1.762426E+03	2.262543E+02	2.763420E+03	8.488155E+09	2.449975E+03	2.863835E+03	0.000000E+00	0.000000E+00	0.000000E+00	2.302147E+05
	Rank(WRST (Std Dev))	4 (1.909140E-01)	12 (1.997505E+05)	4 (2.483959E+05)	12 (1.221471E+10)	4 (1.997505E+03)	8 (3.070511E+03)	11 (4.696290E+09)	7 (1.428289E+03)	9 (3.445546E+03)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)	1 (2.299705E+05)
f_2	Mean	5.984047E-06	3.797939E+05	5.926491E+10	5.926491E+10	1.162843E+06	4.208332E-04	1.156005E+34	2.300306E+15	2.611443E+19	0.000000E+00	0.000000E+00	7.802038E-14	6.431997E+35
	Rank(WRST (Std Dev))	4 (4.31802E-06)	6 (1.210756E+06)	13 (2.710756E+39)	13 (2.520132E+11)	8 (5.866101E+06)	5 (1.422770E-03)	11 (7.234917E+34)	9 (1.230974E+16)	10 (1.009131E+20)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)	12 (1.28989E-15)
f_3	Mean	1.958911E-08	5.358612E+03	7.386313E+03	1.999399E+03	2.606957E+04	9.567629E-03	4.162308E+04	3.217390E+03	6.343870E+04	0.000000E+00	0.000000E+00	0.000000E+00	2.649926E+05
	Rank(WRST (Std Dev))	4 (8.948807E-09)	8 (4.294939E+03)	9 (1.262173E+04)	6 (1.999399E+03)	5 (1.143704E+03)	5 (6.907185E-03)	11 (1.225890E+04)	7 (6.193711E+03)	12 (1.340731E+04)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)	7 (4.83411E+04)
f_4	Mean	1.563382E-01	7.034203E+01	4.537046E+02	9.918793E+01	3.353988E+01	2.954592E+01	1.022019E+03	9.074716E+01	9.826454E+01	4.149698E+01	5.856156E+01	5.856156E+01	6.381693E-03
	Rank(WRST (Std Dev))	2 (7.815346E-01)	8 (2.749051E+01)	12 (1.766332E+03)	12 (1.779533E+01)	4 (3.444641E+01)	5 (3.118045E+01)	13 (1.295426E+03)	9 (3.452403E+01)	10 (2.969624E+01)	6 (9.71045E+01)	5 (2.986919E+01)	5 (2.986919E+01)	13 (8.15032E-03)
f_5	Mean	1.942219E+01	1.589549E+02	2.351194E+02	1.596282E+01	6.125573E+01	4.814894E+01	1.905352E+02	1.202725E+02	1.282664E+01	2.250215E+00	1.154465E+01	3.176902E+00	1.006626E+02
	Rank(WRST (Std Dev))	4 (4.213610E+00)	12 (5.186506E+01)	12 (7.879711E+01)	5 (2.359011E+00)	5 (6.125573E+01)	11 (3.184258E+01)	11 (3.184258E+01)	9 (1.202725E+02)	10 (1.282664E+01)	3 (2.250215E+00)	3 (2.250215E+00)	3 (2.250215E+00)	8 (1.006626E+02)
f_6	Mean	1.128864E+00	9.728998E+00	5.760280E+01	1.041336E-06	2.682464E-04	8.427001E-03	4.404123E+01	2.711286E+01	3.147412E+01	0.000000E+00	1.313101E-08	5.349996E-14	1.348658E-09
	Rank(WRST (Std Dev))	4 (0.083412E+01)	9 (9.728998E+00)	13 (5.760280E+01)	13 (1.041336E-06)	6 (2.682464E-04)	7 (8.427001E-03)	10 (4.404123E+01)	10 (2.711286E+01)	11 (3.147412E+01)	1 (0.000000E+00)	4 (5.349996E-14)	2 (5.349996E-14)	6 (7.79999E-09)
f_7	Mean	4.931856E+01	4.725432E+02	5.040936E+02	4.783596E+01	1.082332E+02	8.086794E+01	3.410663E+02	2.826301E+02	2.612108E+02	3.360975E+01	4.336022E+01	4.336022E+01	1.762416E+02
	Rank(WRST (Std Dev))	5 (6.604428E+00)	12 (1.168230E+02)	13 (5.040936E+02)	4 (4.783596E+01)	7 (1.082332E+02)	6 (8.086794E+01)	10 (3.410663E+02)	9 (2.826301E+02)	9 (2.612108E+02)	1 (3.360975E+01)	2 (4.336022E+01)	2 (4.336022E+01)	8 (1.762416E+02)
f_8	Mean	1.412451E+01	4.858693E+01	1.620788E+02	1.620788E+02	1.763849E+01	5.874148E+01	1.569058E+02	9.880226E+01	8.162524E+01	2.700222E+00	1.354633E+01	1.354633E+01	1.041229E+02
	Rank(WRST (Std Dev))	4 (8.586930E+00)	12 (3.944996E+01)	12 (1.566051E+02)	12 (1.566051E+02)	5 (1.763849E+01)	3 (5.874148E+01)	12 (1.569058E+02)	12 (9.880226E+01)	12 (8.162524E+01)	1 (2.700222E+00)	1 (1.354633E+01)	1 (1.354633E+01)	10 (1.041229E+02)
f_9	Mean	6.641871E-01	4.214683E+03	4.182518E+03	4.214683E+03	4.214683E+03	4.214683E+03	4.214683E+03	4.214683E+03	4.214683E+03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank(WRST (Std Dev))	7 (5.80883E+00)	12 (4.671021E+02)	12 (4.182518E+03)	12 (4.214683E+03)	12 (4.214683E+03)	12 (4.214683E+03)	12 (4.214683E+03)	12 (4.214683E+03)	12 (4.214683E+03)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)
f_{10}	Mean	1.896266E+03	4.214683E+03	4.214683E+03	4.214683E+03	4.214683E+03	4.214683E+03	4.214683E+03	4.214683E+03	4.214683E+03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank(WRST (Std Dev))	5 (3.317186E+02)	10 (7.816016E+02)	10 (4.214683E+03)	10 (4.214683E+03)	10 (4.214683E+03)	10 (4.214683E+03)	10 (4.214683E+03)	10 (4.214683E+03)	10 (4.214683E+03)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)
f_{11}	Mean	4.822852E+01	9.792322E+01	4.053098E+02	4.964001E+01	6.188070E+01	3.584801E+01	1.734666E+03	8.703640E+01	2.825288E+02	5.577215E+00	1.239968E+01	1.954861E+01	1.684375E+03
	Rank(WRST (Std Dev))	1 (6.769430E+01)	3 (3.231460E+01)	6 (8.826432E+02)	2 (5.61209E+01)	2 (6.188070E+01)	2 (3.584801E+01)	12 (1.734666E+03)	9 (8.703640E+01)	10 (2.825288E+02)	1 (5.577215E+00)	2 (1.239968E+01)	2 (1.954861E+01)	1 (1.684375E+03)
f_{12}	Mean	3.363442E+02	7.523383E+05	1.032225E+08	2.719608E+04	3.363876E+05	8.201571E+08	2.327738E+04	1.439919E+06	4.184291E+02	3.754138E+02	5.645356E+02	5.645356E+02	1.853178E+07
	Rank(WRST (Std Dev))	1 (4.97155E+02)	12 (2.462893E+05)	12 (1.095627E+08)	8 (2.719608E+04)	12 (3.363876E+05)	12 (8.201571E+08)	10 (2.327738E+04)	9 (1.439919E+06)	11 (4.184291E+02)	2 (3.754138E+02)	2 (5.645356E+02)	2 (5.645356E+02)	4 (1.853178E+07)
f_{13}	Mean	9.833025E+01	2.462893E+04	3.900841E+07	6.027222E+03	1.270473E+08	3.374574E+03	1.201860E+04	1.958999E+04	1.592853E+01	1.659200E+01	1.349452E+01	1.349452E+01	5.769335E+06
	Rank(WRST (Std Dev))	2 (2.767901E+01)	12 (1.796796E+04)	12 (3.900841E+07)	6 (6.027222E+03)	12 (1.270473E+08)	8 (3.374574E+03)	12 (1.201860E+04)	12 (1.958999E+04)	1 (1.592853E+01)	1 (1.659200E+01)	1 (1.349452E+01)	1 (1.349452E+01)	1 (5.769335E+06)
f_{14}	Mean	6.636587E+01	2.095472E+04	2.396158E+01	2.396158E+01	9.877418E+03	3.975100E+05	8.079636E+01	3.185777E+03	2.271460E+01	2.271460E+01	2.267955E+01	2.267955E+01	1.510212E+05
	Rank(WRST (Std Dev))	6 (1.554954E+01)	12 (1.960029E+04)	12 (2.396158E+01)	12 (2.396158E+01)	6 (9.877418E+03)	12 (3.975100E+05)	9 (8.079636E+01)	10 (3.185777E+03)	2 (2.271460E+01)	2 (2.271460E+01)	2 (2.267955E+01)	2 (2.267955E+01)	1 (1.510212E+05)
f_{15}	Mean	1.650462E+01	2.870376E+04	1.922796E+02	1.922796E+02	6.934866E+03	2.143619E+06	7.331446E+02	8.739936E+03	3.129771E+02	2.919147E+00	4.889691E+02	4.889691E+02	1.916358E+06
	Rank(WRST (Std Dev))	7 (7.033117E+00)	12 (1.610975E+03)	12 (1.922796E+02)	12 (1.922796E+02)	6 (6.934866E+03)	12 (2.143619E+06)	10 (7.331446E+02)	9 (8.739936E+03)	11 (3.129771E+02)	1 (2.919147E+00)	3 (4.889691E+02)	3 (4.889691E+02)	1 (1.916358E+06)
f_{16}	Mean	3.518981E+02	1.255662E+03	1.286211E+03	1.646149E+02	5.014051E+02	4.720756E+02	1.360746E+03	1.034404E+03	1.042673E+03	2.060433E+01	3.366964E+02	3.366964E+02	1.886236E+02
	Rank(WRST (Std Dev))	5 (1.756092E+02)	12 (2.784675E+02)	11 (1.286211E+03)	4 (1.646149E+02)	5 (5.014051E+02)	12 (4.720756E+02)	12 (1.360746E+03)	12 (1.034404E+03)	12 (1.042673E+03)	1 (2.060433E+01)	2 (3.366964E+02)	2 (3.366964E+02)	1 (1.886236E+02)
f_{17}	Mean	1.854769E+02	8.234072E+02	6.074097E+02	3.433031E+02	1.234770E+02	5.968078E+02	6.220736E+02	3.671575E+02	3.072688E+01	2.919113E+01	3.146980E+01	3.146980E+01	1.763337E+02
	Rank(WRST (Std Dev))	7 (7.633948E+01)	13 (3.380666E+02)	13 (6.074097E+02)	5 (3.433031E+02)	5 (1.234770E+02)	12 (5.968078E+02)	12 (6.220736E+02)	12 (3.671575E+02)	10 (3.072688E+01)	2 (2.919113E+01)	2 (3.146980E+01)	2 (3.146980E+01)	12 (1.763337E+02)
f_{18}	Mean	2.850882E+01	2.127784E+05	5.608562E+03	2.722108E+05	1.383381E+06	4.302743E+03	2.360743E+03	9.594670E+04	2.586032E+01	2.091940E+01	2.346201E+01	2.346201E+01	2.380117E+06
	Rank(WRST (Std Dev))	5 (1.643335E+00)	12 (2.196501E+05)	8 (5.608562E+03)	12 (2.722108E+05)	12 (1.383381E+06)	12 (4.302743E+03)	12 (2.360743E+03)	9 (9.594670E+04)	2 (2.586032E+01)	2 (2.091940E+01)	2 (2.346201E+01)	2 (2.346201E+01)	1 (2.380117E+06)
f_{19}	Mean	8.532338E+01	5.958385E+03	3.613613E+06	8.429318E+03	5.659126E+01	1.723770E+08	2.789896E+03	9.387670E+03	7.370806E+01	5.713960E+03	9.951532E+00	9.951532E+00	1.487179E+06
	Rank(WRST (Std Dev))	6 (2.562886E+01)	12 (5.958385E+03)	12 (3.613613E+06)	12 (8.429318E+03)	5 (5.659126E+01)	12 (1.723770E+08)	12 (2.789896E+03)	9 (9.387670E+03)	11 (7.370806E+01)	2 (5.713960E+03)	1 (9.951532E+00)	1 (9.951532E+00)	1 (1.487179E+06)
f_{20}	Mean	1.199925E+02	6.586391E+02	5.322117E+02	4.044238E+01	1.695176E+02	6.254483E+02	6.254475E+02	4.417173E+02	3.601297E+01	3.411999E+01	7.77026E+01	7.77026E+01	6.284465E+02
	Rank(WRST (Std Dev))	5 (6.972066E+01)	13 (2.743036E+02)	10 (5.322117E+02)	10 (4.044238E+01)	6 (1.695176E+02)	12 (6.254483E+02)	12 (6.254475E+02)	11 (4.417173E+02)	8 (3.601297E+01)	2 (3.411999E+01)	2 (7.77026E+01)	2 (7.77026E+01)	1 (6.284465E+02)
f_{21}	Mean	2.138411E+02	2.824243E+02	4.290777E+02	2.155385E+02	2.613813E+02	3.899238E+02	2.909581E+02	3.075996E+02	1.912119E+02	2.126622E+02	2.082440E+02	2.082440E+02	2.829275E+02
	Rank(WRST (Std Dev))	4 (2.18879E+01)	12 (2.824243E+02)	12 (4.290777E+02)	5 (2.155385E+02)	5 (2.613813E+02)	12 (3.899238E+02)	12 (2.909581E+02)	12 (3.075996E+02)	11 (1.912119E+02)	2 (2.126622E+02)	2 (2.082440E+02)	2 (2.082440E+02)	1 (2.829275E+02)
f_{22}	Mean	6.593500E+02	1.722861E+03	3.438187E+03	1.001444E+02	1.010295E+02	1.013404E+02	3.96871E+03	3.454612E+03	1.005515E+02	1.000000E+02	1.000000E+02	1.000000E+02	7.325994E+03
	Rank(WRST (Std Dev))	8 (2.19996E+02)	9 (1.722861E+03)	10 (3.438187E+03)	6 (1.001444E+02)	6 (1.010295E+02)	6 (1.013404E+02)	12 (3.96871E+03)	12 (3.454612E+03)	11 (1.005515E+02)	1 (1.000000E+02)	1 (1.000000E+02)	1 (1.000000E+02)	1 (7.325994E+03)
f_{23}	Mean	3.739848E+02	4.815682E+02	8.005385E+02	3.611988E+02	4.181344E+02	3.951582E+02	6.381376E+02	4.918576E+02	5.470798E+02	3.519423E+02	3.549051E+02	3.552445E+02	

Table 12

Experimental results of ELVS-PSOIMLSM versus non-PSO algorithms on 50-dimensional test functions over 51 runs, including mean values, ranks, standard deviations, and Wilcoxon rank sum test results.

f_i	Criteria	ELVS-PSOIMLSM	IWA 2024	NBA 2024	APD 2024	PBO 2024	ABDA 2024	SG20 2023	OPTA 2020	WVO 2019	ESOWiCMAR 2017	LSHARE-csflgm 2017	LSHARE-SPACMA 2007	CMA-ES 2001
f_1	Mean	7.305756E-03	3.889857E+03	1.035070E+10	1.477960E+03	2.232096E+03	1.649401E+03	2.663999E+10	2.404931E+04	3.990760E+03	0.000000E+00	0.000000E+00	0.000000E+00	1.770393E+04
	Rank(WRST (Std Dev))	4 (3.829534E-03)	8 (6.295696E+03)	12 (1.813977E+10)	8 (1.895193E+03)	7 (2.981225E+03)	6 (1.418987E+03)	13 (7.512149E+09)	11 (2.354747E+04)	9 (4.483719E+03)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)	10 (2.658464E+04)
f_2	Mean	8.390439E-06	1.860428E+41	9.580474E+70	3.690501E+12	4.612608E+22	6.275280E+04	7.375325E+59	1.504032E+28	2.562023E+47	0.000000E+00	0.000000E+00	0.000000E+00	4.543618E+60
	Rank(WRST (Std Dev))	4 (8.046668E-06)	9 (1.28612E+42)	13 (6.789402E+71)	6 (1.035946E+13)	7 (2.35596E+23)	5 (3.584866E+05)	12 (4.886292E+60)	8 (9.950490E+28)	10 (1.225311E+48)	3 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)	12 (3.972602E+61)
f_3	Mean	3.211592E-07	2.260579E+04	3.389075E+04	2.589952E+04	1.485575E+05	8.553954E-01	9.183013E+04	1.227201E+04	1.548726E+05	0.000000E+00	0.000000E+00	0.000000E+00	5.125669E+05
	Rank(WRST (Std Dev))	4 (6.434209E-08)	7 (7.717481E+03)	9 (4.287075E+04)	9 (5.018635E+03)	11 (1.087984E+04)	5 (4.034641E-01)	10 (1.441310E+04)	6 (6.455975E+03)	12 (2.077402E+04)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)	13 (1.102219E+05)
f_4	Mean	5.148651E-01	7.798058E-01	1.927883E+03	1.234438E+02	9.050279E-01	7.970614E-01	3.247067E+03	1.987806E+02	1.709076E+02	0.000000E+00	0.000000E+00	0.000000E+00	3.564634E+00
	Rank(WRST (Std Dev))	1 (3.869691E+00)	6 (4.711931E+01)	12 (1.647038E+03)	6 (5.110716E-01)	9 (5.110716E-01)	8 (4.83122E+01)	13 (1.585304E+03)	11 (4.949149E+01)	10 (4.872434E+01)	4 (2.545103E+01)	5 (4.113303E+01)	3 (3.852930E+01)	2 (4.897060E-01)
f_5	Mean	2.920496E-01	3.557242E+02	3.935629E+02	5.942474E+01	1.549400E+02	1.466294E+02	3.693608E+02	2.349652E+02	2.550992E+02	0.000000E+00	0.000000E+00	0.000000E+00	2.634654E+02
	Rank(WRST (Std Dev))	4 (8.537699E+00)	11 (2.762178E-01)	13 (9.303649E+01)	13 (8.499106E+00)	5 (2.766303E+01)	6 (2.568316E+01)	12 (4.273737E+01)	3 (3.607162E+01)	8 (4.685718E-01)	2 (2.362984E+00)	2 (6.999696E+00)	2 (2.362984E+00)	10 (1.034999E+02)
f_6	Mean	3.586812E+00	2.352812E+01	6.659498E+01	1.051073E-03	2.459438E-02	6.219969E-02	5.632408E-01	4.164476E-01	4.410768E-01	1.425966E-07	7.305995E-07	2.370281E-10	8.444614E-09
	Rank(WRST (Std Dev))	8 (9.782146E-01)	2 (2.037326E-01)	4 (6.384546E+00)	13 (4.398041E-03)	6 (4.311248E-02)	12 (2.184708E-02)	10 (7.746321E+00)	10 (8.453959E-00)	11 (1.843906E-00)	4 (7.537888E-07)	4 (1.691906E-09)	1 (1.691906E-09)	5 (5.39293E-08)
f_7	Mean	1.089934E+02	9.013808E+02	1.001755E+03	1.116703E+02	2.764779E+02	7.078348E+02	1.670595E+02	7.202014E+02	5.183763E+02	5.812965E-01	7.543399E-01	5.699538E-01	3.514692E+02
	Rank(WRST (Std Dev))	4 (1.369466E+01)	12 (1.441315E+02)	10 (1.266912E+02)	5 (9.464848E+00)	7 (4.542837E+01)	6 (2.601823E-01)	8 (1.098700E+02)	9 (1.278529E+02)	11 (1.358491E+02)	2 (1.437464E+00)	2 (3.819219E+00)	2 (1.137030E+00)	8 (4.322899E+01)
f_8	Mean	2.780322E+01	3.607032E+02	3.974994E+02	5.174668E+02	1.717466E+02	1.386717E+02	3.791488E+02	2.005164E+02	2.421329E+02	2.523908E-01	2.523908E-01	5.911227E-01	2.020509E+02
	Rank(WRST (Std Dev))	7 (9.084646E+00)	13 (3.743750E+01)	11 (6.636796E+01)	8 (8.153407E+00)	7 (2.628484E+01)	12 (2.253198E+01)	10 (4.874126E+01)	9 (3.074198E+01)	8 (4.799703E+01)	3 (3.449914E+00)	3 (3.449914E+00)	3 (1.912926E+00)	10 (2.728602E+01)
f_9	Mean	1.379289E+01	1.177419E+04	1.441535E+04	8.747082E+00	1.887052E+03	2.185755E-01	2.643015E+03	6.190412E+03	5.991704E+03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank(WRST (Std Dev))	6 (9.216326E+00)	11 (6.961181E+02)	13 (5.328813E+03)	6 (6.073504E+00)	11 (1.07796E+03)	12 (1.668535E+01)	10 (6.504730E+03)	8 (1.230526E+03)	9 (3.212576E+03)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)
f_{10}	Mean	3.844812E+03	7.389141E+03	7.527912E+03	4.232675E+03	4.697136E+03	4.670873E+03	8.128416E+03	6.428279E+03	3.195796E+03	3.099611E+03	3.195796E+03	3.746065E+03	1.342941E+04
	Rank(WRST (Std Dev))	6 (6.640485E+02)	8 (8.634932E+02)	9 (1.791000E+03)	4 (4.313288E+02)	6 (6.727713E+02)	5 (6.859876E+02)	10 (8.257857E+02)	12 (2.559962E+03)	11 (1.534888E+03)	3 (4.973060E+02)	3 (3.231362E+02)	3 (7.662567E+02)	4 (4.782966E+02)
f_{11}	Mean	9.698998E-01	1.421512E+02	9.214997E+02	1.002891E+02	1.465704E+02	9.528697E-03	1.767274E+02	1.354019E+03	2.754222E+01	2.119195E-01	2.119195E-01	3.068257E-01	1.268219E+04
	Rank(WRST (Std Dev))	7 (1.521913E+01)	4 (4.613165E-01)	7 (2.236272E+03)	7 (2.426311E+01)	5 (3.564585E+01)	12 (2.643070E+03)	4 (4.756772E+01)	10 (1.6380041E+03)	9 (1.153488E+03)	2 (2.131311E+00)	2 (4.301706E+00)	2 (4.301706E+00)	6 (8.28574E+03)
f_{12}	Mean	9.299074E+02	1.414066E+06	3.633106E+09	8.119138E+05	1.818404E+06	8.841966E+05	8.717098E+09	2.471283E+05	9.768157E+06	1.919246E+03	1.919246E+03	1.679743E+03	1.793825E+07
	Rank(WRST (Std Dev))	1 (2.146421E+02)	4 (7.391505E+05)	11 (9.078948E+06)	8 (1.097884E+06)	5 (3.634667E+05)	6 (3.428934E+05)	13 (6.246324E+09)	10 (2.908214E+05)	9 (4.973970E+06)	3 (3.31588E+02)	3 (3.31588E+02)	3 (3.828723E+02)	7 (7.653388E+06)
f_{13}	Mean	1.836338E+02	4.106548E+01	2.107155E+08	2.011568E+03	2.804780E+03	3.770054E+03	2.342657E+09	5.673436E+03	2.070327E+04	4.545110E+01	6.980173E-01	1.677320E+01	7.099999E+06
	Rank(WRST (Std Dev))	4 (1.106946E+01)	8 (7.077964E+03)	13 (5.037082E+09)	2 (2.016851E+03)	2 (2.858544E+03)	4 (4.625667E+03)	5 (4.657562E+09)	6 (5.371925E+03)	11 (1.135646E+04)	2 (2.402441E+01)	3 (3.572902E+01)	3 (1.893911E+01)	4 (4.484548E+06)
f_{14}	Mean	1.029131E+02	4.669256E+04	2.713818E+05	3.787912E+02	2.70314E+05	1.596636E+02	2.13270E+06	1.174196E+03	2.207475E+06	3.049613E+01	2.660679E-01	2.839966E-01	1.708705E+05
	Rank(WRST (Std Dev))	4 (2.472799E+01)	12 (2.39916E+04)	13 (2.39916E+04)	6 (3.077949E+02)	12 (2.812258E+02)	10 (4.21296E+05)	11 (3.101534E+06)	8 (8.220672E+06)	9 (1.724198E+04)	3 (3.229729E+06)	3 (2.155505E+00)	3 (2.499514E+00)	4 (3.42646E+05)
f_{15}	Mean	6.065492E-01	1.494645E+04	1.704676E+07	9.96411E+02	7.73166E+03	5.465741E+03	1.842799E+03	8.989026E+03	3.017690E+02	2.643955E-01	2.643955E-01	3.169426E+01	3.568495E+06
	Rank(WRST (Std Dev))	4 (1.420555E+01)	6 (2.40741E+03)	12 (6.24671E+03)	12 (1.146967E+03)	13 (5.861888E+07)	13 (4.359376E+08)	13 (4.359376E+08)	10 (5.604954E+03)	9 (1.536676E+03)	2 (4.039576E+02)	2 (3.032903E+00)	2 (6.57118E+00)	11 (1.845494E+06)
f_{16}	Mean	5.360188E+02	2.339664E+02	5.883504E+02	1.044555E+03	9.427740E+02	2.350390E+03	1.825133E+02	3.385493E+02	3.054878E+02	3.054878E+02	3.054878E+02	4.095105E+02	1.977222E+03
	Rank(WRST (Std Dev))	4 (3.232004E+02)	4 (4.158406E+02)	4 (4.158406E+02)	12 (1.072017E+03)	12 (1.080551E+02)	6 (2.494788E+02)	13 (2.848742E+02)	9 (4.487169E+02)	8 (4.102546E+02)	2 (1.042806E+02)	2 (1.042806E+02)	2 (1.042806E+02)	6 (4.687736E+02)
f_{17}	Mean	8.124501E+02	1.861873E+03	1.789898E+02	3.962250E+02	8.297370E+02	6.708695E+02	1.707847E+03	1.498268E+02	1.942963E+02	2.389208E+02	2.389208E+02	3.008252E+02	1.761954E+03
	Rank(WRST (Std Dev))	6 (2.257236E+02)	13 (4.516753E+02)	13 (4.516753E+02)	4 (1.311466E+02)	5 (1.311466E+02)	5 (1.311466E+02)	10 (4.041339E+02)	9 (3.704241E+02)	8 (3.194805E+02)	2 (7.884687E+01)	2 (6.209844E+01)	2 (1.180527E+02)	11 (2.175334E+02)
f_{18}	Mean	3.527883E+01	2.602640E+05	2.713111E+06	5.093026E+04	1.970345E+06	9.720951E+02	1.019994E+07	2.409295E+04	2.623915E+05	2.354242E+01	2.354242E+01	2.354242E+01	4.520543E+06
	Rank(WRST (Std Dev))	4 (8.83296E+00)	8 (1.11327E+06)	11 (3.440519E+04)	9 (8.01367E+05)	10 (1.135390E+06)	10 (9.980207E+03)	13 (1.754234E+07)	11 (4.160573E+04)	9 (2.212277E+05)	3 (6.695482E+00)	3 (2.137762E+00)	3 (5.174596E+00)	12 (9.794477E+06)
f_{19}	Mean	9.842577E+01	2.257338E+04	2.633073E+07	6.021919E+03	1.789995E+04	1.185951E+04	8.311479E+07	8.985429E+07	1.790457E+04	2.623106E+01	2.623106E+01	2.996690E+01	1.954899E+06
	Rank(WRST (Std Dev))	2 (2.807327E+01)	4 (1.072579E+04)	12 (1.272579E+04)	5 (2.113515E+03)	8 (5.753646E+07)	8 (5.753646E+07)	13 (2.121535E+08)	12 (9.113727E+07)	10 (1.492486E+04)	2 (4.899960E+00)	2 (3.29529E+00)	2 (3.71919E+00)	11 (7.944796E+05

Table 13

Experimental results of ELVS-PSOIMLSM versus non-PSO algorithms on 100-dimensional test functions over 51 runs, including mean values, ranks, standard deviations, and Wilcoxon rank sum test results.

	Criteria	ELVS-PSOIMLSM	FYA 2024	IBA 2024	AO 2024	PRO 2024	ABOA 2024	SGSO 2023	GFA 2020	WVO 2019	EvoMCMAR 2017	LSHADE-cnfgs 2017	LSHADE-SPACMA 2017	CMA-ES 2001
f_1	Mean	6.769721E-02	8.860970E+04	6.094122E+10	3.779904E+03	1.317300E+06	1.075016E+04	9.123407E+10	1.920899E+05	1.305066E+04	1.057736E-08	0.000000E+00	6.408817E-15	1.548491E+04
	Rank(WRST (Std. Dev.))	4 (5.470968E-02)	5 (7.72334E+05)	12 (7.112042E+10)	6 (3.543597E+03)	11 (3.534169E+06)	6 (3.137505E+03)	13 (2.107899E+05)	1 (1.402666E+04)	2 (2.217398E-08)	1 (0.000000E+00)	2 (7.141559E-15)	1 (5.82087E+04)	
	Mean	5.179986E-06	1.477088E+4	1.799562E+169	2.808266E+43	4.552297E+87	4.305866E+33	5.688639E+140	1.030629E+70	8.301424E+138	7.365474E-08	7.528023E+02	3.534824E+00	1.545020E+105
f_2	Rank(WRST (Std. Dev.))	2 (0.027875E-06)	7 (1.048532E+05)	13 (2.001670E+44)	6 (2.001670E+44)	12 (3.13804E+88)	9 (3.062206E+34)	12 (3.776452E+141)	8 (6.89059E+70)	11 (5.528402E+139)	1 (1.64031E-07)	1 (3.587115E+03)	1 (1.687110E+01)	1 (1.10254E+106)
	Mean	4.087182E-05	9.983388E+4	1.301257E+05	1.344929E+05	4.893819E+05	2.287328E+02	2.24131E+05	4.315481E+04	4.213502E+05	2.000340E-05	0.000000E+00	0.000000E+00	1.18873E+06
	Rank(WRST (Std. Dev.))	4 (4.370725E-06)	5 (2.017414E+04)	8 (5.01706E+04)	12 (1.636314E+04)	12 (4.328360E+04)	12 (2.778810E+01)	10 (2.138145E+04)	6 (1.342654E+04)	11 (4.032205E+04)	3 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)	1 (2.344944E+05)
f_3	Mean	2.345073E-01	2.087476E+02	1.109871E+04	2.570894E+02	2.375709E+02	2.542088E+02	1.261021E+04	3.933156E+02	3.650203E+02	1.877485E+02	1.589365E+02	2.050668E+02	4.065414E+01
	Rank(WRST (Std. Dev.))	1 (9.473630E-01)	6 (4.676355E+01)	12 (2.274857E+04)	9 (4.58877E+01)	7 (5.94069E+01)	8 (4.360354E+01)	13 (4.00319E+03)	4 (4.697345E+01)	6 (4.615201E+01)	4 (4.795322E+01)	5 (4.766455E+00)	5 (3.79399E+00)	2 (3.79369E-01)
	Mean	7.068132E+01	8.224929E+02	9.567911E+02	2.577131E+02	4.843601E+02	4.516993E+02	9.843768E+02	6.520045E+02	6.486330E+02	2.828805E+01	5.601439E+01	1.125669E+01	2.731260E+01
f_4	Rank(WRST (Std. Dev.))	5 (9.880606E+00)	11 (4.79201E+01)	12 (2.048332E+02)	6 (2.57693E+01)	5 (4.83050E+01)	7 (3.20545E+01)	6 (1.11779E+01)	9 (5.76083E+01)	8 (5.83774E+01)	3 (4.79679E+00)	4 (5.161451E+00)	2 (4.793653E+00)	2 (4.793653E+00)
	Mean	8.113929E+00	4.979045E+01	7.356463E+01	1.227256E+01	1.867208E+00	2.363281E+00	6.999623E+01	5.215020E+01	5.411080E+01	1.662955E+01	5.979705E+01	1.138668E-13	1.276371E-08
	Rank(WRST (Std. Dev.))	8 (1.713018E+00)	11 (1.674432E+01)	13 (1.487134E+01)	5 (1.50119E-01)	6 (1.50119E-00)	12 (8.132747E-01)	12 (4.534373E+00)	10 (5.276747E+00)	11 (5.276747E+00)	3 (5.77206E-06)	4 (1.855471E-05)	1 (0.000000E+00)	2 (3.451407E-08)
f_5	Mean	3.999236E+02	2.980331E+03	2.644140E+03	3.962979E+02	1.104591E+03	3.858454E+02	2.121195E+03	2.277663E+03	1.604777E+03	1.268932E+02	1.626096E+02	1.118907E+02	1.533632E+02
	Rank(WRST (Std. Dev.))	6 (4.560368E+01)	12 (2.853636E+02)	12 (2.672344E+02)	8 (4.213815E+01)	8 (7.981861E+01)	7 (2.56982E+01)	10 (1.563006E+02)	11 (3.816289E+02)	9 (3.146891E+02)	2 (1.35007E+00)	4 (1.253649E+00)	3 (1.322781E+02)	3 (1.322781E+02)
	Mean	7.706051E-01	9.713076E+02	1.061043E+03	4.939360E+02	2.571732E+02	4.195037E+02	1.043301E+03	6.937538E+02	7.084168E+02	3.98029E+01	5.642463E+01	1.078648E+01	2.680537E+01
f_6	Rank(WRST (Std. Dev.))	2 (1.170049E+01)	7 (7.23370E+01)	12 (1.075964E+02)	8 (3.023326E+01)	5 (3.007701E+01)	7 (4.97298E+01)	10 (6.931324E+01)	9 (7.02942E+01)	11 (1.58958E+02)	4 (9.63708E+00)	5 (6.584853E+00)	2 (3.66099E+00)	2 (3.66099E+00)
	Mean	2.149090E+02	2.180949E+04	3.040135E+04	6.875198E+02	1.242936E+04	4.911426E+03	7.292018E+04	1.700819E+04	1.576790E+04	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
	Rank(WRST (Std. Dev.))	5 (1.494272E+02)	11 (7.723391E+02)	12 (1.119636E+04)	11 (5.68508E+02)	12 (2.38804E+03)	12 (4.265265E+03)	13 (1.157945E+04)	10 (3.39954E+03)	9 (4.447109E+03)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)	1 (0.000000E+00)
f_7	Mean	8.349467E+03	1.456305E+04	1.776991E+04	1.176661E+04	1.276677E+04	1.248403E+04	1.248403E+04	1.633964E+04	1.452604E+04	8.989582E+03	1.036056E+04	2.813023E+04	1.53023E+04
	Rank(WRST (Std. Dev.))	1 (1.212055E+03)	1 (1.252906E+03)	4 (4.946254E+03)	1 (1.05686E+03)	1 (1.07143E+03)	1 (1.426522E+03)	1 (1.354643E+03)	4 (4.019317E+03)	5 (3.75584E+03)	2 (1.691250E+03)	3 (3.071960E+02)	5 (7.96993E+02)	6 (5.37341E+03)
	Mean	1.926170E+02	8.727168E+02	1.887905E+04	4.963401E+02	7.944098E+02	3.948527E+02	5.804965E+04	9.614082E+02	7.403861E+04	7.091782E+01	5.736208E+01	4.872372E+01	6.578111E+05
f_8	Rank(WRST (Std. Dev.))	4 (4.082635E+01)	1 (1.631121E+02)	13 (3.95521E+04)	1 (1.061874E+02)	1 (1.35520E+02)	1 (1.061874E+02)	1 (2.62613E+02)	1 (1.66294E+02)	2 (2.33008E+01)	3 (3.478640E+01)	2 (2.368787E+01)	1 (1.677602E+05)	1 (1.677602E+05)
	Mean	2.665879E+03	2.140687E+06	2.946757E+10	2.479266E+06	1.301656E+07	7.801628E+06	3.407547E+10	1.17287E+07	1.174747E+08	4.130321E+03	4.649918E+03	4.630347E+03	1.242008E+07
	Rank(WRST (Std. Dev.))	1 (3.32218E+02)	1 (3.32218E+02)	13 (3.32218E+02)	1 (3.32218E+02)	1 (3.32218E+02)	1 (3.32218E+02)	1 (3.32218E+02)	1 (3.32218E+02)	1 (3.32218E+02)	1 (3.32218E+02)	1 (3.32218E+02)	1 (3.32218E+02)	1 (3.32218E+02)
f_9	Mean	3.258700E+02	1.118930E+04	1.583773E+09	3.496281E+03	7.62458E+03	7.62458E+03	7.62458E+03	2.81977E+04	2.81977E+04	2.478735E+02	2.478735E+02	1.349131E+02	4.857279E+05
	Rank(WRST (Std. Dev.))	4 (6.129766E+01)	4 (8.424012E+03)	13 (5.636696E+09)	12 (2.28994E+03)	13 (3.50975E+03)	13 (3.50975E+03)	13 (3.50975E+03)	10 (1.04523E+04)	10 (1.04523E+04)	1 (0.04979E+02)	1 (0.04979E+02)	1 (0.04979E+02)	1 (0.04979E+02)
	Mean	1.502520E+02	1.898925E+05	5.933866E+05	2.742368E+05	1.381598E+06	4.990180E+04	4.961659E+06	3.847190E+04	3.514106E+05	4.965505E+01	7.294362E+01	3.039565E+01	3.039565E+01
f_{10}	Rank(WRST (Std. Dev.))	5 (6.48488E+01)	1 (1.898925E+05)	12 (1.302011E+07)	12 (2.28994E+03)	13 (3.50975E+03)	13 (3.50975E+03)	13 (3.50975E+03)	10 (1.04523E+04)	10 (1.04523E+04)	1 (0.04979E+02)	1 (0.04979E+02)	1 (0.04979E+02)	1 (0.04979E+02)
	Mean	9.558778E-01	3.261129E+03	1.063205E+09	9.431233E+02	1.680545E+03	1.637408E+03	2.215312E+03	1.682556E+03	1.682556E+03	9.94542E+01	9.94542E+01	5.834549E+05	5.834549E+05
	Rank(WRST (Std. Dev.))	1 (1.91957E+01)	8 (3.261129E+03)	12 (1.063205E+09)	12 (9.431233E+02)	12 (1.680545E+03)	12 (1.637408E+03)	12 (2.215312E+03)	10 (1.682556E+03)	10 (1.682556E+03)	2 (9.94542E+01)	2 (9.94542E+01)	1 (5.834549E+05)	1 (5.834549E+05)
f_{11}	Mean	1.322085E+03	4.346126E+03	7.490270E+03	1.964851E+03	3.338451E+03	3.274125E+03	6.745591E+03	4.082731E+03	4.414796E+03	1.597472E+03	1.276561E+03	1.280886E+03	4.897548E+02
	Rank(WRST (Std. Dev.))	1 (3.767358E+02)	10 (4.346126E+03)	13 (7.490270E+03)	13 (1.964851E+03)	13 (3.338451E+03)	13 (3.274125E+03)	13 (6.745591E+03)	13 (4.082731E+03)	13 (4.414796E+03)	11 (1.597472E+03)	11 (1.276561E+03)	11 (1.280886E+03)	11 (4.897548E+02)
	Mean	2.342323E+03	3.523457E+03	3.163412E+04	1.554234E+03	2.389751E+03	1.220866E+04	3.580646E+04	3.22259E+03	3.22259E+03	1.320781E+03	1.042247E+03	1.021441E+03	1.021441E+03
f_{12}	Rank(WRST (Std. Dev.))	7 (8.400616E+02)	4 (3.523457E+03)	13 (3.163412E+04)	13 (1.554234E+03)	13 (2.389751E+03)	13 (1.220866E+04)	13 (3.580646E+04)	13 (3.22259E+03)	13 (3.22259E+03)	11 (1.320781E+03)	11 (1.042247E+03)	11 (1.021441E+03)	11 (1.021441E+03)
	Mean	8.884293E+01	2.581435E+05	2.975111E+06	4.921331E+05	1.251968E+05	5.972394E+06	1.187322E+05	3.758613E+05	3.758613E+05	2.471987E+02	2.471987E+02	6.407128E+06	6.407128E+06
	Rank(WRST (Std. Dev.))	2 (2.034530E+01)	1 (2.581435E+05)	13 (2.975111E+06)	13 (4.921331E+05)	13 (1.251968E+05)	13 (5.972394E+06)	13 (1.187322E+05)	13 (3.758613E+05)	13 (3.758613E+05)	11 (2.471987E+02)	11 (2.471987E+02)	11 (6.407128E+06)	11 (6.407128E+06)
f_{13}	Mean	1.211074E+02	5.929126E+03	8.698985E+08	1.511961E+03	2.348806E+03	2.020927E+03	3.990411E+03	7.817179E+04	7.817179E+04	5.501490E+01	7.471376E+01	3.860428E+05	3.860428E+05
	Rank(WRST (Std. Dev.))	4 (3.18518E+01)	12 (5.929126E+03)	13 (8.698985E+08)	12 (1.511961E+03)	12 (2.348806E+03)	12 (2.020927E+03)	12 (3.990411E+03)	13 (7.817179E+04)	13 (7.817179E+04)	11 (5.501490E+01)	11 (7.471376E+01)	11 (3.860428E+05)	11 (3.860428E+05)
	Mean	1.450525E+03	3.604911E+03	3.274111E+03	4.118770E+03	7.761797E+03	1.940646E+03	5.505326E+03	3.282725E+03	3.282725E+03	1.471356E+03	1.290916E+03	2.244101E+03	2.244101E+03
f_{14}	Rank(WRST (Std. Dev.))	5 (3.605730E+02)	12 (3.604911E+03)	12 (3.274111E+03)	12 (4.118770E+03)	12 (7.761797E+03)	12 (1.940646E+03)	12 (5.505326E+03)	13 (3.282725E+03)	13 (3.282725E+03)	11 (1.471356E+03)	11 (1.290916E+03)	11 (2.244101E+03)	11 (2.244101E+03)
	Mean	3.281876E+02	8.227311E+02	1.862245E+02	4.18954E+02	6.86868E+02	5.571939E+02	1.512634E+02	4.974523E+02	1.015076E+03	2.568599E+02	2.78826E+02	2.412816E+02	2.485040E+02
	Rank(WRST (Std. Dev.))	1 (3.72536E+01)	1 (3.72536E+01)	12 (1.862245E+02)	12 (4.18954E+02)	12 (6.86868E+02)	12 (5.571939E+02)	12 (1.512634E+02)	12 (4.974523E+02)	12 (1.015076E+03)	11 (2.568599E+02)	11 (2.78826E+02)	11 (2.412816E+02)	11 (2.485040E+02)
f_{15}	Mean</													

Table 14
The best simulated results achieved by PSO-based variants for the GTD problem from 20 independent runs are reported.

Variables	EVLS-PSOIMLSM	IMPSOwithCMAR	SQPPSO-sono	EAPSO	HMPPSO	SpadePSO	ADPSO	PSO-sono	MFCPSO	AMSEPSO	EOPSO	PPSO	SDPSO	AHPSO	MPSO
x_1	49	57	43	49	43	43	49	43	51	49	43	43	43	43	56
x_2	16	12	16	19	16	19	16	19	30	16	19	19	16	19	13
x_3	19	37	19	16	19	16	19	16	13	19	16	16	19	16	23
x_4	43	54	49	43	49	49	43	49	53	43	49	49	49	49	37
f_{min}	2.700857E-12	8.887614E-10	2.700857E-12	2.700857E-12	2.700857E-12	2.700857E-12	2.700857E-12	2.700857E-12	2.307816E-11	2.700857E-12	2.700857E-12	2.700857E-12	2.700857E-12	2.700857E-12	6.602090E-10

Table 15
The best simulated results achieved by non-PSO algorithms for the GTD problem from 20 independent runs are reported.

Variables	ELVS-PSOIMLSM	IVYA	BKA	APO	PRO	AROA	SCHO	GPEA	WWO	EBOwithCMAR	LSHADE-cnEpSin	LSHADE-SPACMA	CMA-ES
x_1	49	55	43	53	43	49	53	55	49	43	43	49	54
x_2	16	12	19	13	19	19	13	34	16	19	19	19	17
x_3	19	39	16	20	16	16	20	23	19	16	16	16	22
x_4	43	59	49	34	49	43	34	51	43	49	49	43	48
f_{min}	2.700857E-12	3.299923E-09	2.700857E-12	2.307816E-11	2.700857E-12	2.700857E-12	2.307816E-11	6.750811E-10	2.700857E-12	2.700857E-12	2.700857E-12	2.700857E-12	1.166116E-10

Table 16

The best simulated results achieved by PSO-based variants for the PGTD problem from 20 independent runs are reported.

Variables	EVLS-PSOIMLSM	IMPSOwithCMAR	SQPPSO-sono	EAPSO	HMPPSO	SpadePSO	ADPSO	PSO-sono	MFCPSO	AMSEPSO	EOPSO	PPSO	SDPSO	AHPSO	MPSO
N_1	3.700000E+01	3.000000E+01	3.500000E+01	3.900000E+01	4.700000E+01	4.100000E+01	3.700000E+01	3.500000E+01	3.800000E+01	3.700000E+01	2.900000E+01	4.100000E+01	4.800000E+01	4.200000E+01	4.200000E+01
N_2	2.200000E+01	1.700000E+01	1.900000E+01	2.900000E+01	2.500000E+01	2.300000E+01	2.200000E+01	2.100000E+01	2.600000E+01	2.200000E+01	2.000000E+01	2.200000E+01	3.000000E+01	2.500000E+01	2.500000E+01
N_3	2.000000E+01	1.900000E+01	1.900000E+01	2.500000E+01	1.900000E+01	1.800000E+01	2.000000E+01	2.100000E+01	2.300000E+01	2.000000E+01	2.800000E+01	1.800000E+01	2.800000E+01	2.000000E+01	2.000000E+01
N_4	2.400000E+01	2.400000E+01	2.500000E+01	2.400000E+01	2.500000E+01	2.300000E+01	2.400000E+01	2.500000E+01	2.400000E+01	2.400000E+01	2.900000E+01	2.400000E+01	2.400000E+01	2.400000E+01	2.400000E+01
N_5	1.900000E+01	1.600000E+01	2.700000E+01	2.400000E+01	1.700000E+01	1.700000E+01	2.400000E+01	3.000000E+01	2.300000E+01	1.900000E+01	1.900000E+01	2.000000E+01	1.400000E+01	2.100000E+01	2.500000E+01
N_6	8.700000E+01	8.700000E+01	9.100000E+01	8.700000E+01	9.100000E+01	8.300000E+01	8.700000E+01	9.100000E+01	8.700000E+01	8.700000E+01	1.050000E+02	8.700000E+01	1.160000E+02	8.700000E+01	8.700000E+01
p	3.000000E+00	3.000000E+00	3.000000E+00	3.000000E+00	3.000000E+00	3.000000E+00	3.000000E+00	3.000000E+00	3.000000E+00	3.000000E+00	4.000000E+00	3.000000E+00	3.000000E+00	3.000000E+00	3.000000E+00
m_1	2.250000E+00	3.000000E+00	2.750000E+00	2.000000E+00	2.000000E+00	2.250000E+00	2.250000E+00	2.500000E+00	2.000000E+00	2.250000E+00	2.750000E+00	2.250000E+00	2.000000E+00	2.250000E+00	2.250000E+00
m_2	2.000000E+00	2.000000E+00	2.000000E+00	2.250000E+00	2.000000E+00	2.250000E+00	2.000000E+00	2.000000E+00	2.000000E+00	2.000000E+00	1.750000E+00	2.000000E+00	1.750000E+00	2.250000E+00	2.250000E+00
δ_1	-4.100000E+01	-4.100000E+01	-3.300000E+01	-1.862500E-01	-3.300000E+01	-2.762500E+01	-4.100000E+01	-3.300000E+01	-4.100000E+01	-4.100000E+01	-3.187500E+01	-4.100000E+01	-1.262500E+01	-1.862500E+01	-1.862500E-01
δ_2	-3.325000E+01	-2.200000E+01	-1.375000E+01	-2.200000E+01	-2.200000E+01	-1.975000E+01	-3.325000E+01	-2.250000E+01	-3.600000E+01	-3.325000E+01	-2.475000E+01	-2.425000E+01	0.000000E+00	-8.500000E+00	-8.500000E+00
δ_3	-9.200000E+01	-1.040000E+02	-5.800000E+01	-5.350000E+01	-9.800000E+01	-8.725000E+01	-7.200000E+01	-4.600000E+01	-7.600000E+01	-9.200000E+01	-9.925000E+01	-8.800000E+01	-1.115000E+02	-6.700000E+01	-4.900000E+01
δ_4	-3.000000E+00	0.000000E+00	-2.500000E-01	-7.500000E-01	-4.000000E+00	-2.250000E+00	-3.000000E+00	-4.500000E+00	-4.000000E+00	-3.000000E+00	-4.500000E+00	-5.000000E-01	-1.750000E+00	-4.500000E+00	-4.500000E+00
δ_5	-2.659550E+01	-2.120319E+01	-2.526537E+01	-2.738973E+01	-3.485383E+01	-2.992563E+01	-2.659550E+01	-2.499742E+01	-2.692563E+01	-2.659550E+01	-1.214823E+01	-3.005960E+01	-3.504998E+01	-3.052370E+01	-3.052370E+01
δ_6	-3.552370E+01	-3.738973E+01	-4.085383E+01	-2.619358E+01	-4.085383E+01	-3.579165E+01	-3.552370E+01	-3.712178E+01	-2.992563E+01	-3.552370E+01	-2.394722E+01	-3.925575E+01	-4.571024E+01	-3.552370E+01	-3.552370E+01
δ_7	-1.573909E+01	-1.614102E+01	-1.553332E+01	-1.506922E+01	-1.687307E+01	-1.514102E+01	-1.506922E+01	-1.513140E+01	-1.520319E+01	-1.573909E+01	-1.244113E+01	-1.560512E+01	-2.333717E+01	-1.547114E+01	-1.493524E+01
δ_8	-4.286027E+03	-5.459106E+03	-4.290694E+03	-1.364288E+03	-6.427642E+03	-4.618750E+03	-3.407520E+03	-2.907202E+03	-2.307811E+03	-4.286027E+03	-1.498700E+03	-5.045755E+03	-1.192775E+04	-3.915389E+03	-3.247468E+03
δ_9	-1.800000E+01	-2.000000E+01	-2.300000E+01	-8.000000E+00	-2.300000E+01	-1.900000E+01	-1.800000E+01	-1.900000E+01	-1.200000E+01	-1.800000E+01	-1.500000E+01	-2.200000E+01	-2.300000E+01	-1.800000E+01	-1.800000E+01
δ_{10}	-2.000000E+01	-2.600000E+01	-7.000000E+00	-1.000000E+01	-2.700000E+01	-2.100000E+01	-1.000000E+01	-1.000000E+01	-1.200000E+01	-2.000000E+01	-3.300000E+01	-1.800000E+01	-5.100000E+01	-1.600000E+01	-8.000000E+00
h	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
f_{min}	5.262806E-01	5.294236E-01	5.300000E-01	5.267399E-01	5.300000E-01	5.349097E-01	5.262806E-01	5.300000E-01	5.268955E-01	5.262806E-01	5.284211E-01	5.270040E-01	5.273469E-01	5.273469E-01	5.273469E-01

Table 17

The best simulated results achieved by non-PSO algorithms for the PGTD problem from 20 independent runs are reported.

Variables	ELVS-PSOIMLSM	IVYA	BKA	APO	PRO	AROA	SCHO	GPEA	WVO	EBOwithCMAR	LSHADE-cnEpSin	LSHADE-SPACMA	CMA-ES
N_1	3.700000E+01	2.700000E+01	3.500000E+01	3.500000E+01	3.800000E+01	3.600000E+01	2.700000E+01	2.400000E+01	4.000000E+01	3.500000E+01	3.500000E+01	3.900000E+01	3.500000E+01
N_2	2.200000E+01	1.900000E+01	2.400000E+01	2.600000E+01	2.600000E+01	2.500000E+01	1.600000E+01	1.400000E+01	2.500000E+01	2.600000E+01	2.600000E+01	2.200000E+01	1.600000E+01
N_3	2.000000E+01	1.700000E+01	2.600000E+01	2.500000E+01	2.300000E+01	2.900000E+01	1.400000E+01	2.100000E+01	2.100000E+01	2.500000E+01	2.500000E+01	1.900000E+01	1.400000E+01
N_4	2.400000E+01	1.700000E+01	2.600000E+01	2.400000E+01	2.400000E+01	3.000000E+01	1.700000E+01	2.400000E+01	2.400000E+01	2.400000E+01	2.400000E+01	2.400000E+01	2.600000E+01
N_5	1.900000E+01	1.400000E+01	1.800000E+01	1.800000E+01	2.600000E+01	3.500000E+01	1.800000E+01	1.400000E+01	2.300000E+01	1.600000E+01	1.800000E+01	2.700000E+01	3.200000E+01
N_6	8.700000E+01	6.200000E+01	9.400000E+01	8.700000E+01	8.700000E+01	1.080000E+02	6.200000E+01	6.200000E+01	8.700000E+01	8.700000E+01	8.700000E+01	8.700000E+01	9.500000E+01
p	3.000000E+00	3.000000E+00	4.000000E+00	3.000000E+00	3.000000E+00	3.000000E+00	3.000000E+00	3.000000E+00	3.000000E+00	3.000000E+00	3.000000E+00	3.000000E+00	3.000000E+00
m_1	2.250000E+00	2.250000E+00	2.000000E+00	2.000000E+00	2.250000E+00	2.250000E+00	2.250000E+00	2.750000E+00	2.000000E+00	2.000000E+00	2.000000E+00	2.250000E+00	2.750000E+00
m_2	2.000000E+00	2.250000E+00	1.750000E+00	2.000000E+00	2.250000E+00	1.750000E+00	2.000000E+00	2.250000E+00	2.000000E+00	2.000000E+00	2.000000E+00	2.000000E+00	1.750000E+00
δ_1	-4.100000E+01	-7.487500E+01	-5.112500E+01	-4.100000E+01	-1.862500E+01	-2.662500E+01	-9.100000E+01	-7.487500E+01	-4.100000E+01	-4.100000E+01	-4.100000E+01	-4.100000E+01	-4.937500E-01
δ_2	-3.325000E+01	-6.925000E+01	-5.000000E+01	-4.200000E+01	-1.300000E+01	-2.200000E+01	-8.275000E+01	-7.150000E+01	-3.600000E+01	-4.200000E+01	-4.200000E+01	-2.875000E+01	-3.025000E+01
δ_3	-9.200000E+01	-1.142500E+02	-1.080000E+02	-9.600000E+01	-4.450000E+01	-4.150000E+01	-1.100000E+02	-1.142500E+02	-7.600000E+01	-1.040000E+02	-9.600000E+01	-6.000000E+01	-5.900000E+01
δ_4	-3.000000E+00	-2.250000E+00	-2.750000E+00	-2.000000E+00	-4.500000E+00	-3.000000E+00	-3.500000E+00	-1.500000E+00	-2.000000E+00	-2.000000E+00	-2.000000E+00	-3.000000E+00	-3.000000E+00
δ_5	-2.659550E+01	-1.833717E+01	-1.521930E+01	-2.432755E+01	-2.692563E+01	-2.532755E+01	-1.873909E+01	-1.640897E+01	-2.879165E+01	-2.432755E+01	-2.432755E+01	-2.832755E+01	-2.566730E+01
δ_6	-3.552370E+01	-1.947114E+01	-1.958326E+01	-2.619358E+01	-2.992563E+01	-3.691601E+01	-2.506922E+01	-2.506922E+01	-3.365768E+01	-2.619358E+01	-2.619358E+01	-3.738973E+01	-5.364806E+01
δ_7	-1.573909E+01	-1.034679E+01	-1.061270E+01	-1.587307E+01	-1.480127E+01	-1.879165E+01	-9.810889E+00	-1.034679E+01	-1.520319E+01	-1.614102E+01	-1.587307E+01	-1.466730E+01	-1.572947E+01
δ_8	-4.286027E+03	-1.214141E+03	-5.994767E+02	-2.295570E+03	-1.855383E+03	-2.252218E+03	-1.651938E+03	-2.151905E+03	-3.140033E+03	-2.634845E+03	-2.295570E+03	-3.364657E+03	-6.951960E+03
δ_9	-1.800000E+01	-6.000000E+00	-1.100000E+01	-8.000000E+00	-1.200000E+01	-1.500000E+01	-1.200000E+01	-1.200000E+01	-1.600000E+01	-8.000000E+00	-8.000000E+00	-2.000000E+01	-3.600000E+01
δ_{10}	-2.000000E+01	-1.200000E+01	-2.700000E+01	-2.200000E+01	-6.000000E+00	-3.000000E+00	-4.000000E+00	-1.200000E+01	-1.200000E+01	-2.600000E+01	-2.200000E+01	-4.000000E+00	0.000000E+00
h	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
f_{min}	5.262806E-01	5.435512E-01	5.297479E-01	5.257687E-01	5.268955E-01	5.393103E-01	5.370588E-01	5.370588E-01	5.273469E-01	5.257687E-01	5.257687E-01	5.270040E-01	7.056965E-01

Table 18
Computational complexity for EVLS-PSOIMLSM.

Dimension	T_0	T_1	$\hat{T}_2$	$(\hat{T}_2 - T_1)/T_0$
10D	0.0648	0.3087	1.6607	20.8560
30D	0.0648	0.4249	1.9536	23.5826
50D	0.0648	0.6036	3.7683	48.8207
100D	0.0648	1.3957	8.3152	128.2736

Table 19
Computational complexities of EVLS-PSOIMLSM, IMPSOwithCMAR, SQPPSO-sono, EAPSO, HMPPSO, SpadePSO, ADPSO, PSO-sono, MFCPSO, AMSEPSO, EOPSO, PPSO, SDPSO, AHPSO, and MPSO on 30 dimensions.

Algorithm	T_0 (s)	T_1 (s)	$\hat{T}_2$ (s)	$(\hat{T}_2 - T_1)/T_0$	Rank
EVLS-PSOIMLSM	0.0648	0.4249	1.9536	23.5825	10
IMPSOwithCMAR	0.0648	0.4249	3.1146	41.4918	12
SQPPSO-sono	0.0648	0.4249	0.5985	2.6785	1
EAPSO	0.0648	0.4249	4.0824	56.4217	14
HMPPSO	0.0648	0.4249	0.8751	6.9452	4
SpadePSO	0.0648	0.4249	3.3359	44.9066	13
ADPSO	0.0648	0.4249	2.6405	34.1787	11
PSO-sono	0.0648	0.4249	0.8625	6.7504	3
MFCPSO	0.0648	0.4249	6.3797	91.8612	15
AMSEPSO	0.0648	0.4249	1.5647	17.5828	8
EOPSO	0.0648	0.4249	1.3624	14.4629	7
PPSO	0.0648	0.4249	0.8947	7.2472	5
SDPSO	0.0648	0.4249	1.6645	19.1223	9
AHPSO	0.0648	0.4249	1.0366	9.4364	6
MPSO	0.0648	0.4249	0.7662	5.2651	2

5.2. Planetary gear train design problem

The PGTD problem focuses on determining the optimal number of gear teeth in an automatic planetary transmission system for automobiles, aiming to minimize the maximum error in the gear ratio (Zhang et al., 2018). This requires minimizing the objective function $f(x)$, as defined in Eq. (37). The problem involves six design variables, all representing the number of teeth and constrained to integer values. Of these, three are discrete design variables. Additionally, the problem is subject to 11 constraints, including one equality constraint (see Tables 11 and 12).

$$f(x) = \max |i_k - i_{0k}|, k = \{1, 2, \dots, R\}, \quad (37)$$

where $x = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9) = (N_1, N_2, N_3, N_4, N_5, N_6, m_1, m_2, p)$, and $i_1 = \frac{N_6}{N_4}$, $i_{01} = 3.11$, $i_2 = \frac{N_6(N_1N_3+N_2N_4)}{N_1N_3(N_6+N_4)}$, $i_{0R} = -3.11$, $I_R = -\frac{N_2N_6}{N_1N_3}$, $i_{02} = 1.84$.

Subject to:

$$\begin{aligned} g_1(x) &= m_2(N_6 + 2.5) - D_{max} \leq 0, \\ g_2(x) &= m_1(N_1 + N_2) + m_1(N_2 + 2) - D_{max} \leq 0, \\ g_3(x) &= m_2(N_4 + N_5) + m_2(N_5 + 2) - D_{max} \leq 0, \\ g_4(x) &= |m_1(N_1 + N_2) - m_2(N_6 + N_3)| - m_1 - m_2 \leq 0, \\ g_5(x) &= -(N_1 + N_2) \sin(\pi/p) + N_2 + 2 + \delta_{22} \leq 0, \\ g_6(x) &= -(N_6 - N_3) \sin(\pi/p) + N_3 + 2 + \delta_{33} \leq 0, \\ g_7(x) &= -(N_4 + N_5) \sin(\pi/p) + N_5 + 2 + \delta_{55} \leq 0, \\ g_8(x) &= (N_3 + N_5 + 2 + \delta_{35})^2 - (N_6 - N_3)^2 - (N_4 + N_5)^2 \\ &\quad + 2(N_6 - N_3)(N_4 + N_5) \cos\left(\frac{2\pi}{p} - \beta\right) \leq 0, \\ g_9(x) &= N_4 - N_6 + 2N_5 + 2\delta_{56} + 4 \leq 0, \\ g_{10}(x) &= 2N_3 - N_6 + N_4 + 2\delta_{34} + 4 \leq 0, \\ h_1(x) &= \frac{N_6 - N_4}{p} = \text{integer}. \end{aligned}$$

where $\delta_{22} = \delta_{33} = \delta_{55} = \delta_{35} = \delta_{56} = 0.5$, $\beta = \frac{\cos^{-1}((N_4+N_5)^2 + (N_6-N_3)^2 - (N_3+N_5)^2)}{2(N_6-N_3)(N_4+N_5)}$, $D_{max} = 220$ and with bounds:

$$p = (3, 4, 5), m_1, m_2 = (1.75, 2.0, 2.25, 2.5, 2.75, 3.0), 17 \leq N_1 \leq 96,$$

$$14 \leq N_2 \leq 54, 14 \leq N_3 \leq 51, 17 \leq N_4 \leq 46, 14 \leq N_5 \leq 51, 48 \leq N_6 \leq 124.$$

5.3. Computational complexity of algorithms

The computational complexity of the EVLS-PSOIMLSM algorithm was assessed based on the criteria outlined in the CEC2017 benchmark competition. All experiments were conducted using the following system configuration: • CPU-Intel(R) Core(TM) i7-12700H @ 2.30 GHz; • RAM-16 GB; • OS-Windows 10; • Software-Matlab 2020b. The computational complexity of EVLS-PSOIMLSM for various dimensions is summarized in Table 18. In this table, T_0 represents the time required to execute the following specified statements:

$x = 0.55$;

for $i = 1 : 1000000$

$x = x + x$; $x = x/2$; $x = x * x$; $x = \text{sqrt}(x)$; $x = \log(x)$;

$x = \exp(x)$; $x = x/(x + 2)$;

end

Here, T_1 denotes the time required to execute the benchmark function f_{18} independently (without utilizing EVLS-PSOIMLSM) with 2×10^5 evaluations in D dimensions. The T_2 term represents the runtime of the EVLS-PSOIMLSM algorithm on function f_{18} with 2×10^5 evaluations in D dimensions. The time ($\hat{T}_2$) corresponds to the averaged mean values of T_2 obtained from five runs. The units of T_0 , T_1 , and $\hat{T}_2$ are seconds (s). As illustrated in Table 18, T_1 , $\hat{T}_2$, and $\left(\frac{\hat{T}_2 - T_1}{T_0}\right)$ exhibit linear scaling with respect to the number of dimensions.

To ensure a fair comparison of computational complexity, Table 19 presents the computational complexity comparisons between the EVLS-PSOIMLSM algorithm and various PSO-based variants, all of which are evaluated in 30 dimensions. The same system and procedure were consistently applied to calculate the computational complexity for each algorithm under comparison. In Table 19, EVLS-PSOIMLSM ranks

10th, outperforming ADPSO, IMPSOwithCMAR, SpadePSO, EAPSO, and MFCSO. Given the significant improvement in performance and intelligent evolution, the resource expenditure is considered acceptable.

6. Conclusions

This study introduces an ensemble velocity learning strategy with multiple local search mechanisms for particle swarm optimization. The proposed ensemble velocity learning strategy consists of Types 1 to 3. Type 1 has strong exploration capabilities, whereas Type 2 excels in exploitation, and Type 3 lies in the middle. The ensemble velocity learning strategy can select effective combinations based on the landscape of the problems, demonstrating outstanding intelligent evolution. To further enhance the exploitation of the proposed algorithm, the ISCA and SQP methods are integrated, with both methods intervening after sufficient exploration. The mutation strategy helps maintain the particles' diversity to a certain extent, thereby aiding particles in escaping local optima. The ablation tests demonstrate the effectiveness of individual mechanisms, whereas the Friedman ranks statistic from the ablation tests illustrates the degree of contribution of EVLS-PSOIMLSM. The search behavior of EVLS-PSOIMLSM shows flexibility and adaptability in that it chooses proper search patterns when evolving different problems. Experimental results on CEC2017 benchmark functions demonstrate the superior performance of EVLS-PSOIMLSM compared to 14 recently proposed PSO-based variants. It is also evaluated against 11 non-PSO algorithms, showing impressive performance but generally being outperformed by DE-based variants, highlighting areas for improvement. Continuous efforts are needed to reach the top tier of metaheuristics. Additionally, EVLS-PSOIMLSM is applied to real-world GTD and PGTD problems, where it performs impressively compared to both PSO-based and non-PSO-based algorithms. The gap between the PSO-based and DE-based variants has shrunk, and the progress is evident. In the future, an ensemble acceleration coefficient strategy can be investigated, and other mechanisms can be integrated to form more diverse search patterns.

CRedit authorship contribution statement

Libin Hong: Writing – review & editing, Writing – original draft, Validation, Supervision, Software, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization. **Zhantao Gu:** Writing – review & editing, Validation, Software, Investigation, Conceptualization. **Tianxiang Cui:** Writing – review & editing.

Declaration of competing interest

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Data availability

Data will be made available on request.

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